(1964)], is of the order of 10^{15} cm^{-3}. For p-type impurities it is about 10^3 times larger. In practice these values are probably overestimates since the range of the effective potential is reduced by the $1/q$ singularity of $\epsilon(q,0)$, thus increasing the radii of the impurity orbits, particularly for n-type material.

[10] Lindquist and Ewald, Ref. 9.

[11] E. D. Hinkley and A. W. Ewald, Phys. Rev. 134, A1261 (1964); B. L. Booth and A. W. Ewald, Phys. Rev. 168, 796 (1968).

[12] R. J. Wagner and A. W. Ewald, Bull. Am. Phys. Soc. 11, 828 (1966); R. J. Wagner, thesis, Northwestern University, 1967 (unpublished).

[13] Even in regions where (12) no longer holds, so that (14) is incorrect, the $\lambda^{1/2}$ contribution may still be made much larger than $\Delta\epsilon$ and thus observable.

PROTON-PARTICLE, NEUTRON-HOLE MULTIPLETS IN BI^{208}[†]

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The reactions Pb^{207}(He^3, d)Bi^{208} and Bi^{209}(d,t)Bi^{208} are found to excite the single-proton states coupled to the $3p_{1/2}$ neutron hole in Pb^{208} and the single-neutron hole states coupled to the $h_{9/2}$ proton added to Pb^{208}, respectively. Each particle-hole configuration forms a multiplet of states. Nine such multiplets have been identified, most of them for the first time. Tentative spin assignments based on intensities are proposed; a comparison with theoretical calculations gives remarkable agreement.

Numerous recent studies[1] have established that the reaction Pb^{208}(He^3, d)Bi^{209} excites the single-proton states of Bi^{209} with their full spectroscopic strengths. Similar work on Pb^{208}(d, t)Pb^{207} has been carried out indicating pure neutron-hole states in Pb^{207}.[2] It was the purpose of the present investigation to collect information on the strengths of the particle-hole couplings in Bi^{208}. He^3 and deuterons at incident energies of 30 and 20 MeV, respectively, were produced with the Rochester tandem Van de Graaff accelerator. Emergent particles were analyzed with a split-pole spectrograph at a range of forward angles with a resolution of 15 and 10 keV.

The approximate location of particle-hole multiplets can be computed from the known spectra of Pb^{207} and Bi^{209}. In addition to the multiplets based on the Bi^{209} and Pb^{207} ground states, numerous excited-particle, excited-hole multiplets are also expected, as well as multiplets above 2.6 MeV based on the 3^- core-excited state of Pb^{208} or on the even-parity one-particle, two-hole (1p-2h) states of Pb^{207}.[3] None of these latter states is expected to show up strongly in the reactions studied here.

Strong groups of states were seen in each reaction at the excitation energies corresponding to the expected multiplets. In the (d, t) reactions, angular distributions could easily distinguish $l = 1$, $l = 3$, and $l = 6$ transitions.[2] For the (He^3, d) reaction, the differences were less clear over the angular range studied; for angles less than 20°, where the angular distributions are more sensitive to l values, large portions of the spectrum were obscured by impurity groups.

The experimental energy levels with relative intensities at 40° are shown schematically in Fig. 1 for both reactions. From the data we can conclude that the mixing between states in different configurations is small. Therefore, the cross section of each state in the multiplet will be proportional to $(2J+1)$ and relative intensities may be used to assign spins with reasonable confidence. This is certainly the case for the multiplets populated by $l = 1$ and $l = 3$ transitions in the (d, t) reactions where the small admixtures are actually deduced from the angular distributions. Spin assignments made in this way for the 1-9, 5-9, and 3-9 multiplets[4] agree with measurements of Erskine.[5] Those multiplets, as well as the 7-9 one observed here for the first time, agree very well with the shell-model calculations of Kim and Rasmussen,[6] given in Table I. For the 13-9 multiplet, the states are weak and not so clearly resolved. Here the spins could be assigned from intensities to within ±1 unit of angular momentum, and the most probable spin was selected by comparison with the level scheme of Ref. 6. The $J = 11$ and 2 states of this multiplet are predicted at about 2.3 and 2.8 MeV. At these energies, the relatively weak $l = 6$ transitions could well be obscured by the 7-9 multiplet; however, a peak with an angular distribution which was inconsistent with $l = 3$ but consistent with $l = 6$ was observed at 2.437 MeV with about the right intensity for $J = 11^-$, and a broadening

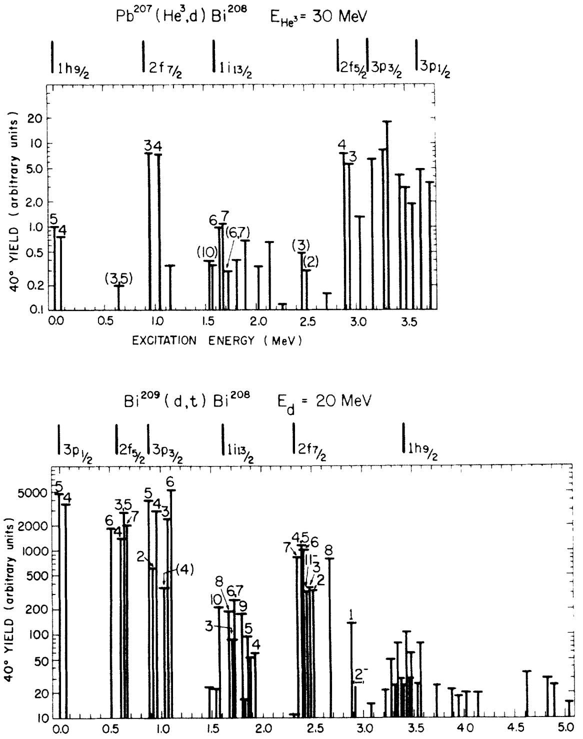

FIG. 1. Schematic spectra giving relative yields in the two reactions. The single-particle level schemes of neutron holes and protons are shown above the relevant graphs. Spin assignments are written over the bars. Numbers in parentheses refer to assignments from multiplets excited only by admixtures.

Table I. Bi^{208} levels to 3-MeV excitation.

Experimental Excitation Energy to ±5 keV (keV)	J^{π}	Configuration[a]	Theoretical Energy[b] (keV)
0	5^+	1-9	0
60	4^+	1-9	81
513	6^+	5-9	529
603	4^+	5-9	596
635	$[3^+$	5-9	630
	5^+	5-9	622
656	7^+	5-9	664
890	5^+	3-9	916
927	2^+	5-9	920
940	3^+	1-7	988
963	4^+	3-9	981
1040	4^+	1-7	1060
1075	3^+	3-9	1046
1108	6^+	3-9	1079
1145		H	
1476		D	
1536		D	
1568		H	
1576	$(10\pm1)^-$ 10^- c	13-9	1687
1633	6^-	1-13	
1666	$(9\pm1)^-$ 8^- c	13-9	1763
1672	7^-	1-13	
1715	$(4\pm1)^-$ 3^- c	13-9	1803
1724	$[{}^{6}_{7}\pm1]^-$ 6^- c	13-9	1847
	7^- c	13-9	1841
1795	$(8\pm1)^-$ 9^- c	13-9	1907
1806		H	
1829		D	
1845	$(5\pm1)^-$ 5^- c	13-9	1822
1878		D	
1887		H	
1931	$(3\pm1)^-$ 4^- c	13-9	1990
2024		H	
2133		H	
2252		H	
2349	$(6 \text{ or } 7)^+$ c	7-9	2373
2393	$[4^+$	7-9	2469
	5^+	7-9	2417
2412	$(6 \text{ or } 7)^+$ 6^+ c	7-9	2484
2437	11^- c	13-9	2239
2465	3^+	7-9	2482
2508	2^+	7-9	2531
2667	8^+	7-9	2637
2697		H	
2885	4^+	1-5	
2890	1^+	7-9	2850
(2910)	2^- c	13-9	2755
2940	3^+	1-5	

[a] An abbreviated notation is used for dominant shell-model configurations. See footnote 4. The letters *H* and *D* refer to weak states seen only in the (He^3, *d*) or only in the (*d*, *t*) reactions.

[b] From Ref. 6.

[c] Uncertainty in spin assignments using only cross sections; choice between allowed assignments was made by reference to theoretical calculations.

in the 2.89-MeV peak is consistent with the expected $J = 2^-$ state. While we observe a cluster of states near the expected position of the 9-9 multiplet, assignments of spins have not been attempted. The $h_{9/2}$ hole state in Pb^{207} has recently been found to be quite badly fragmented.[7] The 1-3 and 1-1 states also seem to be fragmented.

The admixtures in the wave functions which have been extracted from the experimental data are compared below with the calculations of Ref. 6. The numbers separated by commas represent spins-parities, dominant configurations,[4] [theoretical[6] admixtures in squared amplitude (%) of 3-9 and 1-9 states, experimental admixtures (%)]. Each state is separated by a semicolon:

3^+, 5-9, [12,9]; 4^+, 5-9, [8,12];

5^+, 5-9, [9,9]; 6^+, 5-9, [2,4];

3^+, 1-7, [0.1(0.16), <2(2.6)];

4^+, 1-7, [0.3(0.05), 9(0.1)].

The $3^+$and 5^+ states of the 5-9 multiplet were not resolved, and it was assumed that their admixtures were equal. The quantities for the two 1-7 configuration states given in parentheses are admixtures of the 5-9 configuration.

The agreement between theory and experiment is again good. We do observe one discrepancy, a 9% admixture between the $J = 4$ states of the 3-9 and 1-7 multiplets, while the calculations pre-

Table II. Center-of-gravity energies and rms splittings of Bi^{208} multiplets.

Configuration	$\bar{E}$[a] obs (keV)	E from Bi^{209} and Pb^{207}[b] (keV)	Shift ($\bar{E}-E$) (keV) Expt.	(Theory[c])	Rms width[d] (keV)
$p_{1/2}{}^{-1}h_{9/2}$	27	−77	104	(104)	60
$f_{5/2}{}^{-1}h_{9/2}$	633	493	140	(133)	192
$p_{3/2}{}^{-1}h_{9/2}$	1010	820	190	(174)	188
$p_{1/2}{}^{-1}f_{7/2}$	996	820	176	(197)	93
$p_{1/2}{}^{-1}i_{13/2}$	1654	1532	122		39
$i_{13/2}{}^{-1}h_{9/2}$	1886	1680	330	(357)	680
$f_{7/2}{}^{-1}h_{9/2}$	2478	2262	216	(219)	264
$p_{1/2}{}^{-1}f_{5/2}$	2909	2755	154		60

[a] $\bar{E} \equiv \sum_J (2J+1)E_J / \sum_J (2J+1)$ is the energy with respect to the ground state of Bi^{208}.

[b] Computed from the binding energies of the appropriate states in Bi^{209} and Pb^{207}, relative to the ground state of Bi^{208}.

[c] The numbers in parentheses are computed from Ref. 6 and normalized for the ground state.

[d] Width $\equiv [\sum_J (2J+1)(E_J-\bar{E})^2 / \sum_J (2J+1)]^{1/2}$.

dict ~0.3 %. There is also some evidence for perhaps 15% admixture between the $J^\pi = 6^-$ and 7^- members of the 13-9 multiplet and the 1-13 doublet and ~5% mixture between the 2^+ and 3^+ states of the 7-9 and 1-3 or 1-7 configurations.

The centers of gravity of the various multiplets and their rms splittings are given in Table II. It is interesting to note that the energies shift from those calculated from the Pb^{207} and Bi^{209} spectra by 150 ± 50 keV with the exception of the 13-9 multiplet. This is also a feature of the theoretical calulation and may be associated with the fact that this is the only multiplet where both wave functions are nodeless. The mean deviation $|\Delta E|$ between the energies observed and those calculated by Kim and Rasmussen for 31 states was only 35 keV (after a correction for a constant shift of 28 keV). In conclusion, the proton-particle, neutron-hole multiplets in Bi^{208} are remarkably pure and in surprisingly good agreement with theoretical calculations, considering the range of configurations covered.

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[2] G. Muehllehner, A. S. Poltorak, and W. C. Parkinson, Phys. Rev. 159, 1039 (1967).

[3] P. Mukherjee and B. L. Cohen, Phys. Rev. 127, 1284 (1962).

[4] An abbreviated notation is used to save space; $2j_h$-$2j_p$, where j_h and j_p are the angular momenta of the neutron-hole and the proton orbits. For Bi^{208} only the $i_{13/2}$ orbit has even parity; so this notation is unambiguous.

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[7] W. P. Alford and D. Burke, private communication.