PHENOMENA PECULIAR TO NUCLEAR MOLECULAR POTENTIALS IN $O^{16}+O^{16}$ SCATTERING

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Elastic-scattering calculations for $O^{16}+O^{16}$ by real molecular-type potentials were carried out over energies 8-30 MeV (c.m.). The cross sections are too large, although they show the erratic behavior expected in the orbiting energy range. A Blair approximation with an absorbing core of 3- to 4-F radius on these molecular-potential phase shifts gives the correct magnitude and energy dependence of the excitation function from 17 to 26 MeV.

Elastic-scattering data of $C^{12}+C^{12}$ and $O^{16}+O^{16}$ by Bromley et al.[1,2] show some unusual features. The excitation function for energies above the Coulomb threshold deviates from Mott scattering, and for $O^{16}+O^{16}$ shows several pronounced, very sharp maxima with peak-to-valley ratios in excess of 20 at energies from about 16 to 30 MeV (c.m.). The angular distributions have marked deviation from Mott scattering above the Coulomb threshold, where they show erratic behavior as a function of energy. Above the Coulomb threshold, the excitation function is smaller than the Mott result by at least a factor of 10.

Calculations[1,2] using an optical-model potential have not led to good results despite the extensive parameter search carried out. The angular distributions cannot be well fitted over the entire energy range from Coulomb threshold to 35 MeV (c.m.). The calculated excitation function does not, for $O^{16}+O^{16}$, show the large peak-to-valley ratios found experimentally.

The idea of a molecular interaction potential for scattering of heavy nuclei has been suggested for some time.[1-4] Much investigation[4-6] of such potentials, mostly in the context of molecular scattering, has been made, but there are several features peculiar to the nuclear case worth mentioning. The Coulomb field and the complex replusive core associated with the nuclear potential are usually not treated in molecular calculations. The energy range accessible to the heavy-ion accelerators is not high enough to make a semiclassical approach to detailed calculations useful.

Brueckner, Buchler, and Kelly[3] have derived a real $O^{16}+O^{16}$ interaction potential of molecular form (Fig. 1). An investigation of the scattering properties of this kind of potential could provide a way of connecting calculated many-body properties of a system with experimentally derived values. To this end, a numerical integration with proper boundary conditions was carried out for a real molecular-type potential and the Coulomb potential. The residual phase shifts were

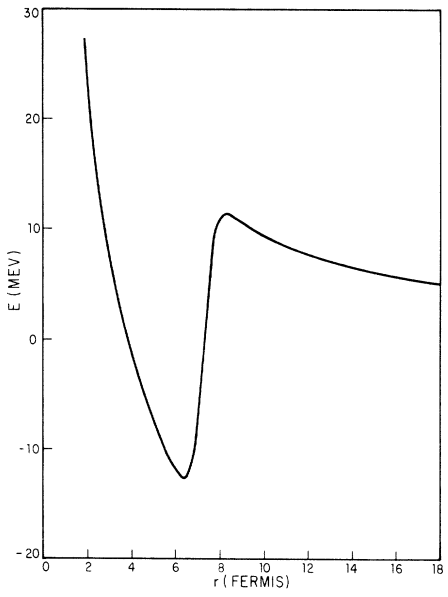

FIG. 1. Nuclear plus Coulomb potential as a function of relative distance r. The nuclear parameters are $A=100$, $B=0.6$, $V_0=30$, $a=0.25$, $R_0=6$, and $C=1.2$.

extracted from the partial waves at a radius large compared with the nuclear radius by comparison with the Coulomb wave function.[7] A partial search using a parametrized form of the nuclear potential was carried out.

The nuclear potential is a Woods-Saxon well with an exponential repulsive core (Fig. 1):

$$V_{\text{nucl}}(r) = Ae^{-Br} - \frac{V_0}{1+\exp[(r-R_0C)/a]}, \qquad (1)$$

where

$$R_0 = 2\times 1.2A^{1/3}.$$

The Coulomb potential is that of a point charge interacting with a sphere of uniform distribution and of charge radius R_0. Values used for the other parameters in V_{nucl} were varied to give a family of potentials resembling those calculated by Brueckner, Buchler, and Kelly.[3]

At energies below the Coulomb threshold, the particle sees only the Coulomb field, and Mott scattering results. As the threshold is approached, there is barrier penetration that depends sensitively on the details of the lip of the well, and the cross section begins to deviate from the Mott results. At an orbiting energy, an impact parameter b exists such that the effective potential $V_{\text{eff}}(r)$ has a local maximum and is tangent to the constant-energy line at the same nonzero radius:

$$V_{\text{eff}}(r) = V_{\text{nucl}}(r) + V_{\text{Coul}}(r) + Eb^2/r^2. \qquad (2)$$

At energies above the lowest orbiting energy, the full well is available to the particle, and reflection of the partial waves occurs at the core. The resulting interference gives rise to peculiar looking angular distributions and to an erratic energy dependence of the excitation function. An estimate of the energy range of this behavior may be made by calculating the energy range over which classical orbiting occurs. In the orbiting energy range an infinite number of classical paths exist having the same asymptotic scattering angle. Since each path carries its own phase (action integral), interference will occur between the paths in the asymptotic region, giving rise to the erratic behavior of the cross sections.

For representative nuclear parameters, the classical orbiting region extends over an energy range of at least 20 MeV (Table I). This large orbiting energy range is due to the rather narrow nuclear edge combined with the 20- to 30-MeV deep nuclear well.

Excitation functions were calculated for a family of real potentials [Eq. (1)]. Figure 2 shows the excitation function both for a Brueckner-type potential and for a purely attractive well compared with the experimental data. Although a real potential with a broad core fits the $O^{16}+O^{16}$ data at energies just above the threshold,[3] this kind of potential fails, as does the real attractive potential at higher energies. This failure is manifest in two ways: The calculated magnitude is too high by about a factor of 20, and the regular peak structure is not found.

There are two striking features of the $O^{16}+O^{16}$ data that must be explained by a potential interaction theory. The small values of the cross section above the Coulomb-threshold energy point to absorption from the elastic channel. The very large peak-to-valley ratios in the excitation function point to an interference explanation of these regularly spaced maxima and minima.[8] If one wants to describe the $O^{16}+O^{16}$ interaction by a standard optical-model potential, then the cross section can be made small by choosing the imaginary potential sufficiently large. The energy dependence of the cross section will vary slowly and will not be able to give the large peak-to-valley ratios found experimentally. However, there is a way out of this dilemma. If the core of the nuclear potential is complex, the low-angular-momentum partial waves will be absorbed. The high-angular-momentum partial waves will have their phase shifts strongly influenced by the presence of the core, their absorption will be small, and violent interference is possible.

A crude approximation can be made by chopping out the low-angular-momentum partial waves within the core radius by setting these phase shifts equal to $i\infty$ in the scattering amplitude. The cross section is calculated with the phase shifts for the high-angular-momentum

Table I. Orbiting-energy range for nuclear plus Coulomb potentials. $R_0 = 6$ F, $C = 1.2$.

A (MeV)	B (F^{-1})	V_0 (MeV)	a (F)	Orbiting range (MeV)	
0	...	20	0.25	11.0	69.7
0	...	20	0.50	9.7	40.0
100	0.6	20	0.25	11.6	68.2
100	0.6	20	0.50	10.1	32.6
100	0.6	30	0.25	11.5	99.8
100	0.6	30	0.50	9.9	46.5

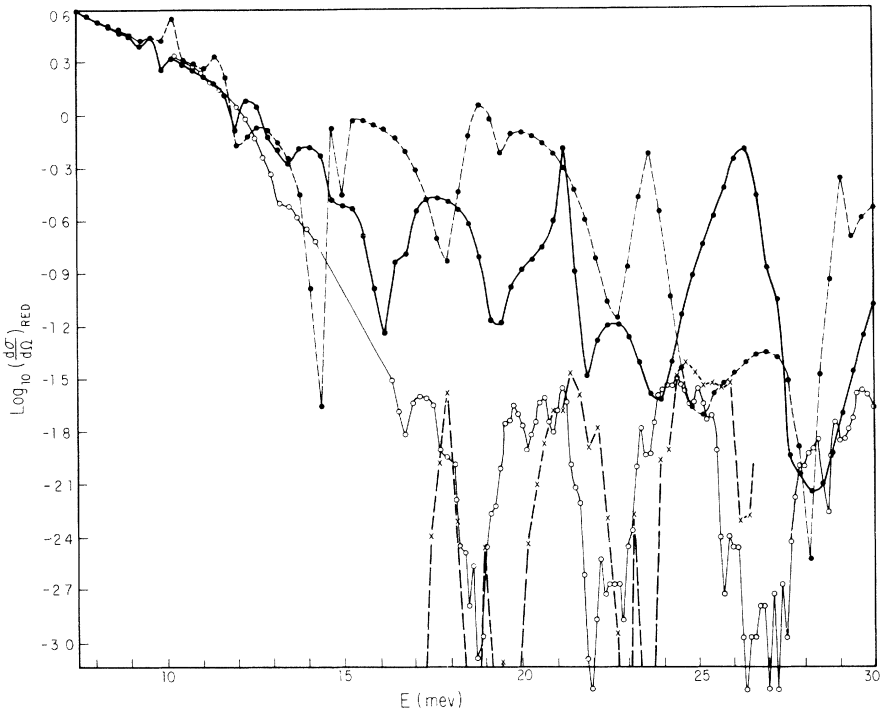

FIG. 2. Calculated and experimental cross section at 90° c.m. for $O^{16}+O^{16}$ elastic channel. Solid line with open circles: experimental values; solid line with closed circles: Woods-Saxon well, $A=0$, $V_0=30$, $a=0.25$, and $C=1.2$; dashed lines with closed circles: core and Woods-Saxon well, $A=100$, $B=0.6$, $V_0=30$, $a=0.25$, and $C=1.2$; dashed line with crosses: black core approximation using above nuclear well. $(d\sigma/d\Omega)_{RED}=(d\sigma/d\Omega)/(1\text{ mb/sr})$. All values are in c.m. system. Calculated or measured points are shown as dots, crosses, or circles. The experimental curve was taken from published graphs, making the estimation of the minima difficult.

partial waves gotten from the real potential with a core of approximately the same radius chosen for the imagninary core. This approximation is a natural extension to the original Blair approximation.[9] The results are presented in Fig. 2 for a potential with a broad core ($B=0.6$ F^{-1}) resembling the Brueckner potential used in obtaining a fit to the O^{16} data at threshold energies. All partial waves with $l \leq 12$ were totally absorbed in the energy range 16-26 MeV. This corresponds to an absorbing core of radius about 4 F. The resulting excitation function both has the right magnitude and shows the sharp peak-to-valley interferences seen in the data. Above 26 MeV this approximation fails, as the remaining partial waves begin to have large phase shifts which give rise to a complicated interference pattern. The same approximation was tried both for potentials having a narrow core ($B=2.0$ F^{-1}) and for those having no core in this energy range; but the phase shifts are already large, and the cross sections show only a complicated, violently fluctuating interference pattern.

The ability of this crude approximation to reproduce both the small cross sections and the large peak-to-valley ratios found above threshold lends support to the idea of a molecular type of interaction with a complex core to describe the $O^{16}+O^{16}$ interaction. An investigation of the scattering from a molecular potential with a complex core will be published elsewhere.

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PION SCATTERING AND THE NEUTRON HALO IN LEAD

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Reanalysis of the 700-MeV $\pi^{\pm}$-Pb inelastic scattering experiment of Abashian, Cool, and Cronin leads to a contradiction with recent suggestions of a neutron-rich surface or "halo" unless one admits neutron distributions with sharp edges and with peaks near the surface.

Does the lead nucleus have a neutron-rich surface region or "halo"? Recent analyses of low-energy proton scattering[1,2] and of isobaric analog state data[3] suggest that for Pb^{208} the rms neutron radius r_n considerably exceeds the proton radius r_p. This evidence directly contradicts the apparently unambiguous result of the earlier $\pi^{\pm}$-Pb inelastic-scattering experiment done by Abashian, Cool, and Cronin.[4] We have undertaken a detailed optical-model analysis of this experiment and confirm the original conclusion $r_n \lesssim r_p$ if we exclude neutron distributions with sharp edges and with large peaks near the surface.

Piccioni[5] observed that at 700 MeV the ratio of total cross sections $\sigma(\pi^+\text{-}n)/\sigma(\pi^+\text{-}p) = \sigma(\pi^-\text{-}p)/\sigma(\pi^-\text{-}n) = 2.6/1$. Thus both π^+ and π^- are strongly absorbed in the interior of a lead nucleus, while in the surface region the π^+ are mainly absorbed by neutrons and the π^- by protons. The quantity

$$q \equiv [\sigma(\pi^-\text{-Pb})/\sigma(\pi^+\text{-Pb})] - 1, \tag{1}$$

where the σ's are absorption cross sections, is sensitive consequently to the properties of the surface region, and in particular, to the ratio of the "maximum" neutron and proton radii.

Abashian, Cool, and Cronin[4] assumed simple uniform distributions for both neutrons and protons and calculated q using a semiclassical approximation and a multiplicative Coulomb correction, as earlier discussed by Courant.[6] They obtained, for $r_p = 5.95$ (all radii are in F),

$$q = +0.044, \quad r_n = r_p; \tag{2}$$

$$q = -0.024, \quad r_n = 1.15 r_p. \tag{3}$$

Similar results were obtained with a larger value of r_p. Their experimental 700-MeV inelastic cross sections gave[4]

$$q_{\text{exp}} = +0.050 \pm 0.011 \tag{4}$$

implying that $r_n \lesssim r_p$.

Greenlees, Pyle, and Tang[2] recently performed an optical-model analysis of low-energy p-Pb^{208} scattering. From the radii of the real and spin-orbit potentials, they extracted an rms matter radius r_m. With the electron-scattering result $r_p = 5.50$, they found

$$r_n = [(A/N) r_m^2 - (Z/N) r_p^2]^{1/2} = (1.09 \pm 0.05) r_p. \tag{5}$$

Isobaric-analog-state data give[3] a smaller ratio, $r_n/r_p = 1.035$. Studies on other nuclei[1] such as the calcium isotopes suggest that in general $N/Z > 1$ implies $r_n/r_p > 1$.

Optical-model analysis.—We computed the exact numerical solution of the optical-model wave equation for $\pi^{\pm}$-Pb scattering with various densities, using a modified version of a code previ-