RENORMALIZATION OF THE $K_{e3}$ FORM FACTOR

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The renormalization of the $K_{e3}$ form factor by the strong SU(3)-nonconserving interaction is calculated from the Fubini-Furlan sum rule and an experimental determination of the derivative of the matrix element of the divergence of the strangeness-changing current. The calculated renormalization is in agreement with that determined from measurements of $f_I$ values in Fermi nuclear $\beta$ transitions and of the absolute rate for $K^{+} \to e^+\pi^0\nu$.

In the universal vector semileptonic interaction it is believed that the strangeness-conserving and -nonconserving components of the current,

$$J^{e(3)}(x) + S^{e(3)}(x) \quad (\Delta Q = \pm 1, \Delta Q = \mp 1),$$

are proportional to components of the unitary spin current with constants of proportionality $G_0 = G \cos \theta$ and $G_0 = G \sin \theta$, respectively, where $G$ is the muon coupling constant. The constant $G_0$ is renormalized only by the electromagnetic interaction, but $G_{0s}$ is renormalized by the SU(3)-nonconserving strong interaction to a value $G \sin \theta_{\rho}$. From measurements of the $f_I$ values for superallowed pure Fermi nuclear $\beta$ transitions, it is found that $\sin \theta_{\rho} = 0.978 \pm 0.001(\pm 0.005)$. The renormalized strangeness-changing coupling constant can be determined from measurements of the absolute rate for $K^+ - e^+\pi^0\nu$. The present average value, $\Gamma = (4.00 \pm 0.09) \times 10^6$ sec$^{-1}$, leads to the result $\sin \theta_{\rho} = 0.222 \pm 0.004$. The difference

$$\sin \theta_{\rho} - \sin \theta = 0.012 \pm 0.007(\pm 0.022) \quad (1)$$

is a measure of the renormalization of $G_{0s}$ by the SU(3)-nonconserving strong interactions. A recent measurement of the $K_{e3}/K_{e3}$ relative branching ratio makes possible an independent estimate, though less direct, of the renormalization of $G_{0s}$, through a sum rule derived by Fubini and Furlan. This estimate is given by Eq. (18) of this note.

The sum rule is derived from the commutation relation for the charges $S^{(3)}$ associated with the currents $S^{(3)}(x)$:

$$[S^{(3)}, S^{(3)}] - G_{0s}^2(Q + Y),$$

where $Q$ is the charge and $Y$ the hypercharge. The expectation value of (2) for a $\pi^+$ state of momentum $\mathbf{p}$ gives the sum rule

$$G^2(\mathbf{p}) + \delta G^2(\mathbf{p}) = G_{0s}^2,$$

where $G(\mathbf{p})$ is the matrix element of $S^{(3)}$ for a $K$ of momentum $\mathbf{p}$ and a $\pi$ of momentum $\mathbf{p}$. In this frame of reference, the center of mass of the lepton pair is stationary and the invariant-mass squared of the leptons is

$$s = [(p^2 + m_K^2)^{1/2} - (p + m_\pi^2)^{1/2}]^2.$$

We have at zero time

$$G(\mathbf{0}) = G_{0s} D(s) [(m_K^2 - m_\pi^2)^2 - s^2]^{-1/2},$$

where $D(s)$ is the matrix element of the divergence of $S^{(3)}(x)$,

$$D(s) = (m_K^2 - m_\pi^2) f_+ + sf_-.$$

The form factors $f_\pm$ are functions of $s$. The function

$$G^2(\mathbf{0}) = \sum_\alpha |(\pi^+) S^{(3)}| \alpha|^2 - \sum_\beta |(\alpha^+) S^{(3)}| \beta|^2$$

is

1553
is nonzero owing to the nonconservation of the

$$S_{\mu}^{(s)}$$. The states \(\alpha\) and \(\beta\) of momentum \(p\) lie outside the octet of pseudoscalar mesons, and have quantum numbers \(S=-1\) and \(I=\frac{1}{2}, \frac{3}{2}\), and \(S=+1\)

and \(I=\frac{3}{2}\), respectively. The limit \(s \to 0\) is approached as \(p \to \infty\), and the renormalized coupling constant is defined at \(s = 0\). It is

$$G(\omega) = G \sin \theta \rho = [G_{0S}^2 - \delta G^2(\omega)]^{1/2}. \tag{8}$$

Measurements of the \(K_{\mu3}-K_{\nu3}\) relative branching ratio give a direct experimental determination of the derivative of \(D(s)\),

$$[1 - \delta G^2(\omega)/G_{0S}^{21}]^{1/2} (\partial D/\partial s)_{s = 0} = 0.285 \pm 0.10. \tag{9}$$

From Eqs. (3), (5), and (9)

$$\left[ (\partial/\partial s) \delta G^2(\vec{p}) \right]_{s = 0} = G_{0S}^2 (-0.046 \pm 0.016) m_{\pi}^{-2} \tag{10}$$

provided \(\delta G(\omega)/G_{0S}^2\) is small compared with unity.

The present calculation of the renormalization of the coupling constant \(G_{0S}\) starts from this new experimental result.

The evaluation of the sum rule (3) depends on the following approximation. It is assumed that the matrix elements of \(S^{(s)}\) which connect the \(\pi^+\) with states of two or more particles are small compared with the matrix elements between single-particle states. The established unstable particle states with spin and parity \(J^P=0^-, \ 1^+\), \(1^-\), and \(2^+\) then form a complete set of states \(\alpha\) and \(\beta\) and saturate the sum rule. There are no established states with \(S=+1\) and \(I=\frac{3}{2}\), and of the states with \(S=-1\) and \(I=\frac{5}{2}\), only those with spin and parity \(J^P=1^+\) can contribute to \(\delta G^2(\vec{p})\) in the chosen frame of reference. There is only one such state, the \(K_A\) with mass \(m_{K_A} = 1320\) MeV.

The approximation therefore implies the condition

$$\delta G^2(\vec{p}) > 0, \quad \text{or} \quad \sin \theta \rho < \sin \theta. \tag{11}$$

States of two pseudoscalar mesons cannot contri-

bute to the sum rule in the chosen frame of reference. The most simple, allowed two-particle state consists of a pseudoscalar meson and a vector meson.

If the amplitude for \(K_A - \nu \bar{\nu}\) is the sum of scalar- and vector-particle pole terms (masses \(m_{\nu}\) and \(m_{\bar{\nu}}\)), the matrix element of \(S_{\mu}^{(4)}\) between the \(\pi^+\) and \(K_A\) states is

$$\left[ \frac{G_{0S}^2}{4E_A^2 \pi^2} \right] \left[ m_{\nu}^{-1} \left[ p_{\nu} \cdot p_{\bar{\nu}} \epsilon \cdot (p_{\nu} + p_{\bar{\nu}}) \right] \left( \delta_{\nu} - \frac{P_{\nu} P_{\bar{\nu}}}{m_{\nu}^2} \right) \left( 1 - \frac{P_{\nu}^2}{m_{\nu}^2} \right)^{-1} \right. \right.$$

$$\left. + (p_{\nu} - p_{\bar{\nu}}) \cdot \epsilon \frac{m_{A} F_{\nu} F_{\bar{\nu}}}{m_{\nu} - m_{\bar{\nu}}} \right] \tag{12}$$

for general values of \(p_A\) and \(p_{\pi}\), where \(\epsilon\) is the polarization vector of the \(K_A\), and \(p_A = p_{\nu} + p_{\bar{\nu}}\).

The form factors \(g\) and \(F\) may be functions of \(P_{\nu}^2\). The function \(\delta G^2(\vec{p})\) is given by the divergence of (12),

$$\delta G^2(\vec{p}) = \frac{G_{0S}^2 |d_A|^2}{4E_A \pi^2 (p_{\nu} \cdot \epsilon)^2}, \tag{13}$$

where

$$d_A = \frac{g}{2m_A^2} \left( m_{\nu}^2 - m_{\pi}^2 - s' \right) + \frac{2m_A F s'}{m_{\nu}^2 - s'}. \tag{14}$$

In the frame of reference in which \(\vec{p}_A = \vec{p}_{\pi} = \vec{p}\),

$$s' = P_{\nu}^2 = [(P_{\nu}^2 + m_A^2)^{1/2} - (P_{\nu}^2 + m_{\pi}^2)^{1/2}]^2. \tag{15}$$

The existence of a scalar-meson pole in \(d_A\) im-

plies a maximum in \(\delta G^2(\vec{p})\) at \(s' = m_{\nu}^2\) provided \(m_{\nu}^2 < m_{A}^2 - m_{\pi}^2\). This would correspond to a minimum of \(D(s)\) on the real axis for \(s < (m_{K} - m_{\pi})^2\) which is unlikely to be in accordance with the expected analyticity of \(D(s)\). This suggests that the approximation of neglecting two or more particle states in the sum rule is not accurate, and it may be conjectured that a correct treatment of these states would result in cancellation of the pole. For the present evaluation of \(\delta G^2\), we as-

sume that \(F = 0\).

If the dependence of \(g\) on \(s'\) is neglected,

$$\left( \frac{\partial}{\partial s'} \delta G^2 \right)_{s' = 0} = - \frac{G_{0S}^2}{4m_A} \left| g(0) \right|^2 \tag{16}$$

(\begin{align*}
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\text{PHYSICAL REVIEW LETTERS} & \\
25 November 1968
\end{align*})
and

\[
\delta G^2(\omega) = \frac{G^2_{0s}}{16} \left| g(0) \frac{\left( m_A^2 - m_\pi^2 \right)}{m_\pi^2} \right|^2 \]

\[= 0.018 G^2_{0s}. \quad (17)\]

From Eq. (8),

\[G(\omega)/G_{0s} = 0.991. \quad (18)\]

This is equivalent to \(\sin \theta_F = \sin \theta = -0.002\), which is completely consistent with the directly measured difference (1).

An independent estimate of \(\delta G^2(\omega)\) can be obtained by assuming that both the \(K_A^0 - \pi\nu\) and \(K - \pi\nu\) amplitudes are given by \(K^*(890)\) pole terms. This gives

\[
\frac{\delta G^2(\omega)}{G^2(\omega)} = \left[ \frac{m_\pi^2}{4m_A^2} \right] \frac{h^2}{f^2},
\]

\[= 0.015 \quad (19)\]

where \(h\) and \(f\) are the coupling constants for \(K_A^0 - (K^*)^0\pi^+\) and \((K^*)^+ - K^0\pi^+\), respectively, and are assumed to be independent of the mass of the \(K^*\). Intermediate scalar mesons cannot contribute to either \(\delta G^2(\omega)\) or \(G^2(\omega)\).

There is a possible second \(K_A\) state at 1230 MeV which is not yet completely established.\(^7\) Owing to the near equality in mass, the existence of such a state would make little difference to (17), but it would increase (19).

A perturbation theory estimate\(^1\) of the contribution of two-particle intermediate states in (3) gives \(G(\omega)/G_{0s} = 0.97\). However, recent estimates by Glashow and Weinberg\(^12\) and by Srivastava\(^13\) suggest a large renormalization, \(G(\omega)/G_{0s} = 0.85\). With the present average value\(^5\) of the absolute rate for the \(K_{e3}^+\) mode, this would lead to \(\sin \theta = 0.261 \pm 0.005\), which is unlikely to be in agreement\(^6\) with determinations from \(f\) values in nuclear \(\beta\) decay.

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\(^2\)J. M. Freeman, J. G. Jenkins, G. Murray, and W. E. Burcham, Phys. Rev. Letters 16, 959 (1966). The error in brackets is given by an estimate of the uncertainty in radiative corrections to the \(f\) values. However, a new measurement of the end-point energy in the \(\beta\) decay of \(^{14}\)O by J. M. Freeman, J. G. Jenkins, D. C. Robinson, G. Murray, and W. E. Burcham, Phys. Letters 27B, 156 (1968), leads to an \(f\) value more in agreement with existing measurements for \(^{20}\)Al.

\(^3\)D. R. Botterill, R. M. Brown, A. B. Clegg, L. F. Corbett, E. Gulligan, J. McL. Emmerson, R. C. Field, J. Garvey, P. B. Jones, N. Middlemas, D. Newton, T. W. Quirk, G. L. Salmon, P. Steinberg, and W. S. C. Williams, Rutherford High Energy Laboratory Report No. RPP/H/34, 1968 (unpublished). The relation between \(\Gamma\) and \(\sin \theta\) is \(\Gamma = \Gamma \sin^2 \theta (1 + 0.274 \lambda \mu \nu / m_\nu^2)\); \(\lambda = 1.42 \times 10^{-11}\) sec\(^{-1}\) given by N. Cabibbo, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif. 1967). It is assumed that \(\lambda = 0.202 \pm 0.008\).


\(^8\)States with spin and parity \(J^P = 1^-\) and \(2^+\) do not contribute to \(\delta G^2(\bar{p})\) in the frame of reference in which initial and final hadrons have equal momentum, \(p_1\), owing to the totally antisymmetric tensor \(\epsilon_{\mu\nu\rho\delta}\) which must be present in the matrix elements of \(S^{(\bar{p})}\). States of two pseudoscalar mesons do not contribute for the same reason.

\(^9\)A minimum on the real axis does not necessarily imply a minimum of \(D(\bar{p})\) in the complex-\(s\) plane, which would violate the maximum-modulus theorem.

\(^10\)We use the value \(f^2/4\pi = 1.6\). For the decay of the \(K_A\), the amplitude is \(\langle E_3 E_2 E_1 \rangle \langle E_1 E_2 E_3 \rangle / (m_A + m_\pi)\), where \(E_1\) is the momentum of the \(K^*(890)\), \(E_2\) and \(E_3\) the polarization vectors of the \(K_A\) and \(K^*\), respectively. For the partial width \(\Gamma(K_A^0 \rightarrow K^{*+}) = 30\) MeV, \(h^2/4\pi = 0.38\). The value for the partial width is given by G. Goldhaber, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif., 1967). The derivative coupling in the \(K_A\) decay amplitude implies that about 1% of the width corresponds to the \(d\) wave final state.


