observations may be made from Figs. 1 and 2: Emission from low exciton number gives spectra with quite high kinetic energies, consistent qualitatively with the high-energy tails observed in neutron spectra and in excitation functions for reactions induced by medium-energy projectiles. The spectra approach the equilibrium distribution as $n \to \bar{n}$. The example of an equilibrium spectrum shown in Fig. 2 was calculated with Eq. (9) for the system described above, with $g = 10$ MeV^{-1} (for which $\bar{n} \simeq 12$).

The spectral distribution is shown in Fig. 2 for $n = 21$, a distribution which gives very much lower kinetic energies than the equilibrium values. Such a situation could arise in heavy-ion reactions, i.e., a situation where the initial exciton number is far in excess of the equilibrium value. For example, in forming $A = 160$ nuclei at 50-MeV excitation via a Ne^{20}-induced reaction with $g \simeq 10$ MeV^{-1}, $\bar{n} \simeq 22$. If the interaction of the projectile with the nucleus is strictly as in a single-particle model, the initial exciton number could be as high as 60, far in excess of the equilibrium value. The dashed curves of Fig. 1 represent the relative emission probabilities approaching $\bar{n}$ from above [e.g., Eq. (9) evaluated from $n = 60$ to $\bar{n} = 22$], showing a possibility of significant precompound emission. In this case, the precompound spectra may show a considerably lower kinetic energy than the equilibrium value, which is the same qualitative result obtained from the old statistical theory when angular momentum effects are considered to lead to rotational cooling. Thus, this model suggests an alternative explanation for such an effect, rendering certain types of heavy-ion experiments ambiguous in interpretation.

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[1]J. J. Griffin, Phys. Rev. Letters 17, 478 (1966).

[2]P. Morrison, in Experimental Nuclear Physics, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1953), Vol. II, p. 48.

[3]V. F. Weisskopf, Phys. Rev. 52, 295 (1937).

[4]A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524 (1955).

SCATTERING OF PROTONS BY DEUTERIUM AND HELIUM*

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We present results of detailed spin- and isospin-dependent analyses of pd and p-^{4}He elastic-scattering intensities, polarizations, and total cross sections. The basic nucleon–nucleon scattering amplitudes used yield nucleon-nucleon observables in excellent agreement with measurements, including those of the elastic-scattering intensity, polarization, and spin correlation. Contributions of multiple scatterings, including multiple charge-exchange collisions, produce considerable structure in the predicted polarizations.

There has been much theoretical and experimental interest lately in scattering of medium- and high-energy particles by few-nucleon systems.[1-11] Extensive measurements were recently made at the Brookhaven Cosmotron where 1-GeV protons collided with a number of different target nuclei.[10] Intensities for elastic scattering by deuterium and ^{4}He were among the observables measured. The most striking and surprising property of these intensities was the virtual absence of a minimum in the pd angular distribution for four-momentum transfers $t \gtrsim -1.4$ (GeV/$c)^2$ contrasted to the appearance of a rather deep and sharp minimum in the p-^{4}He angular distribution near $t \approx -0.24$ (GeV/$c)^2$. There has been no satisfactory explanation of this phenomenon. However, it has been conjectured[6] that the spin dependence of the basic nucleon-nucleon (NN) scattering amplitudes might perhaps help solve this puzzling feature. We wish to present calculations which illustrate the influence of that spin dependence upon pd and p-^{4}He intensities, polarizations, and total cross sections.

Most of the recent analyses[1-8] of collisions between particles with kinetic energies ≥1 GeV and light nuclei have been made by means of the

Glauber approximation[12] with spin- and isospin-independent particle-nucleon scattering amplitudes. The basic techniques for including the spin dependence were presented some time ago,[1,5] and more recently the significance of isospin dependence was investigated.[4,5] It is well known[9] that at 1 GeV np observables, such as the total cross section, the ratio of real to imaginary parts of the forward spin-independent elastic scattering amplitude, and the spin-dependent contributions to the forward elastic scattering intensity, differ rather markedly from the corresponding pp observables. It should be clear, therefore, that multiple charge-exchange collisions[4,5] will be significant in proton-nucleus collisions at 1 GeV. Furthermore, since the pp polarization and spin correlation (C_{nn}) are measured[13] to be quite large near 1 GeV, the significance of spin-independent analyses of proton-nucleus collisions at this energy is unclear,[14] particularly where single- and multiple-scattering amplitudes interfere appreciably to produce minima in the scattering intensities. Consequently, in the present work we shall include both the spin and isospin dependence in the basic NN amplitudes.

The pp scattering amplitudes which we use have been obtained from a detailed phase-shift analysis by Hoshizaki[15,16] of measurements of the pp elastic scattering intensity[17] (including the intensity in the small-angle Coulomb-interference region), polarization,[13] and spin correlation.[13] The quality of the fits to these data given by the phase-shift analysis is illustrated in Fig. 1 by the fit to the elastic scattering intensity.[18] The fits to the polarization and spin-correlation measurements are as good. Hoshizaki obtains a total χ^2 of 55.9 for 60 data points.[16]

Very few np measurements have been made at 1 GeV. There do exist data for the np total cross section, ratio of real to imaginary parts of the forward spin-independent elastic scattering amplitude, and spin-dependent contribution to the forward elastic scattering intensity.[9] For the five np amplitudes we shall assume complex Gaussian functions of the momentum transfer q [say, $a_i \exp(-\alpha_i q^2)$, where a_i and α_i are complex], or q or q^2 multiplying such functions.[19] The data[9] determine two of the amplitudes and the (complex) value of a third in the forward direction. Five complex np parameters remain undetermined.

In order to reduce the number of undetermined np parameters, we first fit the five pp amplitudes obtained earlier from the phase-shift analysis with analytic forms identical to those assumed for the five np amplitudes. This yields values for the ten complex pp parameters corresponding to the ten complex np parameters. We then set two of the five undetermined complex np parameters equal to the values obtained for the corresponding pp parameters, and we set the phases of the remaining three undetermined complex np parameters equal to the values obtained for the phases of the corresponding pp parameters. The only undetermined quantities then remaining are the magnitudes of three of the complex parameters, and these will be the only free parameters in our calculations of the pd and p-^{4}He elastic-scattering intensities, polarizations, and total cross sections.

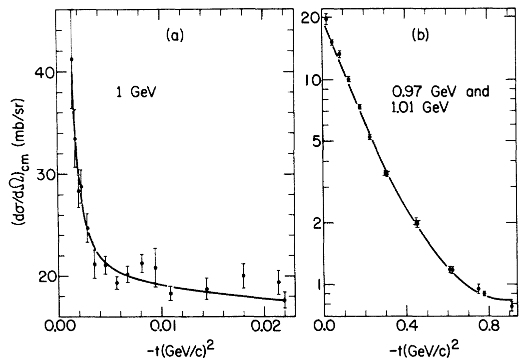

FIG. 1. Proton-proton elastic scattering differential cross sections in the angular regions (a) 3°-12.5° c.m. and (b) 12.3°-90° c.m. The curves are calculated from a phase-shift analysis by Hoshizaki, Ref. 16. The measurements are from (a) Dowell et al., Ref. 17, (b) Dowell et al., Ref. 17, and McFarlane et al., Ref. 17. t is the four-momentum transfer.

The scattering amplitude operator for pd or p-^{4}He elastic collisions may be written as a sum of a single- and a multiple-scattering operator.[20] For p-^{4}He elastic collisions, the single-scattering operator contains a spin-independent amplitude and a spin-dependent amplitude, the latter of which vanishes in the forward direction. For pd elastic collisions, the single-scattering operator contains a spin-independent amplitude and five spin-dependent amplitudes. Two of the spin-dependent amplitudes vanish in the forward direction. The single-scattering operators are rather easy to construct and may be expressed very simply in terms of the NN amplitudes.[1] Therefore, these NN amplitudes, which we have described, may (and should) be used directly in the calculation of the single-scattering amplitudes.

For p-^{4}He elastic collisions, the multiple-scattering operator contains a spin-independent amplitude and a spin-dependent amplitude, the latter of which vanishes in the forward direction. We should point out that the p-^{4}He spin-independent amplitude (as well as the spin-dependent one) is a function of all the NN amplitudes, both the spin-independent and the spin-dependent ones. For pd elastic collisions the multiple-scattering operator contains a spin-independent amplitude and eleven spin-dependent amplitudes. Thus six of the twelve pd scattering amplitudes are multiple-scattering amplitudes and do not contribute at all to single collisions. Six of the eleven pd spin-dependent double-scattering amplitudes vanish in the forward direction. The multiple-scattering operators for pd and p-^{4}He collisions contain the NN amplitudes within quite complicated multidimensional integrals. The calculation of these integrals is simplified if the NN amplitudes can be expressed as Gaussian functions of q or as polynomials in q multiplying such functions. These are the forms taken by the np amplitudes we have described, and the pp amplitudes of Hoshizaki have been previously fitted by these forms. Consequently, we shall use these forms in the evaluation of the double-, triple-, and quadruple-scattering operators.[21]

For the deuteron spatial wave function we use the most accurate fit of Moravcsik to the Gartenhaus S-state wave function.[22] The ground-state density for ^{4}He is written as a product of single-nucleon densities $\rho_0 \exp(-r_i^2/R^2)$, with $R = 1.39$ F which is the value obtained by applying center-of-mass and finite-proton-size corrections to recent measurements of the ^{4}He charge distribution.[23]

The results of our calculations for pd and p-^{4}He elastic scattering intensities are shown in Fig. 2, where the contributions from only single scattering are also presented for comparison. It is seen that the ^{4}He calculation produces quite a deep and sharp minimum, whereas the deuteron calculation does not. If the spin-dependent amplitudes were set equal to zero, the calculated pd intensities at $-t = 0.34$ (GeV/c)^2 and $-t = 0.62$ (GeV/c)^2 would be ~0.008 and ~0.15 mb/sr, respectively, resulting in a very deep minimum. The underestimate of the p-^{4}He calculation at scattering angles greater than those near the secondary maximum is typical of calculations using Gaussian single-nucleon densities. More accurate (and complicated) densities tend to increase the calculated intensities for 0.3 (GeV/c)$^2 \lesssim -t \lesssim 0.6$ (GeV/c)^2.[6] It is worth pointing out that the multiple-scattering effects in p-^{4}He scattering are very large, even near the forward direction, so that great care should be taken in any single-scattering impulse-approximation analysis of, say, π-^{4}He scattering which may be used in attempting to determine the rms radius of the ^{4}He nucleus.

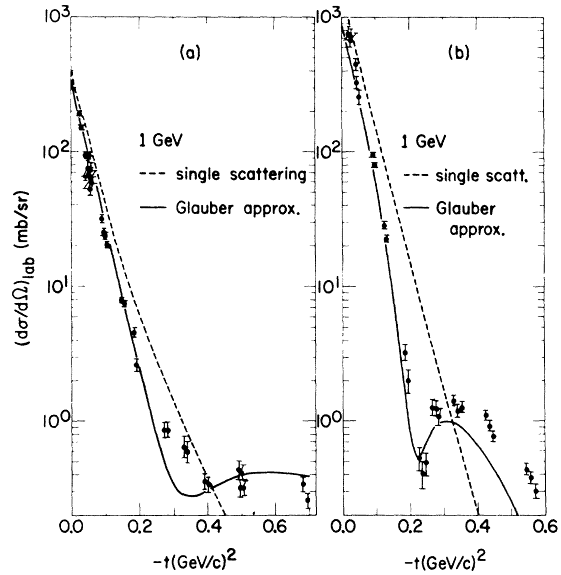

FIG. 2. Elastic scattering differential cross sections in the angular region 0°-26° lab for (a) pd collisions and (b) p-^{4}He collisions. All curves are calculated from a spin- and isospin-dependent NN scattering amplitude. The dashed curves are obtained by considering only single-scattering effects, whereas the solid curves include (a) double-scattering effects and (b) double-, triple-, and quadruple-scattering effects. The measurements are from (a) Bennett et al., Ref. 10, and (b) Palevsky et al., Ref. 10.

Measurements of pd and p-^{4}He polarizations would be extremely valuable for a more complete understanding of scattering by these target nuclei.[24] As one might expect, the influence of multiple collisions, which is so marked in the elastic scattering intensities, is also quite appreciable in the polarizations. In Fig. 3 we present calculations of pd and p-^{4}He polarizations at 1 GeV together with the contributions of single scattering only. We note that the inclusion of multiple-scattering effects results in polarizations with considerable structure, the principal features of which are maxima at relatively small momentum transfers followed by minima in regions where the single- and double-scattering amplitudes interfere appreciably. In the p-^{4}He polarization the interference minimum near $-t$

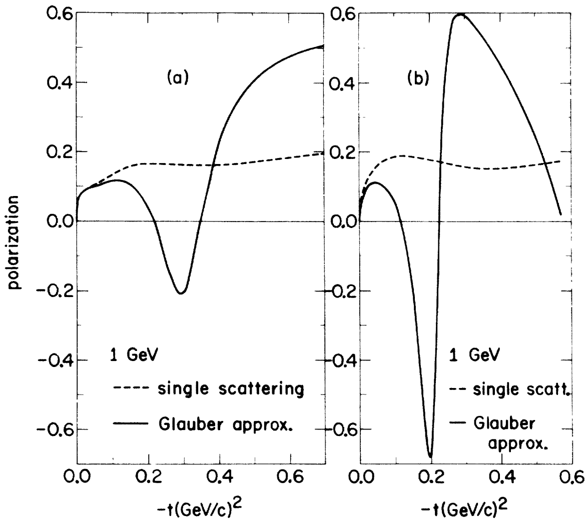

FIG. 3. Polarizations in the angular region 0°-26° lab for (a) *pd* collisions and (b) p-^{4}He collisions. All curves are calculated from a spin- and isospin-dependent *NN* amplitude. The dashed curves are obtained by considering only single-scattering effects, whereas the solid curves include (a) double-scattering effects and (b) double-, triple-, and quadruple-scattering effects.

≈ 0.19 $(\text{GeV}/c)^2$ is followed by a maximum near $-t \approx 0.28$ $(\text{GeV}/c)^2$ produced mainly by the double-scattering amplitudes. The same is true of the *pd* polarization at momentum transfers greater than those shown in Fig. 3(a). For $-t \gtrsim 0.6$ $(\text{GeV}/c)^2$ the p-^{4}He polarization might be expected to reveal an additional minimum.

Although the precise quantitative predictions of the polarizations depend upon the *NN* parameters, some of which may not be very accurately known, even crude *pd* and p-^{4}He polarization measurements at 1 GeV could test the interesting qualitative features of the calculations. More accurate polarization measurements would be invaluable in helping to determine some of the *NN* parameters with greater precision. For a better understanding of *pd* and p-^{4}He scattering, it would be more useful to have *pd* and p-^{4}He intensity and polarization measurements at one or two energies than to have only intensity measurements at a number of energies.

By virtue of the optical theorem it is easy to determine the theoretical total cross sections. The *pd* total cross section is calculated to be 83.74 mb when both single and double scatterings are included and the *NN* spin dependence neglected, and 83.46 mb when both single and double scatterings and the *NN* spin dependence are considered. The measured value is[25] 83.04 ± 0.06 mb (error quoted is statistical only). The p-^{4}He total cross section is calculated to be 143.7 mb when single through quadruple scatterings are included and the *NN* spin dependence neglected, and 137.5 mb when single through quadruple scatterings and the *NN* spin dependence are considered. The measurements of Igo et al.[10] yield 152 ± 8 mb. We see that the effect of the *NN* spin dependence on the *pd* total cross section is to decrease it by only ~0.3 %, whereas its effect on the p-^{4}He cross section is to decrease it by ~4 %. We might mention that no parameters were adjusted to try to fit the *pd* and p-^{4}He total cross-section measurements. The nucleon-nucleus amplitudes used were simply those previously obtained by fitting the elastic scattering intensities.

After the completion of this work, an attempt to explain the *pd* and p-^{4}He elastic scattering intensities was published.[26] Unfortunately the basic *NN* amplitudes used are inconsistent with *NN* measurements. Furthermore, two of the five *NN* amplitudes (denoted as G_{NN} and H_{NN} by Kujawski, Sachs, and Trefil[26]) were taken to be equal since it was claimed that there is no evidence to the contrary. In addition a third amplitude (denoted by B_{NN} in Ref. 26) was taken equal to $\frac{1}{2}(G_{NN}+H_{NN})$ since the equality is valid to $O(\theta)$, where θ is the scattering angle. These assumptions may be tested by the phase-shift analysis of Hoshizaki[15,16] and are found to be inaccurate.[27] The discrepancies arising from these assumptions may help account for the appearance of a maximum in the calculated *pd* elastic scattering intensity of Ref. 26 at a momentum transfer [$-t \approx 0.39$ $(\text{GeV}/c)^2$] where the data [see Fig. 2(a) of the present paper] perhaps show signs of a shallow minimum.

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[1]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[2]V. Franco, Phys. Rev. Letters 16, 944 (1966).

[3]V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966).

[4]C. Wilkin, Phys. Rev. Letters 17, 561 (1966).

[5]R. J. Glauber and V. Franco, Phys. Rev. 156, 1685 (1967).

[6]R. H. Bassel and C. Wilkin, Phys. Rev. 174, 1179

(1968).
[7]R. J. Glauber, in High Energy Physics and Nuclear Structure, edited by G. Alexander (North-Holland Publishing Company, Amsterdam, The Netherlands, 1967), p. 311; W. Czyż and L. Lesniak, Phys. Letters 24B, 227 (1967); R. Bassel and C. Wilkin, Phys. Rev. Letters 18, 871 (1967); L. Bertocchi and A. Capella, Nuovo Cimento 51A, 369 (1967); W. Czyż and L. Maximon, to be published; D. Harrington, to be published.
[8]See also V. Franco, Los Alamos Scientific Laboratory Report No. LADC-9663 (to be published) for a theory of nucleus-nucleus collisions and applications to *dd* scattering.
[9]D. V. Bugg et al., Phys. Rev. 146, 980 (1966); L. M. C. Dutton, R. J. W. Howells, J. D. Jafar, and H. B. Van der Raay, Phys. Letters 25B, 245 (1967).
[10]H. Palevsky et al., Phys. Rev. Letters 18, 1200 (1967); G. W. Bennett et al., Phys. Rev. Letters 19, 387 (1967); G. J. Igo et al., Nucl. Phys. B3, 181 (1967); see also E. Coleman, R. M. Heinz, O. E. Overseth, and D. E. Pellett, Phys. Rev. 164, 1665 (1967).
[11]E. T. Boschitz et al., Phys. Rev. Letters 20, 1116 (1968).
[12]R. J. Glauber, in Lectures in Theoretical Physics, edited by W. E. Brittin et al. (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.
[13]G. Cozzika et al., Phys. Rev. 164, 1672 (1967).
[14]We do not wish to imply that such analyses are without meaning. Many interesting qualitative properties of the collision processes and target nuclei have been found in those analyses.
[15]Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) 31, 615 (1964), and 31, 1162 (1964); N. Hoshizaki, Rev. Mod. Phys. 39, 700 (1967).
[16]N. Hoshizaki, private communication.
[17]J. D. Dowell et al., Nuovo Cimento 18, 818 (1960), and Phys. Letters 12, 252 (1964); W. K. McFarlane et al., Nuovo Cimento 28, 943 (1963).
[18]Similar fits may be seen in Ref. 15.
[19]Which of the three forms we use is determined by the angle dependence of a given amplitude near the forward direction. Each of the five amplitudes is parametrized with two complex numbers.
[20]The *pd* multiple-scattering operator consists of a double-scattering contribution, whereas the corresponding p-^{4}He operator consists of double-, triple-, and quadruple-scattering contributions.
[21]The accuracy of this approach is aided by the property of multiple scattering, described in Ref. 3 for *pd* collisions, that n-tuple scatterings with a resultant net four-momentum transfer t consist mainly of n single scatterings each involving four-momentum transfers $\approx t/n^2$. Thus, a double scattering at t depends mainly on the NN amplitude near $\frac{1}{4}t$.
[22]M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).
[23]R. F. Frosch et al., Phys. Rev. 160, 874 (1967).
[24]Such measurements are presently being made at Space Radiation Effects Laboratory with 600-MeV protons.
[25]Bennett et al. (Ref. 10) obtained from data of Bugg et al. (Ref. 9).
[26]E. Kujawski, D. Sachs, and J. Trefil, Phys. Rev. Letters 21, 583 (1968).
[27]For example, at $-t=0.1$ (GeV/c)^2, $H_{NN}/G_{NN}\approx 2+2i$ and $\frac{1}{2}(G_{NN}+H_{NN})/B_{NN}\approx -4$. To approximate each of these two complex ratios by +1 seems quite inaccurate. Furthermore, a fourth amplitude was obtained by taking a Regge-pole analysis (which ignores isospin dependence) of pp and $\bar{p}p$ data between 5 and 25 GeV and extrapolating it down to 1 GeV.

TEST OF THE EQUIVALENCE PRINCIPLE FOR UNSTABLE ELEMENTARY PARTICLES*

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It is argued that a sufficiently fast, charged, nongravitating particle would radiate strongly in a classical gravitational field. Subject to some assumptions, the possibility that the muon does not couple to gravitation is ruled out on the basis of existing data.

I would like to present a calculation which predicts that a sufficiently energetic, electrically charged, nongravitating[1] particle, in the presence of a classical gravitational tensor potential, would radiate photons in a manner similar to that of the Čerenkov effect. The predicted radiation rate is sufficiently high that the very existence of a particle with velocity above a certain threshold for practical laboratory times constitutes evidence that this particle couples to gravitation.

I think that this effect is of little importance in the case of the particles which comprise "normal" matter, and the concern here is with the unstable particles, such as the muon. For such particles, to my knowledge there are at present no "direct" measurements of gravitational properties, e.g., the observation of deflection in the earth's gravitational field, nor in my opinion are any likely until some unforseen technological advance occurs. Some indirect evidence exists and is discussed at the end of this report.

For the purpose at hand, one must assume for