be published).
[4]K. W. Ford and J. A. Wheeler, Ann. Phys. (N.Y.) 7, 159 (1959).
[5]R. J. Munn, E. A. Mason, and F. J. Smith, J. Chem. Phys. 41, 3978 (1964).
[6]E. A. Mason, R. J. Munn, and F. J. Smith, J. Chem. Phys. 44, 1967 (1966).
[7]N. F. Mott and H. S. W. Massey, Theory of Atomic Collisions (Oxford University Press, New York, 1965), 3rd ed., pp. 65-66.
[8]B. Block and F. B. Malik, Phys. Rev. Letters 19, 239 (1967).
[9]D. D. Kerlee, J. S. Blair, and G. W. Farwell, Phys. Rev. 107, 1343 (1957).

PION SCATTERING AND THE NEUTRON HALO IN LEAD

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Reanalysis of the 700-MeV $\pi^{\pm}$-Pb inelastic scattering experiment of Abashian, Cool, and Cronin leads to a contradiction with recent suggestions of a neutron-rich surface or "halo" unless one admits neutron distributions with sharp edges and with peaks near the surface.

Does the lead nucleus have a neutron-rich surface region or "halo"? Recent analyses of low-energy proton scattering[1,2] and of isobaric analog state data[3] suggest that for Pb^{208} the rms neutron radius r_n considerably exceeds the proton radius r_p. This evidence directly contradicts the apparently unambiguous result of the earlier $\pi^{\pm}$-Pb inelastic-scattering experiment done by Abashian, Cool, and Cronin.[4] We have undertaken a detailed optical-model analysis of this experiment and confirm the original conclusion $r_n \lesssim r_p$ if we exclude neutron distributions with sharp edges and with large peaks near the surface.

Piccioni[5] observed that at 700 MeV the ratio of total cross sections $\sigma(\pi^+\text{-}n)/\sigma(\pi^+\text{-}p) = \sigma(\pi^-\text{-}p)/\sigma(\pi^-\text{-}n) = 2.6/1$. Thus both π^+ and π^- are strongly absorbed in the interior of a lead nucleus, while in the surface region the π^+ are mainly absorbed by neutrons and the π^- by protons. The quantity

$$q \equiv [\sigma(\pi^-\text{-Pb})/\sigma(\pi^+\text{-Pb})]-1, \tag{1}$$

where the σ's are absorption cross sections, is sensitive consequently to the properties of the surface region, and in particular, to the ratio of the "maximum" neutron and proton radii.

Abashian, Cool, and Cronin[4] assumed simple uniform distributions for both neutrons and protons and calculated q using a semiclassical approximation and a multiplicative Coulomb correction, as earlier discussed by Courant.[6] They obtained, for $r_p = 5.95$ (all radii are in F),

$$q = +0.044, \quad r_n = r_p; \tag{2}$$

$$q = -0.024, \quad r_n = 1.15 r_p. \tag{3}$$

Similar results were obtained with a larger value of r_p. Their experimental 700-MeV inelastic cross sections gave[4]

$$q_{\text{exp}} = +0.050 \pm 0.011 \tag{4}$$

implying that $r_n \lesssim r_p$.

Greenlees, Pyle, and Tang[2] recently performed an optical-model analysis of low-energy p-Pb^{208} scattering. From the radii of the real and spin-orbit potentials, they extracted an rms matter radius r_m. With the electron-scattering result $r_p = 5.50$, they found

$$r_n = [(A/N)r_m^2 - (Z/N)r_p^2]^{1/2} = (1.09 \pm 0.05) r_p. \tag{5}$$

Isobaric-analog-state data give[3] a smaller ratio, $r_n/r_p = 1.035$. Studies on other nuclei[1] such as the calcium isotopes suggest that in general $N/Z > 1$ implies $r_n/r_p > 1$.

Optical-model analysis.—We computed the exact numerical solution of the optical-model wave equation for $\pi^{\pm}$-Pb scattering with various densities, using a modified version of a code previ-

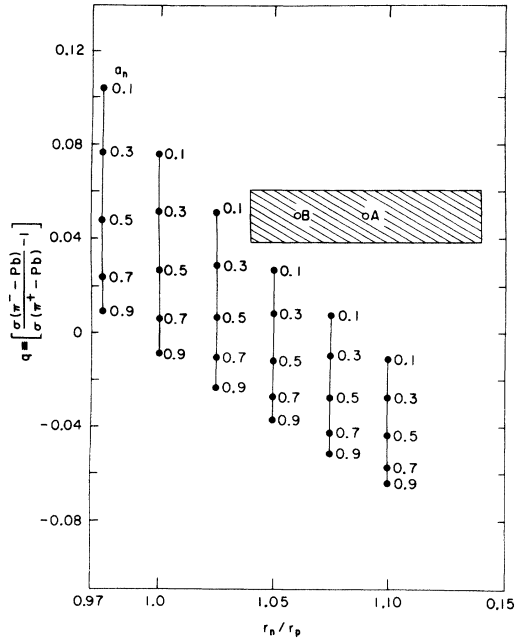

FIG. 1. Ratios of $\pi^{\pm}$-Pb absorption cross sections for Saxon densities. Throughout $R_p = 6.628$, $a_p = 0.5348$ ($r_p = 5.50$); R_n and a_n are varied together for fixed values of r_n/r_p. Numbers next to black circles give a_n. Open circles labeled A and B refer to the neutron densities shown in Fig. 2. The shaded box indicates the limits on q_{exp} and on r_n/r_p from low-energy proton scattering.

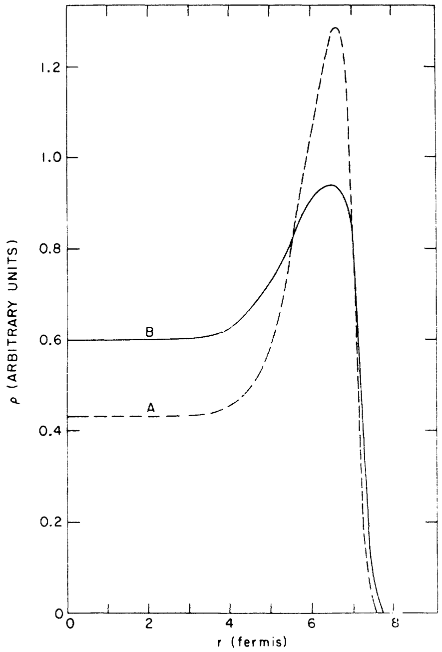

FIG. 2. Two densities fitting the $\pi^{\pm}$-Pb ratio with $r_n/r_p = 1.09$ for density A and $r_n/r_p = 1.06$ for density B.

ously applied to low-energy pion-nucleus scattering.[7] The optical potentials $V^{\pm}$ are related to π-p and π-n phase shifts[8] by[7,9]

$$V^{\pm}(\vec{r}) = Zb_p^{\pm}\rho_p(\vec{r}) + Nb_n^{\pm}\rho_n(\vec{r}). \tag{6}$$

Here

$$b_i^{\pm} = 2\pi f_i^{\pm}(0)[(\mu^2 + M^2 + 2E_\pi M)/M^2](k/p),$$
$$i = p, n; \tag{7}$$

$f(0)$ is the forward-scattering amplitude in the pion-nucleon c.m. frame, μ and M are the pion and nucleon masses, E_π is the total laboratory pion energy, k/p is the ratio of momentum in the pion-nucleon center of mass to that in the laboratory, and $\rho_n(\rho_p)$ is the neutron (proton) density. $V^{\pm}$ plus the Coulomb potential was inserted into a Klein-Gordon equation as the time component of a four-vector, and the $(V^{\pm})^2$ term was dropped.

Three types of neutron densities were employed: (I), Fermi, $\{1+\exp[(r-R)/a]\}^{-1}$; (II), Fermi plus surface Gaussian, $\exp[-(r-R')^2/c^2]$; (III), Fermi plus Fermi times surface Gaussian. In most of our calculation the best electron-scattering proton distribution[10] was used, a Fermi distribution with $R_p = 6.628$ and $a_p = 0.5348$ corresponding to $r_p = 5.50$.

For a Fermi neutron distribution we found that with a fixed r_n a considerable range of q values can be obtained by varying the parameters R_n and a_n. For example, with $r_n = 1.05r_p$, q changes from about +0.03 to −0.04 as a_n is increased from 0.1 to 0.9. Thus the π^+ cross section grows as ρ_n becomes more diffuse and achieves a larger "maximum radius." If we set $a_n = a_p = 0.5348$, then we find $r_n = (0.975 \pm 0.010)r_p$, in agreement with the uniform distribution result $r_n \lesssim r_p$. Keeping $a_p = 0.5348$ and decreasing a_n, slightly larger r_n values lead to q's still within the error limits. Extrapolating to the uniform distribution limit $a_n = 0$ gives $r_n = (1.035 \pm 0.01)r_p$, still in disagreement with (5) but in agreement with the isobaric analog state data. (See Fig. 1.)

Neutron distributions of types (II) and (III) have larger r_n values than uniform distributions of the same maximum radius if the parameters are suitably chosen. They can reproduce q_{exp} for $r_n = 1.09r_p$; however, this is so only if they have very large, narrow peaks at the surface and sharp edges. Distributions with somewhat broader peaks and with (peak/center) ratios ~1.5/1 will also fit q_{exp} for $r_n = 1.06r_p$ (Fig. 2).

Thus if one allows only ρ_n's with no surface peaks and with moderately diffuse edges, $a_n \geq a_p \approx 0.5$, then one confirms the earlier conclusion $r_n/r_p \lesssim 1$, i.e., there is no neutron halo.

Corrections to simple optical model.—It is important to estimate the effect of corrections to the simple model used above in assessing the validity of the analysis.

One of the remarkable features of the $\pi^\pm$-Pb experiment is the insensitivity of its interpretation to fine details, e.g., to the precise optical parameters used. Doubling Reb_i changes cross sections by about 1% and q negligibly. To alter q by 0.01 or 0.02 one must change Imb_i by about 10 to 20%. Hence the $(V^\pm)^2$ term dropped in the Klein-Gordon equation, which is about 2% of the principal term, is completely negligible. Equation (6) is derived using a forward-scattering approximation for the π-nucleon amplitude, $(\vec{q}'|t|\vec{q}) \times \rho(\vec{q}'-\vec{q}) \approx (\vec{q}|t|\vec{q})\rho(\vec{q}'-\vec{q})$; correcting for this alters the effective radii by about 1% and is also unimportant.

More significant effects arise from nucleon Fermi motion[11] and correlations.[11,12] The former smooths out the energy dependence of the optical parameters and increases them at 700 MeV. Correlations also increase the b's. Together these changes are $\lesssim 20\%$ and tend to increase q slightly, according to our calculations based on rough estimates of these corrections. For the case of Fermi densities of similar shape, we find now

$$r_n/r_p = 0.980 \pm 0.015 \quad (a_n = a_p = 0.5348) \tag{8}$$

instead of 0.975 ± 0.010 as before.

For π-nucleus interactions at several hundred MeV, the optical model has been shown by Crozon et al.[11] to predict correctly absorption and diffraction cross sections.[11] These calculations used recent nuclear density and π-nucleon data; the latter were adjusted for Fermi motion and correlations. No adjustable parameters were introduced. By contrast, optical-model analysis of low-energy proton-nucleus scattering requires considerable parameter fitting. Thus its interpretation is much more ambiguous.

While it is conceivable that some subtle effect has not been accounted for by this optical-model analysis, we are led to the conclusion that the $\pi^\pm$ experiment of Abashian, Cool, and Cronin[4] provides strong evidence against a neutron halo in lead. Additional experiments of this type would yield valuable information concerning the nuclear surface region.

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[1]A. Bromley and J. Weneser, Comments Nucl. Particle Phys. 1, 174 (1967), give a brief review of the recent work relating to r_n/r_p determinations and list further references.

[2]G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. Letters 17, 33 (1966), and to be published.

[3]J. A. Nolan, J. P. Schiffer, and N. Williams, unpublished. A very recent version [Phys. Letters 27B, 1 (1968)], published after submittal of this paper, gives the later result $r_n/r_p = 1.012 + 0.005$, which is consistent with our result.

[4]A. Abashian, R. Cool, and J. W. Cronin, Phys. Rev. 104, 855 (1956).

[5]O. Piccioni, private communication quoted in Ref. 4 and in E. D. Courant, Phys. Rev. 94, 1081 (1954).

[6]Courant, Ref. 5.

[7]E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 162, 1683 (1967), and Phys. Rev. (to be published).

[8]L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965).

[9]K. M. Watson, Rev. Mod. Phys. 30, 565 (1958).

[10]B. C. Clark, R. Herman, and D. G. Ravenhall, to be published.

[11]M. Crozon et al., Nucl. Phys. 64, 567 (1965).

[12]M. A. B. Bég, Phys. Rev. 120, 1867 (1960).