IS CP INVARIANCE VIOLATED?

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The well-known fact that the existing quantum mechanical formalism is inconsistent with the energy-time uncertainty-relation postulate is shown to play a crucial role in the logical deduction leading to the conclusion that CP invariance is violated in some neutral kaon decays. It is therefore argued that if one believes in the existence of the energy-time uncertainty relation, then one has to accept that none of the existing experimental facts can unambiguously be interpreted as evidence against the universal validity of CP invariance.

The well-established fact that both the short-lived and the long-lived components of neutral kaons can decay into two pions,[1] and the results of the recent measurements[2] which indicate the existence of the charge asymmetry in the decays $K_L \to \pi^\pm l^\mp \nu_l$, are widely accepted as evidence against the universal validity of CP invariance. In particular, the existence of the charge asymmetry in the decays $K_L \to \pi^\pm l^\pm \nu_l$ is claimed to be a direct proof[3] of CP noninvariance. In this paper we shall show that the latter is not true, and that the conclusion about the violation of CP invariance in the above-stated neutral kaon decays can be arrived at only by a logical deduction based on the existing quantum mechanical formalism (QMF). The well-known fact[4] that this formalism, as well as the underlying quantum mechanical theory of measurement (QMTM), are inconsistent with the energy-time uncertainty-relation (ETUR)[5] postulate will be shown to play a crucial role in this deduction. Consequently, we shall arrive at the following conclusion: If one believes in the existence of the ETUR, then one has to be very skeptical about the validity of the conclusion that CP invariance is violated in some neutral kaon decays. Some experiments which might clarify the situation will be discussed.

Let us introduce the energy eigenstates $|K^0, E\rangle$ and $|\bar{K}^0, E\rangle$, with the internal quantum numbers identical to those of the neutral kaon $|K^0\rangle$ and antikaon $|\bar{K}^0\rangle$ states, respectively. Then one can write

$$|K^0\rangle = \int_0^\infty \rho(E)|K^0, E\rangle dE, \tag{1}$$

$$|\bar{K}^0\rangle = \int_0^\infty \rho(E)|\bar{K}^0, E\rangle dE. \tag{2}$$

Let us further introduce the states $|K_L\rangle$ and $|K_L'\rangle$, which have the same decay law and the same energy distribution, identical, respectively, to the decay law and the energy distribution of the long-lived component of neutral kaons,

$$|K_L\rangle = \int_0^\infty \rho_L(E)|K_L, E\rangle dE, \tag{3}$$

$$|K_L'\rangle = \int_0^\infty \rho_L'(E)|K_L, E\rangle dE, \tag{4}$$

where the states $|K_L, E\rangle$ are given as

$$|K_L, E\rangle = a_L(E)|K^0, E\rangle + \bar{a}_L(E)|\bar{K}^0, E\rangle. \tag{5}$$

The fact that the two states have the same decay law and the same energy distribution implies that the following condition must be fulfilled:

$$|\rho_L(E)|^2 = |\rho_L(E)|^2. \tag{6}$$

In a similar way we introduce the states $|K_S\rangle$ and $|K_S'\rangle$, which have the same decay law and the same energy distribution, identical, respectively, to the decay law and the energy distribution of the short-lived component of neutral kaons:

$$|K_S\rangle = \int_0^\infty \rho_S(E)|K_S, E\rangle dE, \tag{7}$$

$$|K_S'\rangle = \int_0^\infty \rho_S'(E)|K_S, E\rangle dE, \tag{8}$$

where the states $|K_S, E\rangle$ are given as

$$|K_S, E\rangle = a_S(E)|K^0, E\rangle + \bar{a}_S(E)|\bar{K}^0, E\rangle. \tag{9}$$

As before, the following condition must be fulfilled:

$$|\rho_S(E)|^2 = |\rho_S'(E)|^2. \tag{10}$$

The expansion matrix of the equations (5) and (9) is unitary.

The existing experimental facts concerning neutral kaons indicate that each of the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ is a superposition of two other states, which have pure exponential decay laws, and that the states which enter the superposition representing the state $|K^0\rangle$ have the same decay laws and, probably, the same energy distributions as the corresponding states which enter the superposition representing the state $|\bar{K}^0\rangle$. Therefore, one may assume that, in the general case, the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are given by

$$|K^0\rangle = \alpha|K_L\rangle + \beta|K_S\rangle, \tag{11}$$

$$|\bar{K}^0\rangle = \gamma|K_L'\rangle + \delta|K_S'\rangle. \tag{12}$$

We wish to emphasize that the existing experimental evidence is insufficient to decide whether or not the states $|K_L\rangle$ and $|K_S\rangle$ are identical to the states $|K_L'\rangle$ and $|K_S'\rangle$, respectively. The possibility that they might not be identical plays an important role in our subsequent discussion.

Let us now discuss implications (with respect to CP invariance) of the fact that both the long-lived and the short-lived components of neutral kaons can decay into two pions. The widely accepted conclusion that this fact is evidence against the universal validity of CP invariance can be reached only by a logical deduction based on the existing QMF. The only convincing way of arriving at this conclusion is to demonstrate that this experimental fact implies that the time distributions of the decay rates $\Gamma(K^0\rightarrow 2\pi)$ and $\Gamma(\bar{K}^0\rightarrow 2\pi)$ are different. If one relies on the existing QMF, then such demonstration is a straightforward procedure.[6] Now, the important question is whether the failure of the formalism to take ETUR into account is relevant to this kind of demonstration. Here we shall show that in the demonstration a crucial role is played by that part of the formalism which explicitly denies the existence of the ETUR.

To show this let us write down the expression for the difference D of the time distributions of the decay rates $\Gamma(K^0\rightarrow 2\pi)$ and $\Gamma(\bar{K}^0\rightarrow 2\pi)$, taking explicitly into account that the states $|K^0\rangle$ and $|\bar{K}^0\rangle$, as well as $|K_L\rangle$ and $|K_S\rangle$, being unstable, have no well-defined values of energy, and assuming CPT invariance.[7] The expression is

$$D \equiv \Gamma(K^0\rightarrow 2\pi) - \Gamma(\bar{K}^0\rightarrow 2\pi) = \int_0^\infty\int_0^\infty G(t',E')\theta_1(t',t;\Delta t)\theta_2(E',E;\Delta E)dt'dE', \tag{13}$$

where θ_1 and θ_2 are, respectively, the time and the energy resolution functions of the given experimental arrangements. The function $G(t,E)$ is defined as

$$G(t,E) = \Omega(E)\{(|a|^2+|b|^2)2|A_S(E)||A_L(E)|\cos[\Delta m t+\eta(E)]$$
$$\times\exp[-\tfrac{1}{2}(\lambda_S+\lambda_L)t] + (|a|^2-|b|^2)[\exp(-\lambda_S t)|A_S(E)|^2 + \exp(-\lambda_L t)|A_L(E)|^2]\}. \tag{14}$$

Here we have used the following notation:

$$A_{S,L}(E) = \langle 2\pi,E|T|K_{S,L}\rangle = \rho_{S,L}(E)W_{S,L}(E);$$

$$W_{S,L}(E) = \langle 2\pi,E|T|K_{S,L};E\rangle;$$

$$\eta(E) = \arg[A_S(E)A_L^*(E)]; \quad \Delta m = m_L - m_S,$$

where T is the transition operator, Ω is the phase-space factor, and m_S, λ_S and m_L, λ_L are the mass and the half-width of the states, $|K_S\rangle$ and $|K_L\rangle$, respectively.

The important thing to be emphasized in connection with the expression (13) is that, according to the existing QMTM, we must assume that there is no correlation between the resolution functions θ_1 and θ_2, i.e., that there is no correlation between the uncertainties Δt and ΔE with which time and energy are simultaneously measured. As we shall soon show, this assumption is crucial for the conclusion that CP invariance is violated in the two-pion decays of neutral kaons.

Indeed, in the case of CP invariance D should be zero for all times and energies and under any experimental condition. But, according to the above stated assumption, this means that D should be identically zero for any θ_1 and θ_2, chosen independently. This can be fulfilled only if G is identically zero. However, the experimental fact[1] that none of the amplitudes A_S and A_L is identically zero implies that G is not identically zero. Consequently, one concludes that CP invariance is violated.

Evidently, a crucial point in this logical deduction is the assumption that θ_1 and θ_2 are uncorrelated, for otherwise from the fact that G is not identically zero one could not draw the conclusion that D is not identically zero.

Let us now discuss the charge asymmetry in the decays $K_L\rightarrow\pi^\pm l^\mp\nu_l$. Here we shall make full use of the fact that the existing experimental evidence does not imply $|K_L\rangle = |K_L'\rangle$.

We define the charge asymmetry parameters

Z and $\overline{Z}$ in the following way:

$$Z = \frac{[\Gamma(K_L \to \pi^+ l^- \nu_l) - \Gamma(K_L \to \pi^- l^+ \overline{\nu}_l)]}{[\Gamma(K_L \to \pi^+ l^- \nu_l) + \Gamma(K_L \to \pi^- l^+ \overline{\nu}_l)]},$$

$$\overline{Z} = \frac{[\Gamma(K_L{}' \to \pi^- l^+ \overline{\nu}_l) - \Gamma(K_L{}' \to \pi^+ l^- \overline{\nu}_l)]}{[\Gamma(K_L{}' \to \pi^- l^+ \overline{\nu}_l) + \Gamma(K_L{}' \to \pi^+ l^- \nu_l)]},$$

where Γ's are decay rates and where l stands for the muon or the electron. It is evident, on the basis of (11) and (12), that Z and $\overline{Z}$ are CP conjugated quantities, and only if one finds experimentally that $Z \neq \overline{Z}$ can one claim to have a direct proof of CP noninvariance. Of course, if one assumes, as one usually does, that $|K_L\rangle = |K_L{}'\rangle$, i.e., that $Z = -\overline{Z}$, then in order to have a proof of CP noninvariance it is enough to find experimentally that one of the above defined charge asymmetries (say Z) is different from zero. In two experiments of this kind[2] it is indeed found that $Z \neq 0$, and it was concluded that CP invariance is violated (the equality $|K_L\rangle = |K_L{}'\rangle$ being tacitly assumed). But this kind of proof is not direct, viz., it is based on the assumption $|K_L\rangle = |K_L{}'\rangle$, which might or might not be true. In fact, with the possibility $|K_L\rangle \neq |K_L{}'\rangle$, the nonzero value of Z is not a proof of CP noninvariance at all, unless one demonstrates that $Z = -\overline{Z}$ even if $|K_L\rangle \neq |K_L{}'\rangle$. If one relies on the existing QMF then such a demonstration is a straightforward procedure, if only the condition (6) is fulfilled. But, as we shall soon show, in this demonstration also a crucial role is played by that part of the existing QMF which explicitly denies the existence of the ETUR.

To show this let us write down expressions for Z and $\overline{Z}$, based on the existing QMF. They are[8]

$$Z = \frac{\int_0^\infty \int_0^\infty [\,|A^+(t',E')|^2 - |A^-(t',E')|^2]\theta_1(t',t;\Delta t)\theta_2(E',E;\Delta E)dt'dE'}{\int_0^\infty \int_0^\infty [\,|A^+(t',E')|^2 + |A^-(t',Et)|^2]\theta_1(t',t;\Delta t)\theta_2(E',E;\Delta E)dt'dE'}, \tag{15}$$

$$\overline{Z} = \frac{\int_0^\infty \int_0^\infty [\,|\overline{A}^+(t',E')|^2 - |\overline{A}^-(t',E)|^2]\theta_1(t',t;\Delta t)\theta_2(E',E;\Delta E)dt'dE'}{\int_0^\infty \int_0^\infty [\,|\overline{A}^+(t',E')|^2 + |\overline{A}^-(t',E)|^2]\theta_1(t',t;\Delta t)\theta_2(E',E;\Delta E)dt'dE'}, \tag{16}$$

where we have used the notation

$$A^\pm(t,E) = \rho_L(E) b_L{}^\pm(E) \exp iM_L t); \quad \overline{A}^\pm = \rho_L{}^1(E) b_L{}^\mp \exp iM_L t; \tag{17}$$

$$b_L{}^\pm(E) = \langle \pi^\pm l^\mp \nu_l | T | K_L, E\rangle; \quad M_L = m_L + i\tfrac{1}{2}\lambda_L.$$

From (6), (15), (16), and (17) it is evident that $Z = -\overline{Z}$ in spite of the fact that $|K_L\rangle$ must not necessarily be identical to $|K_L{}'\rangle$. Clearly, this is a consequence of the condition (6) and the fact that only the absolute squares $|\rho_L(E)|^2$ and $|\rho_L{}'(E)|^2$, and not $\rho_L(E)$ and $\rho_L{}'(E)$ themselves, enter expressions (15) and (16), which is an essential property of the existing QMF. Indeed, the appearance of only the absolute squares $|\rho_L(E)|^2$ and $|\rho_L{}'(E)|^2$ in expressions (15) and (16) is a necessary consequence of the assumption that energy can be measured accurately in an infinitely short time, and this assumption is crucial for the consistency of the existing QMF (and of the underlying QMTM).

Thus, the fact that the existing QMF (and the underlying QMTM) is inconsistent with the ETUR postulate is shown to play a crucial role in the logical deduction leading to the conclusion that CP invariance is violated in some neutral kaon decays. It may well be that with a new QMTM (and the corresponding new formalism), consistent with the ETUR postulate, one would come to a different conclusion. Of course, the question of whether CP invariance is violated or not can be answered on an experimental basis. For instance, a sufficiently precise measurement of the time distributions of the decay rates $\Gamma(K^0 \to 2\pi)$ and $\Gamma(\overline{K}^0 \to 2\pi)$, or a measurement of both charge asymmetries, Z and $\overline{Z}$, would provide a direct answer to the above stated question. The equality of the two time distributions, and/or the equality $Z = \overline{Z}$, would be not only the most exciting outcome of these measurements but also the first experimental indication that the ETUR postulate has to be included in the set of quantal postulates and, consequently, that the existing QMTM has to be modified. Unfortunately, at the moment such experiments seem not to be feasible. However, an experiment which is a modification of one of the above mentioned experiments and which might be feasible would also be of interest. Namely, it would be desirable to perform two or more measurements of the charge asymmetry in $K_L \to \pi^\pm l^\mp \nu_l$ decays varying in the initial beam the ratio of the number of K^0 mesons to the number of $\bar{K}^0$ mesons. Any discrepancy between the results of these measurements would be an idication that $|K_L\rangle \neq |K_L'\rangle$ and that the ETUR exists. A similar situation can arise when one tries to determine the imaginary part of the ratio of the amplitudes for $\Delta Q = -\Delta S$ and $\Delta Q = \Delta S$ three-body semileptonic decays of neutral kaons. It has never been suspected that the measured value of this imaginary part could depend on the ratio of the number of K^0 mesons to the number of $\bar{K}^0$ mesons in the initial beam. However, from our point of view such dependence can arise and, as before, would be an indication that $|K_L\rangle \neq |K_L'\rangle$ and that the ETUR exists.

In conclusion let us remark that a more detailed discussion of the problems touched in this Letter will be the subject of forthcoming papers.

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[1]For a summary of the present experimental situation, see, for example, J. W. Cronin, in Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, New York, 1967, edited by C. R. Hagen et al. (Interscience Publishers, Inc., New York, 1968).

[2]S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters 19, 993 (1967); D. Dorfan et al., ibid. 19, 987 (1967).

[3]By the direct proof of CP noninvariance one means a direct experimental demonstration that two CP-conjugated physical quantities are not equal.

[4]For a detailed list of references concerning the energy-time uncertainty relation, see, for example, M. I. Shirokov, Dubna Preprint No. E-2478, 1965 (unpublished). See also J. von Neumann, Mathematical Foundations of Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1955).

[5]Here we speak about the energy-time uncertainty relation in the Bohr-Heinsenberg sense, i.e., in the sense of an uncertainty relation which holds for the measurement process. As for the relation between the lifetime and the level width, which also has the form of the uncertainty relation $\Delta t \Delta E \geq \hbar$, it is a straightforward consequence of the time evolution (according to the Schrödinger equation) of the system without a well-defined value of the energy, and it has nothing to do with the energy-time uncertainty relation in the Bohr-Heisenberg sense.

[6]J. J. Sakurai and A. Wattenberg, Phys. Rev. 161, 1449 (1967).

[7]For simplicity of discussion we shall assume $|K_L\rangle = |K_L'\rangle$ and $|K_S\rangle = |K_S'\rangle$. The main results of this part of our discussion are essentially independent of this assumption. However, when discussing the charge asymmetry in $K_L \to \pi^\pm l^\mp \nu_l$ decays we shall make full use of the fact that the existing experimental evidence does not imply $|K_L\rangle = |K_L'\rangle, |K_S\rangle = |K_S'\rangle$. Note that within the realm of the existing QMF one can show that CP and/or CPT invariance imply $|K_L\rangle = |K_L'\rangle, |K_S\rangle = |K_S'\rangle$. However, this conclusion can be arrived at only by an analysis in which, too, a crucial role is played by that part of the existing QMF which explicitly denies the existence of the ETUR. Consequently, the consistency of our approach to the problem of CP noninvariance requires leaving open the question of whether or not CP and/or CPT invariance imply $|K_L\rangle = |K_L'\rangle, |K_S\rangle = |K_S'\rangle$.

[8]For simplicity of discussion we assume the validity of the $\Delta Q = \Delta S$ selection rule.