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[2]E. A. Peterson, Phys. Rev. Letters 20, 776 (1968); E. A. Peterson and T. F. Walsh, Bull. Am. Phys. Soc. 13, 706 (1968).

[3]S. L. Adler, Phys. Rev. 140, B736 (1965); W. I. Weisberger, Phys. Rev. 143, 1302 (1966).

[4]F. T. Porter, Phys. Rev. 115, 450 (1959).

[5]Our coupling constant $G_{\pi XX_i}$ is defined in the pole term contributions

$$\mathrm{Im}T^{(-)}(q,p,p')$$

$$=\sum_i G_{\pi XX_i}{}^2\pi\bar{U}_{X_i}(P')[\not{q}-M-M_i]U_X(p)\delta((p+q)^2-M_i{}^2)$$

and includes kinematic factors due to normalizations of $\bar{U}_{X_i}U_X$ for various nuclei. See, T. E. O. Ericson, J. Formanek, and M. P. Locher, Phys. Letters 26B, 91 (1967).

[6]Our choice of the path from the physical pion to the pion of zero four-momentum is $(\nu,p^2)\equiv(p\cdot q,p^2)=(\mu M,M^2)$ which has been shown to be a correct one in the π-nucleon scattering [S. Weinberg, Phys. Rev. Letters 17, 616 (1966)].

[7]S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 40A, 1171 (1965).

[8]$M_i{}^2(F_A{}^{XX_i})^2/\pi G_{\pi XX_i}{}^2=M_j{}^2(F_A{}^{XX_j})^2/\pi G_{\pi XX_j}{}^2$ $=\cdots=\sum_i(F_A{}^{XX_i})^2/[\pi\sum_i(G_{\pi XX_i}{}^2/M_i{}^2)]\equiv M^2F_A{}^2/\pi G^2.$

[9]Weinberg, Ref. 6.

[10]The overall atomic-number (A) dependence of $\mathrm{Re}\,a(\pi X)$ is roughly linear for each $I=0$ and $I=\frac{1}{2}$ nucleus, although because of more complicated structure in the A dependence the best linear fit for $6\leq A\leq 24$ nuclei does not exhibit clearly the difference of $I=0$ and $I=\frac{1}{2}$ nuclei. See R. Seki and A. H. Cromer, Phys. Rev. 156, 93 (1967). Here, we take the neighboring nuclei to estimate the isospin shift. The π-mesonic atom data used here are due to G. Backentoss et al., Phys Letters 25B, 365 (1967) (CERN data) and to D. A. Jenkins, R. Kunselman, M. K. Simmons, and T. Yamazaki, Phys. Rev. Letters 17, 1 (1966) (Berkeley data), and their analyses are due to Seki and Cromer (loc. cit.), and R. Seki (to be published), respectively. The π-nucleon datum, $a_1-a_3=(0.279\pm0.010)\mu^{-1}$, is due to a private communication with V. K. Samaranayake. See also V. K. Samaranayake and W. S. Woolcock, Phys. Rev. Letters 15, 936 (1965).

[11]Δ was estimated to be +4.7 and the above-threshold contribution $(f_\pi{}^2/\pi)\int$ to be -0.5. There is a question in this model calculation of Δ: For the physical pion the paper in Ref. 5 gives the estimation that the contribution from below the threshold in the integral of Eq. (1) is about $\frac{1}{10}$ of the one from above the threshold by assuming a phenomenological behavior of $\mathrm{Im}T^{(-)}$ below the threshold.

[12]Seki and Cromer, Ref. 10.

D-WAVE EFFECTS IN HIGH-ENERGY PROTON-DEUTERON SCATTERING

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Effects due to the D-wave component of the deuteron wave function in high-energy proton-deuteron scattering are investigated. Although negligible for most values of momentum transfer, they are quite significant in the "dip" region where the S-wave single- and double-scattering contributions interfere destructively. Here the D-wave contributions are large enough to fill in the dip, turning it into a shoulder, without the need for phase variation or spin dependence in the nucleon-nucleon amplitudes.

The differential cross section for high-energy proton-deuteron scattering[1,2] has a sharp forward peak followed by a region of much slower decrease at higher momentum transfer. The Glauber multiple-scattering theory provides a simple explanation for this structure: The two regions are dominated by single and double scattering, respectively.

The first application[3] of the Glauber theory to this problem, however, predicted a dip due to destructive interference in the transition region, in disagreement with experiment. Later calculations have attempted to explain the absence of this dip in terms of a conjectured rapid phase variation[2] or spin dependence[4] of the nucleon-nucleon scattering amplitude. We should like to show here that the well-known D-wave component of the deuteron wave function, which has been ignored in previous calculations, produces modifications which fill in the dip quite neatly even in the absence of the phase variation or spin dependence mentioned above.

The small-angle differential cross section for proton-deuteron scattering for momentum transfer Δ, summed and averaged over final and initial deuteron spins, is given by[5]

$$\pi^{-1}d\sigma/d\Delta^2=\tfrac{1}{3}\sum_{M,M'}|F_{M'M}(\vec{\Delta})|^2. \qquad (1)$$

The Glauber multiple-scattering expansion[6,7] gives

$$F_{M'M}(\vec{\Delta}) = \int d^3r\, \psi_{M'}^{\dagger}(\vec{r}) F(\vec{\Delta},\vec{r}) \psi_M(\vec{r}), \tag{2}$$

where $F(\vec{\Delta},\vec{r})$ is related to the nucleon-nucleon scattering amplitude $f(\Delta)$ by

$$F(\vec{\Delta},\vec{r}) = 2f(\vec{\Delta}) \exp(-\tfrac{1}{2}i\vec{r}\cdot\vec{\Delta}) + (2\pi)^{-1}i\int d^2\delta\, f(\tfrac{1}{2}\vec{\Delta}+\vec{\delta}) f(\tfrac{1}{2}\vec{\Delta}-\vec{\delta}) \exp(-i\vec{r}\cdot\vec{\delta}). \tag{3}$$

The deuteron wave function $\psi_M(\vec{r})$ can be written[8] in terms of radial functions and spin-one spinors $\chi_{1,M}$ as

$$\psi_M(\vec{r}) = (4\pi)^{-1/2} r^{-1}[u(r) + (8)^{-1/2} S_{12}(\hat{r}) w(r)]\chi_{1,M}, \tag{4}$$

where

$$S_{12}(\hat{r}) = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2 \tag{5}$$

is the standard tensor operator.

Using straightforward projection operator and trace techniques one finds

$$\pi^{-1} d\sigma/d\Delta^2 = |F_0(\Delta)|^2 + |F_2(\Delta)|^2, \tag{6}$$

where

$$F_0(\Delta) = 2f(\Delta)S_0(\tfrac{1}{2}\Delta) + (2\pi)^{-1}i\int d^2\delta\, f(\tfrac{1}{2}\vec{\Delta}+\vec{\delta}) f(\tfrac{1}{2}\vec{\Delta}-\vec{\delta}) S_0(\delta) \tag{7}$$

and[9]

$$|F_2(\Delta)|^2 = \tfrac{3}{4}|2f(\Delta)S_2(\tfrac{1}{2}\Delta)|^2 + \tfrac{1}{4}|2f(\Delta)S_2(\tfrac{1}{2}\Delta) + (2\pi)^{-1}i\int d^2\delta\, f(\tfrac{1}{2}\vec{\Delta}+\vec{\delta}) f(\tfrac{1}{2}\vec{\Delta}-\vec{\delta}) S_2(\delta)|^2. \tag{8}$$

The functions S_0 and S_2 are the deuteron's "spherical" and "quadrupole" form factors,[10] respectively, given by

$$S_0(\delta) = \int_0^\infty dr[u^2(r) + w^2(r)] j_0(\delta r) \tag{9}$$

and

$$S_2(\delta) = \int_0^\infty dr\, 2w(r)[u(r) - (8)^{-1/2}w(r)] j_2(\delta r). \tag{10}$$

Except for the small w^2 term in S_0, $\pi|F_0(\Delta)|^2$ is the differential cross section obtained if the D-wave component of the deuteron is omitted. The calculations described below show that the D-wave contribution $\pi|F_2(\Delta)|^2$ can generally be ignored except in the transition, or dip, region where the single- and double-scattering contributions to F_0 tend to cancel.

We have made numerical calculations with[11]

$$f(\Delta) = i(4\pi)^{-1}\exp[-5(\mathrm{GeV}/c)^{-2}\Delta^2]40 \text{ mb}, \tag{11}$$

and taking S_0 and S_2 from the "Potential No. 2" table of Glendenning and Kramer.[8] According to these authors the form factors should be fairly model independent up to $\Delta^2 \approx 0.35$ $(\mathrm{GeV}/c)^2$, which is just in the dip region. The results of our calculations are shown in Fig. 1. It is easy to see that the only significant effect of the D-wave contribution is to fill in the dip in $|F_0|^2$, converting it to a rather sudden break in slope. It should also be pointed out that the double-scattering contribution to $|F_2|^2$ changes it by only about 20% in the dip region.

Giving the nucleon scattering amplitude a real part equal to α times its imaginary part would change our results mainly by adding α^2 times the square of the S-wave double-scattering amplitude to the quantity $[4\pi|f(0)|^2]^{-1}d\sigma/d\Delta^2$ plotted in Fig. 1. Since a reasonable value for α^2 would seem to be about 0.1-0.2, this would raise the curve in the double-scattering region by 10 or 20% and round off the break somewhat, leaving the qualitative shape of the curve unchanged. The important point is that when the dip is filled by the D-wave contribution, the relative influence of other modifications taking into account the phase or spin-dependence of the nucleon-nucleon amplitudes is greatly reduced.

It seems clear from the above results that the effect of the D-wave component should be investigated in every case of high-energy scattering from deuterons. Its importance may vary somewhat from case to case, depending in part upon

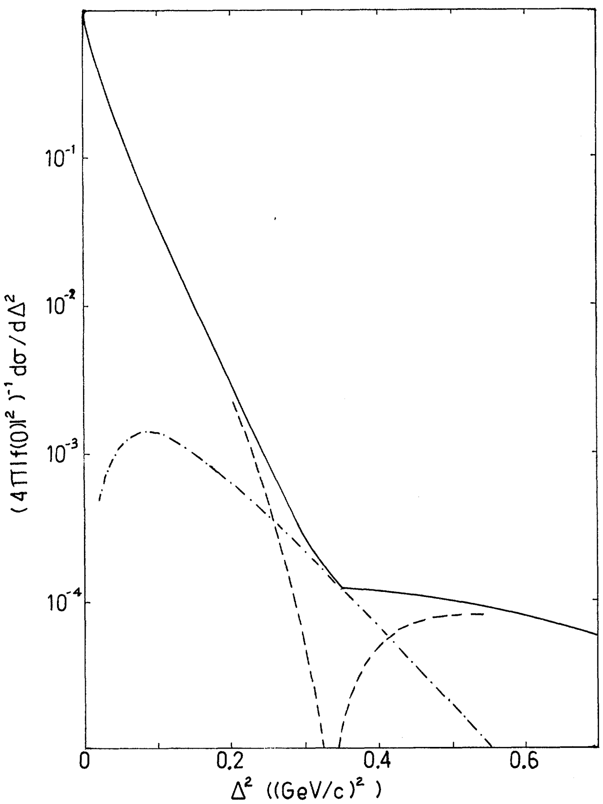

FIG. 1. Calculated proton-deuteron differential cross section. The dashed curve is the S-wave or "spherical" contribution, the dash-dotted curve the D-wave or "quadrupole" contribution, and the solid curve is their sum. The parametrization assumed for the nucleon-nucleon amplitude is given in the text.

whether the total cross sections on the individual nucleons are greater or less than the 40 mb assumed above. Obviously inclusion of D-wave effects is especially important in any attempt to determine the phase or spin dependence of the amplitudes for scattering on individual nucleons from the shape of the differential cross section for scattering on deuterons in the dip region.[12]

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Note added in proof.—After submitting this Letter the author received a preprint by E. Coleman and T. G. Rhoades (University of Minnesota) in which essentially the same results are obtained. These authors also treat D-wave effects in pion-deuteron scattering.

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[4]E. Kujawski, D. Sachs, and J. S. Trefil, Phys. Rev. Letters 21, 583 (1968).

[5]For simplicity we ignore the possibility of spin or isospin dependence in the nucleon-nucleon scattering amplitudes.

[6]R. J. Glauber, Lectures in Theoretical Physics (Interscience Publishers, Inc., New York, 1959), p. 315.

[7]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[8]J. H. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), p. 100.

[9]In deriving the expression for $|F_2|^2$ we have assumed the product of the two f's in the integral over $\vec{\delta}$ to be independent of the direction of $\vec{\delta}$. This is true for the Gaussian form of f which we assume in our calculations.

[10]N. K. Glendenning and G. Kramer, Phys. Rev. 126, 2159 (1962).

[11]This parametrization of f is meant to be illustrative, not accurate. It will not necessarily produce good fits to proton-deuteron scattering data, especially in the few-GeV/c region.

[12]The D-wave component has also been found to be surprisingly important in calculations which include Fermi-motion corrections to pion-deuteron scattering: G. Fäldt and T. E. O. Ericson, to be published.