MODIFIED ORDINARY MODE IN MAGNETIZED PLASMAS WITH RELATIVE STREAMING

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It is shown that in the presence of relative streaming motion, linearly polarized electromagnetic waves of frequencies below the plasma frequency can propagate across the magnetic field. An instability criterion is given for this "modified ordinary mode."

It is well known that in a magnetized plasma, a linearly polarized electromagnetic wave propagating across the magnetic field is unaffected by the field and has a cutoff frequency at the plasma frequency.[1] The purpose of this Letter is to draw attention to the fact that in the presence of relative streaming between the charged particles, the above is no longer true. Waves with frequencies below the plasma frequency can propagate in the plasma and it is possible to arrange conditions such that no low-frequency cutoff exists. Moreover, the wave can become unstable under appropriate conditions.

We consider here the simplest model possible: a system composed of two identical electronic plasmas (infinite ion mass) moving with equal and opposite velocities $+u$ and $-u$, respectively, in the direction of the external magnetic field B_0, which is taken to be the y axis. Each of the streams has a plasma frequency $\frac{1}{2}\omega_{pe}^2$ so that the plasma frequency of the system is ω_{pe}^2. We consider waves with propagation vector $\vec{k}$ in the x axis. We first obtain the essential results by using the simple cold-plasma model and will subsequently discuss the effect of temperature. Looking for perturbation solutions of the form $\exp[i(\omega t+kx)]$ in the linearized cold-plasma equations (continuity, momentum, and Maxwell), we arrive at the following dispersion relation for linearly polarized electromagnetic waves with electric vector in the y axis:

$$k^2c^2=\frac{(\omega^2-\Omega_e^2)(\omega^2-\omega_{pe}^2)}{\omega^2-[\Omega_e^2-\omega_{pe}^2(u^2/c^2)]}, \qquad (1)$$

where Ω_e is the electron gyrofrequency.

Propagation is possible only if k^2c^2 is positive. If there is no streaming, $u=0$ and we recover the familiar result that the ordinary mode cannot propagate below the plasma frequency. If there is streaming but no magnetic field, the denominator of Eq. (1) is positive and again propagation is possible only for $\omega>\omega_{pe}$. In the general case in which both u and Ω_e are nonzero, the wave is no longer "ordinary" since it is affected by the magnetic field even for perpendicular propagation. We shall refer to it as the modified ordinary (hereafter abbreviated as MO) mode. To find its allowable frequencies, we consider three separate cases. The propagation and evanescent bands are as follows:

Case 1. $\omega_{pe}^2<\Omega_e^2$; $\Omega_e^2-\omega_{pe}^2>\omega_{pe}^2u^2/c^2$.

$0\le\omega^2\le\omega_{pe}^2$, evanescent;

$\omega_{pe}^2\le\omega^2\le\Omega_e^2-\omega_{pe}^2u^2/c^2$, propagation;

$\Omega_e^2-\omega_{pe}^2u^2/c^2\le\omega^2\le\Omega_e^2$, evanescent;

$\Omega_e^2\le\omega^2$, propagation.

Case 2. $\omega_{pe}^2<\Omega_e^2$; $\Omega_e^2-\omega_{pe}^2<\omega_{pe}^2u^2/c^2$.

$0\le\omega^2\le\Omega_e^2-\omega_{pe}^2u^2/c^2$, evanescent;

$\Omega_e^2-\omega_{pe}^2u^2/c^2\le\omega^2\le\omega_{pe}^2$, propagation;

$\omega_{pe}^2\le\omega^2\le\Omega_e^2$, evanescent;

$\Omega_e^2\le\omega^2$, propagation.

Case 3. $\omega_{pe}^2>\Omega_e^2$; $\omega_{pe}^2<\Omega_e^2c^2u^2$.

$0\le\omega^2\le\Omega_e^2-\omega_{pe}^2u^2/c^2$, evanescent;

$\Omega_e^2-\omega_{pe}^2u^2/c^2\le\omega^2\le\Omega_e^2$, propagation;

$\Omega_e^2\le\omega^2\le\omega_{pe}^2$, evanescent;

$\omega_{pe}^2\le\omega^2$, propagation.

It is seen from the above that instead of the simple situation in which ω_{pe} is the cutoff frequency, we have bands of propagating and evanescent frequencies. The most interesting feature is that propagation is possible for $\omega<\omega_{pe}$. Referring to case 3, we see that it is possible to arrange conditions such that the allowable frequencies extend down to zero.

If $\omega_{pe}^2 > (c/u)^2\Omega_e^2$, the wave becomes unstable. To see this, we note that Eq. (1) can be written as

$$\omega^4 - \omega^2(\Omega_e^2 + \omega_{pe}^2 + k^2c^2) + k^2c^2\Omega_e^2 + \omega_{pe}^2\Omega_e^2 - k^2u^2\omega_{pe}^2 = 0. \quad (2)$$

It is clear from (2) that a negative root for ω^2 appears, corresponding to an instability, when

$$k^2c^2\Omega_e^2 + \omega_{pe}^2\Omega_e^2 - k^2u^2\omega_{pe}^2 < 0. \quad (3)$$

Thus waves with infinite wave number begin to become unstable when

$$\omega_{pe}^2 > (c/u)^2\Omega_e^2. \quad (4)$$

It can be shown that the maximum growth rate occurs at $k = \infty$ and is

$$(\omega_i)_{\max} = [(u\omega_{pe}/c)^2 - \Omega_e^2]^{1/2}. \quad (5)$$

It should be pointed out that all of the above results carry over to the case of small deviation from perpendicular propagation by making the substitutions $u \to u \sin\theta$ and $\Omega_e \to \Omega_e \sin\theta$, where θ is the angle between the propagation vector and the magnetic field.

We have so far used a cold-plasma model. Since interpenetrating beams of zero temperature are subject to the electrostatic two-stream instability for any value of the streaming velocity u,[2,3] the question arises as to whether conditions can be created under which the predictions concerning the MO wave can be checked experimentally. To shed light on this question, we consider the effect of temperature. It is well known that if, in the model considered, each of the electron streams has a Maxwellian distribution with thermal velocity v_T, then the two-stream instability occurs only if $u > 1.31v_T$.[4] Meanwhile, because the stable electromagnetic waves have phase velocities much greater than v_T, their propagation characteristics are not significantly affected by thermal motion. Thus in order to check the predictions concerning the stable MO wave without exciting the two-stream instability, one must arrange conditions such that $u < 1.31v_T$. This requirement can easily be realized in practice.

Using the warm-plasma equations,[5] we have also investigated the effect of temperature on the stability criterion of the MO wave. The minimum value of u required for instability is found to be

$$u_{\min} = [v_T^2 + c^2\Omega_e^2/\omega_{pe}^2]^{1/2}. \quad (6)$$

Thus temperature has a stabilizing effect. Furthermore, if everything matches the assumptions of the theory, it is possible to excite the MO instability alone if we choose u such that $[v_T^2 + c^2\Omega_e^2/\omega_{pe}^2]^{1/2} < u < 1.31v_T$. In practice, however, this is a stringent condition and it is difficult to excite the MO instability without also triggering the two-stream instability. Hence the experimental test of the instability criterion is difficult. (This statement probably applies to all attempts to test instability theories.) Nevertheless, we venture to suggest the detection of coherent (nonthermal) radiation[6] as a possible means of identifying the existence of the unstable MO wave. Consider first the case $B_0 = 0$. If $u > 1.31v_T$, theory predicts the excitation of both the two-stream and the MO instabilities. Since the former is a longitudinal wave in the absence of an external magnetic field, a mechanism for its conversion into transverse oscillations is necessary in order to enable these waves to propagate in free space. According to Ginzburg and Zhelesniakov,[7] this conversion is effected in a homogeneous plasma by the interaction of the longitudinal waves with the density fluctuation. The conversion efficiency is small and the transverse waves produced by this mechanism are unpolarized. On the other hand, the MO wave is a polarized electromagnetic wave and it can propagate in free space without the need of a conversion mechanism. Thus for the $B_0 = 0$ case, the detection of a polarized component in the direction perpendicular to the stream motion is an indication of the existence of the MO instability.

Next we apply an external magnetic field B_0. If its strength is such that the condition $1.31v_T < u < [v_T^2 + c^2\Omega_e^2/\omega_{pe}^2]^{1/2}$ is satisfied, theory predicts the suppression of the MO instability and only the electrostatic two-stream instability remains. When this happens, the intensity of the radiation should be significantly reduced.

In conclusion, we have exhibited an electromagnetic mode in streaming plasmas possessing two interesting features: propagation across the magnetic field below the plasma frequency and the occurrence of instability. The characteristics of the stable waves of this mode can be checked experimentally under conditions easily realized in practice. We have also suggested a possible method for the more difficult task of detecting the instability. It is hoped that this Letter will draw the attention of experimenters and that the predictions reported will be subsequently verified.

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[1]K. G. Budden, Radio Waves in the Ionosphere (Cambridge University Press, New York, 1961), pp. 61, 65.

[2]See, for example, T. H. Stix, The Theory of Plasma Waves (McGraw-Hill Book Company, New York, 1962), pp. 109-114.

[3]For our model in which the magnetic field is aligned with the direction of stream motion, it can be shown that the most unstable electrostatic waves are those with $\vec{k} \parallel \vec{u}$ and that no such unstable waves exist with $\vec{k} \perp \vec{u}$.

[4]J. D. Jackson, J. Nucl. Energy C1, 186 (1960).

[5]A fuller account of the thermal stabilization will be available in a subsequent article.

[6]M. A. Heald and C. B. Wharton, Plasma Diagnostics with Microwaves, (John Wiley & Sons, Inc., New York, 1965), pp. 285-286, 300-302.

[7]V. L. Ginzburg and V. V. Zhelezniakov, Astron. Zh. 35, 694 (1958), and 36, 233 (1959).

PHONON SPECTRUM CHANGES IN SMALL PARTICLES AND THEIR IMPLICATIONS FOR SUPERCONDUCTIVITY*

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We have used molecular-dynamic techniques on bulk crystals and particles several atom-layers thick to study phonon behavior. It is found that the surface modes in the small particles produce changes in the phonon spectrum quantitatively large enough to explain the observation by Strongin and co-workers of large increases in the superconducting transition temperature of extremely thin films.

Strongin and co-workers[1] have observed increases in the transition temperature T_c of thin composite films consisting of alternating layers of dissimilar metals. It was estimated that the regions of film which exhibit the greatest increase in T_c are within a few monolayers of the metal-vacuum or metal-metal surface, and it was suggested that the increase in T_c is a consequence of an increase in the electron-phonon coupling which results from a lowering of the phonon frequencies in the ultrathin films. Strongin et al. applied the McMillan[2] expression for T_c in the form

$$T_c = \frac{\Theta_D}{1.45} \exp\left[-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right],$$

where Θ_D is the Debye Θ, μ^* is the Coulomb pseudopotential (~0.1), and the quantity λ involves the phonon frequency spectrum and the electronic matrix elements (approximately proportional to $1/\langle\omega^2\rangle_M$). They estimated that a frequency shift of the McMillan average squared frequency,

$$\langle\omega^2\rangle_M = \langle\omega\rangle/\langle 1/\omega\rangle,$$

of about 30 % would account for their Al and Sn results. This change in $\langle\omega^2\rangle_M$ could arise in several ways: an irregular arrangement of the atoms when first deposited on a foreign substrate, a density decrease in the first few layers, or the introduction of surface modes as the crystal becomes more two-dimensional. While all these effects are qualitatively possible explanations, it requires a realistic calculation to distinguish the major effect. Using the molecular-dynamic technique to simulate the atomic motions, we have calculated the frequency spectrum in bulk crystals and small particles. We find that the low-frequency modes in small particles result in a change in shape of the frequency spectrum which is mainly responsible for the large reduction in