EXPERIMENTAL COMPARISON BETWEEN ON- AND OFF-MASS-SHELL INELASTICITIES IN π^-p SCATTERING*

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We have investigated the mass spectra of the systems recoiling against peripherally produced Δ^{++} for the reactions $pp\to\Delta^{++}p\pi^-$, $pp\to\Delta^{++}p\pi^-\pi^0$, and $pp\to\Delta^{++}n\pi^+\pi^-$ at 6.6 GeV/c. The experimental cross-section ratios of these systems are found to be comparable with the known on-mass-shell cross section ratios $\sigma(\pi^-p\to\pi^-p)$:$\sigma(\pi^-p\to\pi^-p\pi^0)$:$\sigma(\pi^-p\to\pi^-n\pi^+)$ as a function of mass of the system $m\lesssim 1.8$ GeV. This result is compatible with a one-pion-exchange interpretation for the production of the peripheral Δ^{++} component in these five-body states and furthermore suggests that $\pi\pi$ and $K\pi$ inelastic scattering may be realistically studied in the same way.

In earlier analyses[1,2] evidence was presented that the reaction $pp\to\Delta^{++}p\pi^-$ at 6.6 GeV/c proceeds via a one-pion-exchange (OPE) mechanism with $\pi^\pm p$ quasielastic scattering at each vertex. In this Letter we report a study of four- and five-body final states in $pp\to\Delta^{++}X$ in which an interpretation of X production as off-mass-shell π^-p inelastic scattering is shown to compare favorably with the known on-mass-shell π^-p inelastic scattering. We have analyzed the reactions

$$pp\to pp\pi^+\pi^- \quad (7515 \text{ events}), \tag{1}$$

$$pp\to pn\pi^+\pi^+\pi^- \quad (7302 \text{ events}), \tag{2}$$

$$pp\to pp\pi^+\pi^-\pi^0 \quad (6098 \text{ events}), \tag{3}$$

produced by 6.6-GeV/c incident protons in the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber. The production cross sections for Reactions (1)-(3) are 2.70 ± 0.16, 2.47 ± 0.15, and 2.15 ± 0.13 mb, respectively.[3]

We find that Reactions (1)-(3) are dominated by the processes

$$pp\to\Delta^{++}p\pi^- \quad (80\,\%), \tag{4}$$

$$pp\to\Delta^{++}n\pi^+\pi^- \quad (50\,\%), \tag{5}$$

$$pp\to\Delta^{++}p\pi^-\pi^0 \quad (60\,\%) \tag{6}$$

with the indicated percentages. The production cross sections for Reactions (4)-(6) are thus 2.16 ± 0.18, 1.24 ± 0.12, and 1.29 ± 0.12 mb, respectively. For the Δ^{++} selection

$$1.14<M<1.30 \text{ GeV} \tag{7}$$

the mass of X in $pp\to\Delta^{++}X$ is plotted in Fig. 1 versus the lower of the two possible values of the momentum transfer t from the beam or target proton. If both $p\pi^+$ mass combinations for an event satisfy selection (7), an event is plotted twice.

We confine our attention henceforth only to those extreme peripheral events with

$$t<0.2 \text{ GeV}^2. \tag{8}$$

This t_{max} is indicated in Figs. 1(a)-1(c). In the case that two combinations for an event satisfy both selections[4] (7) and (8), we select that combination with the smaller t. The projections of $p\pi^+$ effective mass M and the mass m of X in $pp\to\Delta^{++}X$ are shown in Fig. 2. The contents of any mass bin in Figs. 2(d)-2(f) are simply related to $d^2\sigma/dtdm$ for the reaction in question via the aforementioned event/mb information.

The ratios of the experimental differential cross sections

$$\left(\frac{d^2\sigma}{dtdm}\right)_{p\pi^-\pi^0}\Big/\left(\frac{d^2\sigma}{dtdm}\right)_{p\pi^-}, \tag{9}$$

$$\left(\frac{d^2\sigma}{dtdm}\right)_{n\pi^+\pi^-}\Big/\left(\frac{d^2\sigma}{dtdm}\right)_{p\pi^-} \tag{10}$$

are shown as a function of m in Figs. 3(a) and 3(b). In evaluating the ratios for Fig. 3(a), the $p\pi^-\pi^0$ mass spectrum has been uniformly corrected for an unseparable 10% ω^0 contamination.

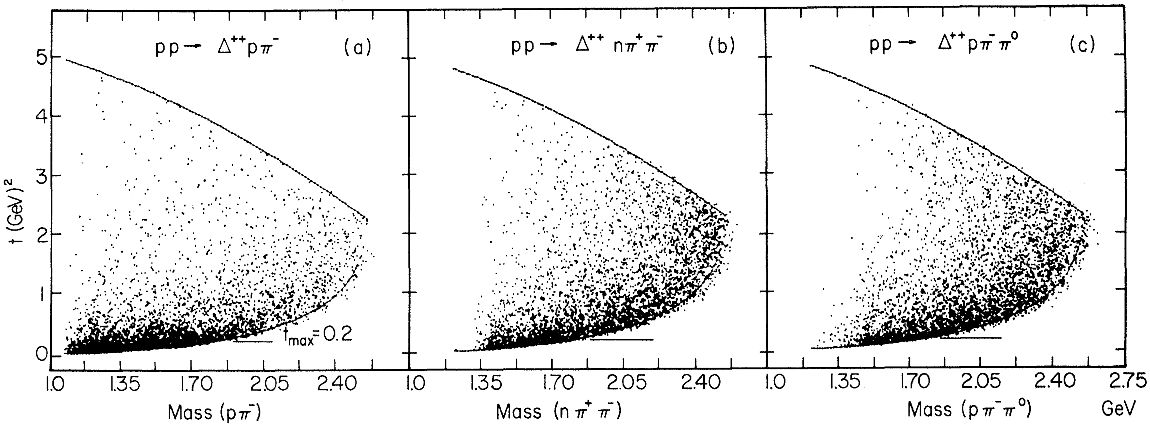

FIG. 1. Chew-Low plots of the type $pp \rightarrow \Delta^{++}X$ for the indicated reactions. The Δ^{++} selection is 1.14-1.30 GeV. If both $p\pi^+$ mass combinations satisfy this selection, an event is plotted as two points.

[Corrections for other background such as, e.g., possible non-Δ^{++} events in Fig. 2(c) are extremely unreliable because of the unknown m and t dependence of this background and have not been made.]

Motivated by the success of the OPE interpretation[1,2] of Reaction (4), which may be described as having $p \rightarrow \Delta^{++}\pi^-$ dissociation at one vertex and π^-p elastic scattering at the other, we now suggest as a possible interpretation for Reactions (5) and (6) that $\pi^-p \rightarrow n\pi^+\pi^-$ and $\pi^-p \rightarrow p\pi^-\pi^0$, respectively, occur at the second vertex. As a means of testing this hypothesis, we compare the experimental ratios in Fig. 3 (interpreted now as off-mass-shell inelastic/elastic π^-p cross-section ratios) with the known experimental on-mass-shell inelastic/elastic ratios for π^-p scattering. The shaded bands in Fig. 3 represent a summary of these known inelastic/elastic ratios.[5,6] The two sets of experimental ratios agree quite well, except possibly for the upper end of the $p\pi^-\pi^0/p\pi^-$ ratio. This possible discrepancy may be due to the aforementioned possible non-Δ^{++} background in Fig. 2(c) or may be just of statistical origin. In general the agreement is good, however.

Within the context of the OPE model with p

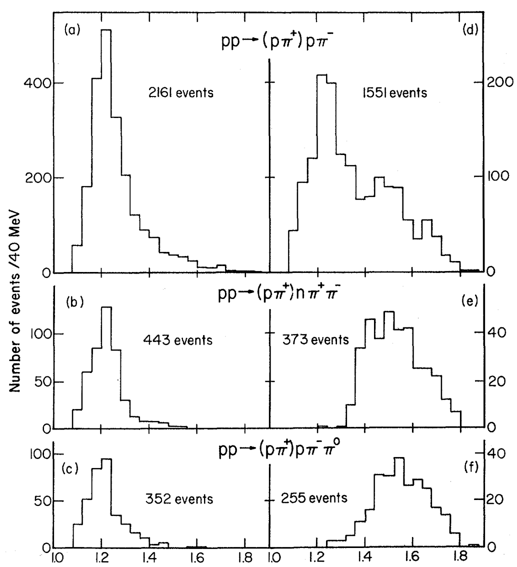

FIG. 2. (a)-(c) $p\pi^+$ mass projections for momentum transfer to the $p\pi^+$ system $t < 0.2$ GeV^2. Only one $p\pi^+$ combination is plotted for each event (the combination with lower t). (d)-(f) Mass projection for the X system in $pp \rightarrow \Delta^{++}X$ for the indicated reactions using the 1.14- to 1.30-GeV Δ^{++} selection and for $t < 0.2$ GeV^2.

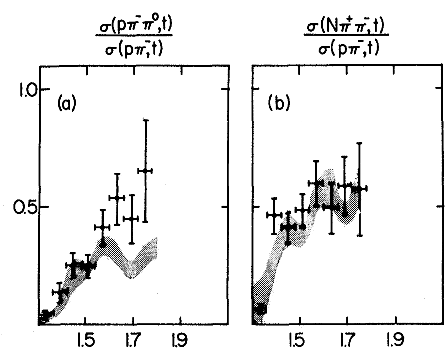

FIG. 3. (a) Ratio of the cross section for $(pp \rightarrow \Delta^{++}p\pi^-\pi^0)/(pp \rightarrow \Delta^{++}p\pi^-)$ as a function of the mass of the non-Δ^{++} final-state system, for momentum transfer to the $\Delta^{++}t < 0.2$ GeV^2. (b) The same for $(pp \rightarrow \Delta^{++}n\pi^+\pi^-)/(pp \rightarrow \Delta^{++}p\pi^-)$. The shaded bands represent a summary of the known experimental ratios $(\pi^-p \rightarrow \pi^-\pi^0p)/(\pi^-p \rightarrow \pi^-p)$ and $(\pi^-p \rightarrow n\pi^+\pi^-)/(\pi^-p \rightarrow \pi^-p)$, respectively.

$\to\Delta^{++}\pi^-$ dissociation at one vertex and π^-p scattering at the other, the noninterference expression for the differential cross section in the three variables t, M, m is[7]

$$\frac{d^3\sigma}{dt dM dm}=\frac{2}{4\pi^3 m_p{}^2 P_{\rm lab}{}^2}M^2 Q_t \sigma(M,t)$$
$$\times\frac{1}{(t+\mu^2)^2}m^2 q_t\sigma(m,t). \qquad (11)$$

$P_{\rm lab}$ is the laboratory beam momentum, m_p is the proton mass, Q_t and q_t are the momenta of the incident protons at the M and m vertex, respectively, as seen in that center of mass, and $\sigma(M,t)$ and $\sigma(m,t)$ are the off-mass-shell vertex cross sections, which for $t\to-\mu^2$ become the on-mass-shell cross sections $\sigma(M)$ and $\sigma(m)$, respectively. Integrating Eq. (11) over the M selection in Eq. (7), the agreement between the two sets of experimental data in Fig. 3 is seen to imply that

$$\sigma(m,t)_{p\pi^-\pi^0}:\sigma(m,t)_{n\pi^+\pi^-}:\sigma(m,t)_{p\pi^-}$$
$$\approx\sigma(m)_{p\pi^-\pi^0}:\sigma(m)_{n\pi^+\pi^-}:\sigma(m)_{p\pi^-}.$$

We conclude that the data are consistent with production via OPE processes with $p\to\Delta^{++}\pi^-$ at one vertex and π^-p elastic or inelastic scattering at the other. This result suggests that $\pi\pi$ and $K\pi$ inelastic scattering may be studied in the same way.

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[1]Eugene Gellert, Gerald A. Smith, Stanley Wojcicki, Eugene Colton, Peter E. Schlein, and Harold K. Ticho, Phys. Rev. Letters 17, 885 (1966).

[2]Eugene Colton, Peter E. Schlein, Eugene Gellert, and Gerald A. Smith, University of California at Los Angeles Report No. UCLA-1023 Rev., 1968 (unpublished).

[3]The slightly different events/mb ratios for these three reactions arise because of the difference in background contaminations. We estimate the background in Reactions (1)-(3) to be ~2, ~8, and ~3%, respectively. For the low-t Δ^{++} selections considered in this Letter, the contaminants are reduced to negligible amounts.

[4]This occurs 9.5, 5.1, and 2.3% of the time for Reactions (4)-(6), respectively.

[5]For a summary of π^-p elastic scattering, see, e.g., M. N. Focacci and G. Giacomelli, CERN Report No. 66-18 (unpublished).

[6]For a summary of π^-p inelastic scattering, see, e.g., M. G. Olsson and G. B. Yodh, Phys. Rev. 145, 1309 (1966).

[7]See, e.g., E. Ferrari and F. Selleri, Nuovo Cimento Suppl. 24, 453 (1962).

LOCAL TENSOR FIELDS AND THE ENERGY-MOMENTUM OPERATOR*

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It is shown that for any local tensor field $f_{\mu\nu}(x)$ with certain natural properties, the space integral of $f_{0\nu}$ is proportional to the energy-momentum operator P_ν. General conditions are established under which the agreement of a charge operator with the space integral of a local density on the one-particle subspaces implies their agreement on all of Hilbert space.

Recently, Sugawara[1] has proposed a theory in which the (putative) energy-momentum tensor density $\theta_{\mu\nu}(x)$ is constructed directly from currents $J_\mu{}^a(x)$ satisfying a current algebra. In this connection, Callan and Gross[2] have suggested tests of these ideas by showing that the identification of $\int d^3x\,\theta_{0\mu}$ with the momentum operator P_μ, when applied to one-particle matrix elements, leads to interesting sum rules. In this note we wish to point out that for any tensor field $f_{\mu\nu}(x)$, satisfying rather general conditions, the space integral[3]

$$A_\nu\equiv\int f_{0\nu}(x)d^3x$$

is necessarily proportional to P_ν. More generally, we also discuss circumstances under which the agreement of a formal charge with a generator[4] on the one-particle subspace imply their agreement on the whole Hilbert space. Quite apart from its relevance to any specific theory of currents, our investigation serves to clarify the role of various properties, particularly lo-