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[5] There are three independent isospin amplitudes for the reactions $K^+d \rightarrow KNN\pi$ (and $K^+N \rightarrow KN\pi$), and six for the reactions $K^+d \rightarrow KNN\pi\pi$ (and $K^+N \rightarrow KN\pi\pi$). Equations (2a) and (2b) are independent of the relative importance of the amplitudes. It is a lengthy process to derive the relations using Clebsch-Gordan coefficients. The method of Shmushkevich enables one to write them almost at once. See I. Shmushkevich, Dokl. Akad. Nauk SSSR 103, 235 (1955); N. Dushin and I. Shmushkevich, Dokl. Akad. Nauk SSSR 106, 801 (1956) [translation: Soviet Phys.—Doklady 1, 94 (1956)]; G. Pinski, A. J. Macfarlane, and E. C. G. Sudarshan, Phys. Rev. 140, B1045 (1965).

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STUDY OF AN $I=0$ $\pi^-\pi^+$ RESONANCE AT A MASS OF 1.06 GeV*

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We have observed a resonance in the $\pi^-\pi^+$ final state with $I=0$, $M=1.06\pm0.015$ GeV, and $\Gamma<0.07$ GeV; our analysis favors $l\geq2$. It is produced primarily at a four-momentum transfer squared less than 0.1 GeV^2, and observed most clearly in the region where the cosine of the $\pi\pi$ scattering angle is less than −0.75.

As part of a continuing study of $\pi\pi$ interactions[1-3] we have observed an $I=0$ enhancement at 1.060 ± 0.015 GeV in the $\pi^-\pi^+$ final state. This enhancement appears to be incompatible with the S^*, an $l=0$ effect observed in the $K\bar{K}$ system near threshold,[4,5] which, if interpreted as a resonance, has a width ≥0.08 GeV. An enhancement in the $\pi^-\pi^+$ mass spectrum in this mass region has been reported previously.[6,7] This enhancement was particularly evident for events with $\cos\theta$ less than zero[8] (where θ is the angle between the outgoing π^- and the incident π^- in the di-pion rest system). A review of these experiments is contained in Ref. 8 where the effect was considered to be the $\pi\pi$ decay mode of the S^*. The following analysis is based on data from a Purdue-Notre Dame-Stanford Linear Accelerator Center Collaboration.[9] The reaction channels and numbers of events obtained are

$$\pi^-+p\rightarrow\pi^-+\pi^++n,\quad 7916\text{ events;}\qquad(1)$$

$$\pi^-+p\rightarrow\pi^-+\pi^0+p,\quad 4979\text{ events.}\qquad(2)$$

The average beam momentum is 4.0 GeV/c and a detailed analysis of the data can be found in Ref. 9.

Figure 1 is a plot of $\cos\theta$ vs $\sqrt{s}$, where $\sqrt{s}$ denotes the $\pi^-\pi^+$ effective mass. Inspection of Fig. 1 reveals that, in addition to the production of the ρ^0 and f^0, a cluster of events can be seen near $\sqrt{s} \approx 1.06$ GeV. This is particularly true for $\cos\theta < -0.75$.

The effective mass of the $\pi^-\pi^+$ system is shown in Fig. 2(a) for $\cos\theta < -0.75$, and in Fig. 2(b) for $\cos\theta < -0.75$ and $\Delta^2 < 0.1$ GeV^2 (where Δ^2 is the square of the four-momentum transfer to the $\pi\pi$ system). In both of these plots a clear peak is seen between the ρ^0 and f^0 resonances. The solid line on Fig. 2(a) shows a fit to the data, using phase space plus Breit-Wigner shapes for the ρ^0, f^0, and this enhancement; standard parameters were used for the ρ^0 and f^0 resonances, but the width and position of this enhancement were allowed to vary. This technique was used also for the data in Fig. 2(b), except that a phase-space contribution was not required. In both cases the mass of the enhancement was found to be 1.05 ± 0.015 GeV. The width for $\Delta^2 < 0.1$ GeV^2 was found to be 0.04 GeV, comparable to our experimental resolution. This width is also acceptable for the data in Fig. 2(a), if the difficulties in fitting the f^0 region are ignored. The $\pi\pi$ effective mass for other regions of $\cos\theta$ and $\Delta^2 < 0.1$ GeV^2 are shown in Figs. 2(c) and 2(d). The region $\cos\theta > 0.75$ [Fig. 2(c)] shows no enhancement. This may be partly due to the larger background resulting from the asymmetric decay of the ρ^0. The solid line shown in Fig. 2(c) is the fit obtained from the backward decay events, with the ρ^0 and f^0 contributions scaled to fit the data in this figure. The region $0.5 > \cos\theta > -0.5$ [Fig. 2(d)] does have an enhancement lying between the ρ^0 and f^0. A fit using three Breit-Wigner shapes yields here a mass of 1.08 ± 0.01 GeV, and width $\Gamma \leq 0.04$ GeV. We conclude that it is

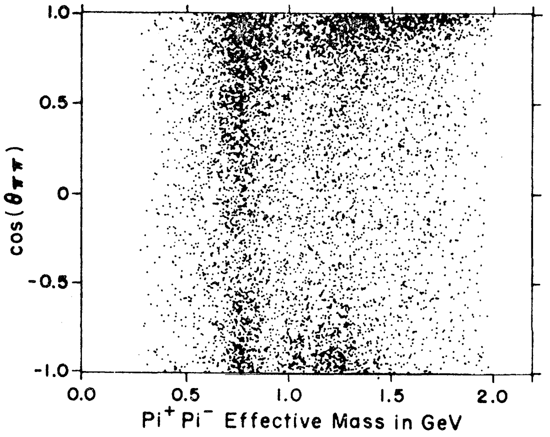

FIG. 1. Cosine of the scattering angle in the di-pion rest frame plotted against the effective mass of the $\pi^-\pi^+$ system.

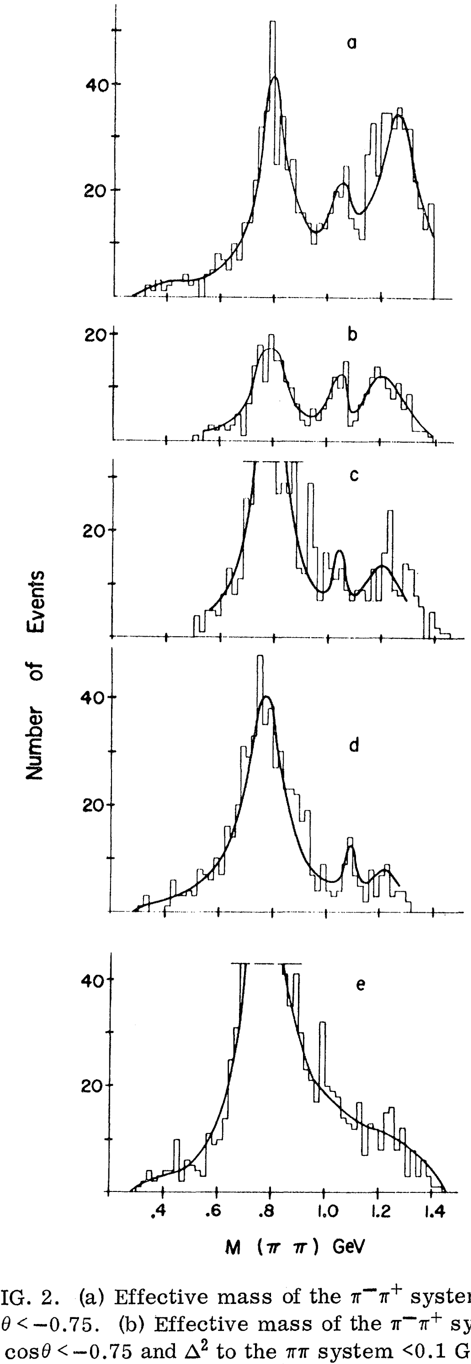

FIG. 2. (a) Effective mass of the $\pi^-\pi^+$ system for $\cos\theta < -0.75$. (b) Effective mass of the $\pi^-\pi^+$ system for $\cos\theta < -0.75$ and Δ^2 to the $\pi\pi$ system <0.1 GeV^2. (c) Effective mass of the $\pi^-\pi^+$ system for $\cos\theta > 0.75$ and $\Delta^2 < 0.1$ GeV^2. (d) Effective mass of the $\pi^-\pi^+$ system for $0.5 > \cos\theta > -0.5$ and $\Delta^2 < 0.1$ GeV^2. (e) Effective mass of the $\pi^-\pi^0$ system for $\Delta^2 < 0.1$ GeV^2.

difficult to obtain a determination of this width because of the mass-peak shifts observed in different regions of $\cos\theta$. It is possible that this resonance is wider than our resolution, or that it is narrow with interference effects causing the observed shift. If we assume that the enhancements seen are all part of the same effect, we find a mass of 1.06 ± 0.015 GeV and $\Gamma \leq 0.07$ GeV. A total production cross section of 40 ± 10 μb is obtained for the enhancement, 30 μb of which occurs at $\Delta^2 < 0.1$ GeV^2.

The region $\cos\theta < -0.75$ has particular kinematic properties, but no evidence has been found to indicate that the enhancement is kinematical in origin. No properties of the $n\pi$ channels appear to be able to account for such a narrow peak in the $\pi\pi$ spectrum.

More positive evidence of resonant behavior comes from the production characteristics in this mass region. A significant change occurs in the Δ^2 distribution at the region of this enhancement. This distribution sharply rises toward small Δ^2 values, in a way very similar to the behavior at the ρ^0 and f^0 regions. Moreover, the $\pi\pi$ decay asymmetry parameter changes from a level of 0.5 ± 0.05 to a small negative value as the resonance is reached. This indicates a rapid change in at least one of the partial waves. These facts are consistent with, and indicate the presence of, a resonance (hereafter referred to as P) at $M = 1.06 \pm 0.015$ GeV and with a width $\Gamma \leq 0.07$ GeV.

The events from Reaction (2) show no enhancement in this region for any angular or four-momentum transfer cuts. Similar $\pi^-\pi^0$ mass spectra are fitted using just ρ^- production plus a smooth background. Figure 2(e) shows the $\pi^-\pi^0$ mass distribution with $\Delta^2 < 0.1$ GeV^2. If this resonance is an isovector, and one-pion exchange (OPE) is the production mechanism, we would expect an enhancement with a cross section of ≈ 20 μb in the $\pi^-\pi^0$ spectrum. From our data, we can rule out an $I = 1$ assignment by ~2.5 standard deviations.[10] We cannot determine if $I = 2$; but if we assume that $I = 0$ and the decay is strong, then this resonance has $IJ^{PG} = 0(\text{even})^{++}$.

OPE has been shown[1-3,9] to dominate ρ and f^0 production, and to yield the unitarity limit for the cross section at the pole.[11] The Δ^2 distribution in the region of this resonance [Fig. 3(b)] is somewhat sharper than in the ρ^0 and f^0 regions; however, we assume that OPE dominates here also.

If this is the case, then we can calculate the expected cross sections at the pole for different l values. If we further assume that the extrapolation to the pole is the same for the P and f^0, we can compare cross sections in the physical region and attempt to rule out $l = 0$.

On the basis of the production and decay angular distribution, the differential cross section for the production of a di-pion system can be written as[12]

$$\frac{d^3\sigma}{ds\,d\Delta^2\,dt} = \frac{1}{(4\pi)^3 8 s^{1/2} q q_L{}^2}\,\frac{\Delta^2}{(\Delta^2+\mu^2)^2}\,f'(\Delta^2)\,|T_{4\pi}(s, t, \Delta^2)|^2, \tag{3}$$

where symbols q_L, q, f', and t are the incident momentum of the π^- in the laboratory frame, the π^- momentum in the di-pion rest frame, the pion-nucleon coupling constant, and the usual variable t which is proportional to $\cos\theta$. $|T_{4\pi}(s, t, \Delta^2)|^2$ is proportional to the di-pion angular distribution $d\sigma_{\pi\pi}/d\Omega$ at $\Delta^2 = -\mu^2$. Equation (3) does not contain absorption corrections, but this fact will not affect our arguments.

Let us initially assume that P is an inelastic S-wave resonance with elasticity η, whereby the partial-wave expansion through the real part of the $\pi\pi$ scattering phase shifts $\delta_J{}^I$ is given by

$$\begin{aligned} d\sigma/d(\cos\theta) = (2\pi/9q^2)\}&\sin^2\delta_0{}^2 + 1 + \eta^2 - 2\eta\cos 2\delta_0{}^0 + 2\sin^2\delta_0{}^2 - 2\eta\sin(\delta_0{}^2 - 2\delta_0{}^0)\sin\delta_0{}^2 \\ &+ \{18\cos(\delta_0{}^2 - \delta_1{}^1)\sin\delta_0{}^2\sin\delta_1{}^1 + 18[\sin^2\delta_1{}^1 - \eta\sin(\delta_1{}^1 - 2\delta_0{}^0)\sin\delta_1{}^1]\}P_1(\theta) \\ &+ \{20\cos(\delta_0{}^2 - \delta_2{}^0)\sin\delta_0{}^2\sin\delta_2{}^0 + 20[\sin^2\delta_2{}^0 - \eta\sin(\delta_2{}^0 - 2\delta_0{}^0)\sin\delta_2{}^0]\}P_2(\theta) \\ &+ [180\cos(\delta_1{}^1 - \delta_2{}^0)\sin\delta_1{}^1\sin\delta_2{}^0]P_1(\theta)P_2(\theta) + 81\sin^2\delta_1{}^1 P_1(\theta)^2 + 100\sin^2\delta_2{}^0 P_2(\theta)^2\{ \end{aligned} \tag{4}$$

Let us neglect off-mass-shell and absorption effects, and the change in the boundary of the Chew-Low plot. Then using Eqs. (3) and (4), we have computed R, the ratio of S- to D-wave total cross sections at $\delta_0{}^0 = \delta_2{}^0 = \frac{1}{2}\pi$ and $\eta = 1$, and obtained a value of $R = 0.2$. An evaluation of R under the above conditions and for the restricted region $\cos\theta < -0.75$ yields

$$R = \frac{(sq)_P}{(sq)_{f^0}} \frac{\int d\sigma(P) d(\cos\theta)}{\int d\sigma(f^0) d(\cos\theta)} = 0.17, \tag{5}$$

where we have used a Breit-Wigner form for the ρ^0, together with a scattering length approximation[13] for $\delta_0{}^2$. A measurement of this ratio with the restrictions $\cos\theta < -0.75$ and $\Delta^2 < 0.1$ GeV^2, together with a correction factor of 1.80 for the different minimum-Δ^2 cutoffs at the P and f^0 regions, gives $R = 0.7 \pm 0.2$. This ratio is inconsistent with the above S-wave interpretation and favors $l = 2$, although we cannot rule out values of $l > 2$ (and even) if the resonance width is substantially smaller than our resolution. The assignment $l = 2$ is also consistent with the decay angular distribution shown in Fig. 3(a), for the region $1.02 < M(\pi^+\pi^-) \leq 1.10$ GeV and $\Delta^2 < 0.1$ GeV^2. The solid line shown in Fig. 3(a) is a theoretical calculation normalized to the total number of events, obtained from the same approach as that of Eq. (4), except that an $l = 2$ P resonance is assumed. This calculation is in reasonable agreement with the data. Distributions in $\cos\theta$ over the entire $\pi^+\pi^-$ effective mass range have been fitted[9] using Lengendre polynomial expansions. The results [Fig. 3(c)] show a rise in the D-wave contribution in the P region; also, contributions from $l > 2$ partial waves are found to be absent. However, we note that the nonobservation of a peak in the effective-mass distribution in the forward decay direction may mean that this resonance decays asymmetrically, favoring the $\cos\theta = -1$ region. This would be characteristic of an S-wave resonance interfering with the P wave. Thus, from the shape of a decay angular distribution it is not always possible, without exact knowledge of the other partial waves, to prefer $l = 2$ over $l = 0$. But from the above cross-section argument, $l = 2$ is favored. This agrees with the spark-chamber experiment by Whitehead et al.[6] which is also inconsistent with $l = 0$.

If $l = 0$, this resonance could be the $\pi\pi$ decay mode of the S^*,[4,5,8] which is an enhancement near threshold in the $K\bar{K}$ system. A recent experiment[14] has studied this threshold enhancement in

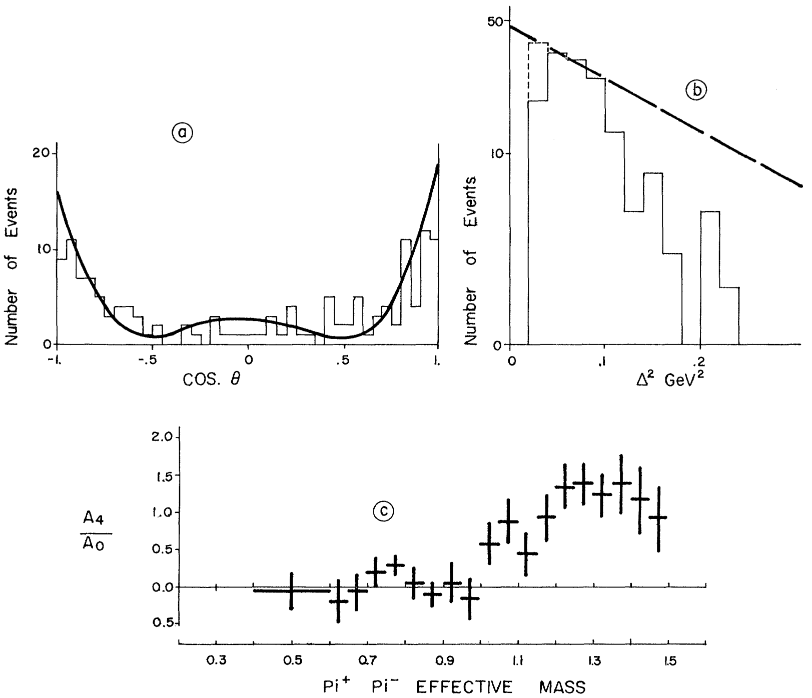

FIG. 3. (a) Decay angular distribution for the region $1.02 < M_{\pi\pi} < 1.1$ GeV and $\Delta^2 < 0.1$ GeV.^{2} (b) Distribution of four-momentum transfer squared for the region $1.02 < M_{\pi\pi} < 1.1$ GeV. (c) Coefficient A_4/A_0 for a Legendre-polynomial fit to the decay angular distributions, as a function of $M_{\pi\pi}$ for $\Delta^2 < 0.2$ GeV^2.

π^-p interactions at 4 GeV/c, affording direct comparison with our data. In this experiment, if the $K\bar{K}$ enhancement is assumed to be resonant, a width of $\Gamma > 0.12$ GeV is found; this width is not consistent with our data. Further, if we use the S^* production cross section[8,14] in the $K\bar{K}$ mode at 4 GeV/c, and assume that the P is the $\pi\pi$ decay mode of the S^*, we obtain $\sigma(\pi\pi)/\sigma(K\bar{K}) = 1.1 \pm 0.4$. This would increase the strength of our above cross-section argument against an $l=0$ assignment, as η would be <1.0. The latest experimental Δ^2 distribution for S^* production[14] is shown by the dashed line on Fig. 3(b). Once again, the S^* and P appear to have different production characteristics. These facts indicate that the P and S^* are not the different decay modes of the same resonance, although the presence of an S^* may well complicate[15] the competing $\pi\pi$ channel.

Thus we conclude that we observe an $IJ^{PG} = 0(\text{even})^{++}$ resonance at $\sqrt{s} = M_P = 1060 \pm 15$ MeV with a width of $\Gamma \leqslant 70$ MeV. The observed enhancement is consistent with having a value of $l=2$. In this case, the resonance could be the lowest lying particle on the Pomeranchuk (P) Regge trajectory. This would imply a trajectory slope of $\alpha_P'(0) \approx 1\ \text{GeV}^2$, which is a value larger than currently accepted[16,17] in theoretical analyses where Pomeranchukon exchange is treated by simple pole-term singularity. The second (P') Regge trajectory appears to be described well by the f^0 meson as a member of the 2^+ SU(3) nonet.[18]

We would like to acknowledge the help of S. L. Kramer on certain aspects of the analysis.

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