SOLAR ROTATION AND NEUTRINO FLUX*

G. Shaviv and Edwin E. Salpeter
Physics Department, Laboratory of Nuclear Studies,
and Center for Radiophysics and Space Research, Cornell University, Ithaca, New York
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If the sun possesses a fast-rotating core, mixing in its interior may be a possibility. We calculate the solar neutrino flux as a function of fractional mass q_m in the mixed zone for "slow," "fast," and "very fast" mixing. Only for "slow" mixing and large q_m could the neutrino flux be decreased appreciably, but such large values of q_m are unlikely.

A recent experiment by Davis, Harmer, and Hoffman,[1] designed to detect solar neutrinos by their absorption in a ^{37}Cl target, has given an upper limit to the neutrino-capture counting rate at the earth of 0.3 (in units of $10^{-35}/^{37}$Cl atom sec). Theoretical values for the fluxes φ and absorption cross sections σ for the various types of solar neutrinos have been re-examined recently by Bahcall, Bahcall, and Shaviv,[2] and the most probable theoretical value for the counting rate $\sum\varphi\sigma$ is 0.76 (in the above units). Because of a very large cross section, this rate comes mainly from the ^{8}B neutrinos whose flux is very small and extremely[3] temperature sensitive ($\varphi_8 \propto T^{14}$). Because of this sensitivity the theoretical value can easily be in error by a factor of 2 or 3, and there is not necessarily any discrepancy with the experiment. Nevertheless, Ezer and Cameron[4] suggested the possibility of mixing in the solar interior due to fast differential rotation, which could lower the neutrino flux. In the present paper this effect is examined in more detail than in a preliminary calculation[5]; in particular we estimate the additional uncertainties in the solar neutrino flux due to uncertainties regarding mixing, especially regarding the mixing time.

The situation on solar rotation and mixing is confused at the moment: The sun has an outer convective envelope (containing only ~2% of the sun's mass) which rotates slowly (equatorial period ~26.9 days). Recent observations by Dicke and Goldenberg[6] on solar oblateness suggest, but do not prove, that the sun's interior is rotating very much more rapidly so that the ratio η of centrifugal to gravitational potential might be as large as 10^{-3} at some levels in the sun's interior. If the sun were chemically homogeneous, the "turnover" or mixing time t_m for meridional circulation would be $t_m \sim \eta^{-1} t_0$, where $t_0 \sim 10^6$ yr is $\frac{1}{10}$ of the Kelvin contraction time, so that $t_m < t_a \approx 4.7\times10^9$ yr, the present age of the sun. However, $^1\text{H} \rightarrow {}^4\text{He}$ conversion in the sun's deep interior gives positive variations $\Delta\mu$ in the mean molecular weight from the center outwards ($\Delta\mu \sim 0.1 \gg \eta$ for the present sun, if unmixed). After a short transient[7] in the young sun, meridional circulation is expected to be inhibited (when $\Delta\mu > \eta$), but other types[8] of circulation patterns in an inner zone with some fractional mass q_m and some mixing time t_m cannot yet be ruled out.

If either (a) powerful circulation has stopped differential rotation in the past or (b) circulation is inhibited, then there is no mixing and the composition and neutrino flux are essentially unchanged. To maximize the possible effect on the neutrino flux we consider here the least likely case, (c) rapid rotation and mixing with a constant ^{1}H abundance X_1 in some inner zone with fractional mass q_m (requiring $t_m \ll t_a$). Two further time scales are also important:

(1) The relaxation time t_3 for establishing equilibrium values of the ^{3}He abundance X_3 (an intermediate stage in the p-p reaction chain) is $t_3 \sim 10^6$ to 10^7 yr. For "slow" mixing, $t_m \gg t_3$, each point in the mixed zone has X_3 close to the equilibrium value for the local temperature T (T decreases and X_3 increases from the center outwards). For "fast" mixing, $t_m \ll t_3$, a constant value of X_3 is determined from the total production and destruction rates of ^{3}He over the whole[9] mixing zone.

(2) Most of the mixing zone is in radiative equilibrium with heat-transfer times of order $t_0 \sim 10^6$ yr (discussed above). The circulating material tends to set up the adiabatic temperature gradient if $t_m \ll t_0$; the radiative gradient is unaffected if $t_m \gg t_0$ and increased slightly if $t_m \sim t_0$.

Since the value of t_m is unknown, we have carried out three parallel sets of solar-model calculations for each of a number of values of q_m (fractional mass in the mixed zone), otherwise using the methods and parameters as for the "standard model" of Bahcall, Bahcall, and Shaviv[2]: (i) For the "slow-mixing" case X_1 was kept uniform in the mixed zone, but X_3 at each level was allowed to take on its local equilibrium value wherever radiative equilibrium held ($t_m \gg t_0, t_3$). The ordinary heat-flow equations were used. A small convective core usually appears in the interior of the mixed zone. Throughout this core X_3 was kept constant[9] and the adiabatic temperature gradient was used. (ii) As an intermediate case of "fast mixing" we kept X_3 (as well as X_1) uniform throughout the mixed zone, but still kept the ordinary heat-flow equations (which implies $t_0 \ll t_m \ll t_3$). (iii) As an extreme case ("very fast" mixing) we considered "adiabatic" models where we used the adiabatic temperature gradient throughout the mixed zone as well as keeping X_3 uniform ($t_m \ll t_0, t_3$).

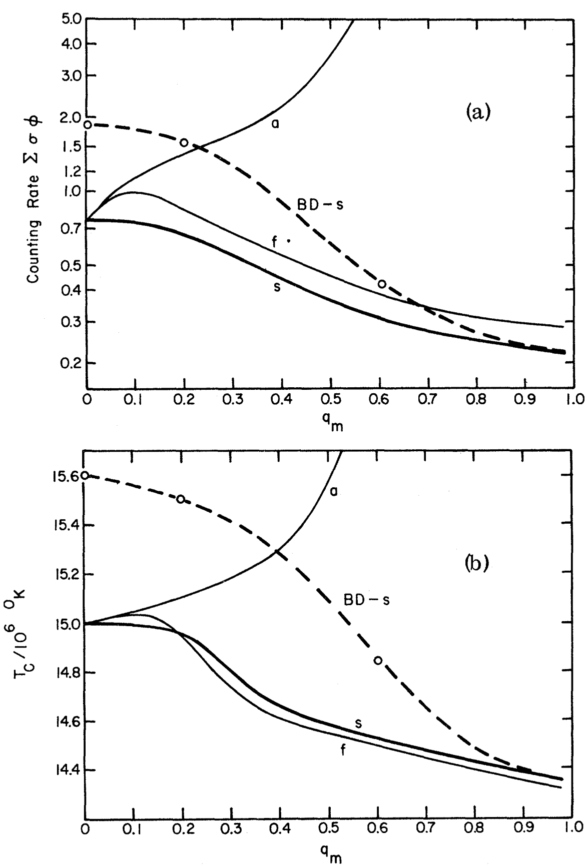

FIG. 1. (a) The total neutrino capture counting rate $\sum(\varphi\sigma)$ in units of $10^{-35}/^{37}$Cl atom sec and (b) the central temperature T_c in units of 10^6 °K plotted against the fractional mass q_m in the mixing zone. The labels s, f, and a refer to "slow mixing," "fast mixing," and "adiabatic," respectively. The dashed curve refers to Brans-Dicke models (circles represent calculated models) with a scalar-field coupling constant of 5, a Hubble constant of 75 km/sec Mpc, a flat universe, and "slow mixing".

Detailed calculations were carried out for q_m = 0, 0.10, 0.20, 0.40, 0.60, and 0.98 and interpolation was used for intermediate values. The main results are given by the solid curves in Fig. 1(a) which plot the total counting rate $\sum\varphi\sigma$ from present-day solar neutrinos (mainly from the ^{8}B neutrinos) against q_m for each of the three cases; the central temperature T_c is sketched in Fig. 1(b) and some auxiliary quantities in Fig. 2. Note the rather complicated shapes of these curves and that the introduction of slow mixing lowers the neutrino flux but further speeding up of the mixing increases the flux again. The features can be understood qualitatively as follows.

The initial hydrogen abundance in the solar-evolution calculation is always adjusted so that for the present sun (with about 5 % of the solar hydrogen converted to helium) the calculated luminosity, and hence the total present rate of nuclear energy production L_{nuc}, equals the observed[10] value $L_\odot$. The flux of ppe neutrinos, produced by electron capture in the dominant

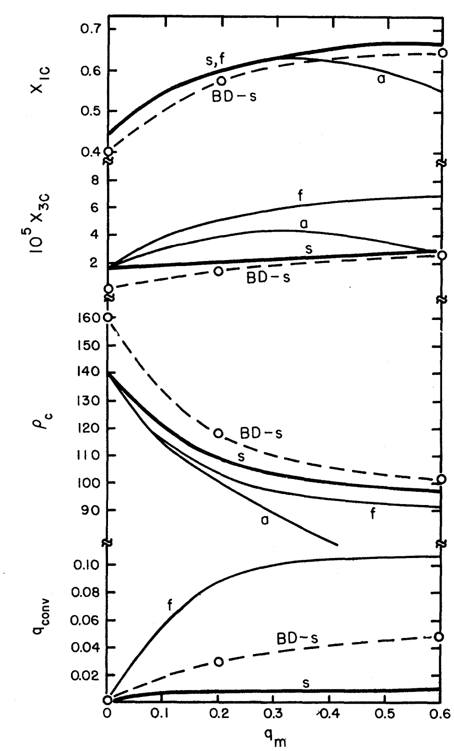

FIG. 2. Various quantities plotted against q_m with labels as in Fig. 1. X_{1c} and X_{3c} are the fractional abundances (by mass) of ^{1}H and ^{3}He, respectively, in the center of the present sun. ρ_c is the central density in g/cm^3 and q_{con} is the fraction of the solar mass in the central convective core.

branch of the pp chain, depends on ρL_{nuc} but not on temperature. The contribution of these neutrinos to the total counting rate $\sum\varphi\sigma$ of about 0.03 in units of $10^{-35}/^{37}\text{Cl}$ atom sec is almost independent of history, mixing, nuclear reaction rates, etc. (as noted in previous papers from specific numerical calculation). However, the main contribution to the counting rate comes from the ^{8}B neutrinos (produced in a minor branch) whose flux, φ_8, is extremely temperature sensitive, is produced in the innermost region of the sun, and depends mainly on the central temperature T_c. On the other hand, L_{nuc} comes from a more extended region (inner 10% of the mass, say, with temperature drop ΔT) and is proportional to ρX_1^2 and to a fairly low power of $T_c-\Delta T$. Since the present L_{nuc} is kept constant, T_c and hence φ_8 will increase if ρ or X_1 are decreased or if the interior temperature gradient (and hence ΔT) are increased.

The $^1\text{H}\to{}^4\text{He}$ conversion in the interior of an unmixed sun leads to a much lower central value X_{1c} of X_1 than its average value; hence X_{1c} is raised by mixing (see Fig. 2). As a function of increasing q_m, X_1 in the hydrogen-burning region increases slowly when $q_m \lesssim 0.2$ while ρ_c decreases, so that (for small q_m) T_c and φ_8 increase much less rapidly than one might expect, even in the slow-mixing case. For the "fast"-mixing case the influx of the nuclear fuel ^{3}He into the center (see X_{3c} in Fig. 2) raises the temperature gradient and ΔT so that T_c and φ_8 actually increase with q_m at first (for $q_m > 0.2$ the inner region is convective and the effect saturates). For the extreme case (iii) with an adiabatic mixed interior this effect is even stronger and also ρ_c decreases, so that T_c and φ_8 actually increase monotonically with q_m.

A fast-rotating core of the sun would provide some evidence for the Brans-Dicke type of cosmology, which involves a time-varying gravitational constant G and implies a larger amount (by a factor ~1.5) of $^1\text{H}\to{}^4\text{He}$ conversion inside the sun in its past. For an unmixed sun this results[11] in lower present values of X_1 and hence an increased flux φ_8. We have repeated our series of "slow-mixing" calculations, using a Brans-Dicke type of G variation (but only for $q_m = 0$, 0.2, and 0.62). The results are given in the dashed curves in the figures. Note that the effect of mixing is stronger here because the inhomogeneity X_1 was greater without mixing. As $q_m \to 1$, the difference between the Brans-Dicke and ordinary models (for slow mixing) disappears since a fully mixed, chemically homogeneous present sun is history independent.

In view of uncertainties in mixing theory we have kept the mixed mass fraction q_m as an open parameter, but too large a value of q_m would be in conflict with stellar evolution evidence: An unmixed star evolves from the main sequence into a red giant when about 0.1 of its mass has been converted into ^{4}He, but for a mixed star with $q_m < 0.1$ this mass fraction and the main-sequence lifetime are likely to increase by a factor $F \sim q_m/0.1$ (this point is currently under investigation). Values of F larger than about 2 (or 3 for Brans-Dicke cosmology) would increase the ages of the oldest star clusters in our galaxy uncomfortably far above the cosmological Hubble time and radioactive time scales. Statistical comparisons of numbers of main-sequence stars with red giants can also be used to infer similar limits on F for the globular cluster[12] $M3$ and for K stars[13] in the solar neighborhood. We consider the possibility of $q_m > 0.25$ (or > 0.4 for Brans-Dicke) as very unlikely.

To summarize our main conclusions, based on the curves in Fig. 1(a): The neutrino counting rate as a function of the mixing time t_m has its greatest decrease when the mixing is "slow" (10^7 yr $\ll t_m \ll 10^{10}$ yr) in which case the maximum decrease (fully mixed) would be by a factor of 3.5 (keeping all other parameters fixed). For extremely fast mixing the neutrino flux would actually increase. If we restrict the mass of the mixed zone so as to avoid conflict with the evolution of old stars in our galaxy, the maximum allowed decrease in the neutrino counting rate is only by a factor of about 1.5 (and the rate with Brans-Dicke and with mixing could not be below the "standard" rate at all). Luckily, then, the uncertainties in mixing theory do not affect the neutrino fluxes very strongly (and the absence of effective mixing in the present sun is in any case the most likely possibility).

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$\pi^- p$ BACKWARD ELASTIC AND INELASTIC SCATTERING AT 2.15 GeV/c *

R. Anthony, C. T. Coffin, E. Meanley, J. Rice, N. Stanton,† and K. Terwilliger
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan
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We have measured differential cross sections for the reactions $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^\pm N^{*\mp}(1236)$ at an incident pion momentum of 2.15 GeV/c and for center-of-mass scattering angles in the range $\cos\theta^* = -0.999$ to -0.950. The elastic differential cross section has a sharp dip in the backward direction, dropping from ~70 μb/sr at $\cos\theta^* = -0.95$ to ~4 μb/sr at $\cos\theta^* = -1$; the $N^{*-}(1236)$ differential cross section is, within our errors, consistent with the same size and shape as that for elastic scattering; the average differential cross section for $N^{*+}(1236)$ in our angular region is less than 18 μb/sr with 97% confidence.

While there have been studies of backward elastic πp scattering at a number of momenta up to 17 GeV/c,[1-6] the backward inelastic πp reactions are still largely unexplored.[1] To examine these inelastic reactions and to continue the backward elastic scattering investigations, we have carried out an optical spark-chamber experiment at the Argonne zero-gradient synchrotron to obtain angular distributions near 180° for the reactions $\pi^- p \rightarrow \pi^\pm N^{*\mp}$ and $\pi^- p \rightarrow \pi^- p$ for π^- beam momenta between 2.15 and 6 GeV/c. In this Letter we present results at 2.15 GeV/c, the minimum of the deep dip in the $\pi^- p$ 180° elastic-scattering excitation function of Kormanyos et al.[4]

Experiment. – The experimental apparatus is shown schematically in Fig. 1. It consists essentially of a magnetic spectrometer which is used to measure the missing mass associated with a pion moving backward in the laboratory. The spark chambers were triggered whenever a backward pion had the proper timing with a beam particle. In order to minimize the accidental coincidence rate, a beam veto counter following a second bending magnet was added to the trigger requirements (neither the veto counter nor the second magnet is shown in Fig. 1.).

The events to be presented here were obtained from scanning 60 000 pictures, about half of all those collected at 2.15 GeV/c. Around 35% of the scanned pictures were measured (with manu-

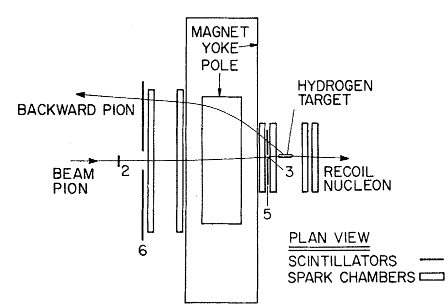

FIG. 1. A schematic diagram of the experimental apparatus.