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[1]D. Kurath and L. Picman, Nucl. Phys. 10, 313 (1959); R. D. Lawson, Phys. Rev. 124, 1500 (1961); J. D. McCullen, B. F. Bayman, and L. Zamick, ibid. 134, B515 (1964); M. R. Gunye, Phys. Letters 27B, 136 (1968).

[2]J. C. Parikh and J. P. Svenne, Phys. Rev. 174, 1343 (1968).

[3]C. M. Shakin, Y. R. Waghmare, M. Tomaselli, and M. H. Hull, Jr., Phys. Rev. 161, 1015 (1967).

[4]C. G. Adler, M. K. Banerjee, and G. J. Stephenson, Jr., Bull. Am. Phys. Soc. 13, 581 (1968), and to be published; M. K. Banerjee, private communication.

[5]E. C. Halbert, J. B. McGrory, and B. H. Wildenthal, Phys. Rev. Letters 20, 1112 (1968); J. B. McGrory, private communication.

[6]A. P. Stamp, Nucl. Phys. A105, 627 (1968); G. Ripka, Fundamentals in Nuclear Theory (International Atomic Energy Agency, Vienna, Austria, 1967).

# EXTENSIONS OF GRIFFIN'S STATISTICAL MODEL FOR MEDIUM-ENERGY NUCLEAR REACTIONS*

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The new statistical model due to Griffin has been extended to include charged-particle emission by evaluation of the transition matrix in terms of a relative velocity and scattering cross section. Additionally a quite different formulation is derived for the equilibrium emission probability, whereas the precompound emission probability is similar to Griffin's result.

A new statistical model for medium-energy reactions has recently been proposed[1] in which a precompound decay probability may be computed at each stage of the approach of the initial system to the long-lived equilibrium state, in addition to the decay probability of the equilibrium state. The new model differs further from the earlier statistical model in that all transitions are assumed to proceed via weak two-body interactions in a single-particle–model nucleus; the details of the two-body interaction are suppressed within an average and constant transition matrix element $|M|$ for transitions in which the particle-plus-hole number remains constant or changes by ±2 units, while it is assumed that $|M|=0$ for all other transitions.

In this note, an extension of the new statistical model is presented in which $|M|$ is evaluated in terms of a relative velocity between particles $v$ and a scattering cross section $\sigma(v)$ for the process. This results in a formulation in which both compound and precompound probabilities have a different velocity dependence than in the earlier formulations, and introduces an inverse cross section permitting application to reactions involving charged-particle emission. The derivations are based on and follow Griffin's work closely; the precompound result differs mainly in the velocity dependence of the outgoing particle, whereas the compound emission probability yields basic disagreement with the earlier formulation.

Derivations are presented first for the precompound emission probability, then for the compound emission probability, following which a short discussion of the application of the model to the interpretation of experimental results is presented. Following Griffin,[1] particles and holes are not differentiated in the model, but are referrred to simply as excitons, represented by exciton number $n$. With an average single-particle–level density $g$, the density of $n$-exciton states at excitation $E$ is

$$\rho_n(E)=g(gE)^{n-1}/n!(n-1)!. \quad (1)$$

The total density of states at excitation $E$ is

$$\rho(E)=\sum_{n\geq 3}\rho_n(E), \quad (2)$$

where $n$ is taken at 2-unit intervals corresponding to the selection rule imposed by the assumed two-body interaction. The emitting-nucleus excitation will be represented by $E$, the residual-nu-

cleus excitation by $U$, particle-channel energy by $\epsilon$, and momentum by $p$.

Assume a nucleus excited initially into a 3-particle and -hole (or 3-exciton) state, progressing via successive two-body collisions to an equilibrium distribution centered about a most probable exciton number $\bar{n}$. Assume that each $n$-exciton state along the way has a small probability for decay into the continuum with respect to the probability of scattering into an $(n+2)$-exciton state, which in turn is large with respect to the probability of scattering into an $n$- or $(n-2)$-exciton state so long as $n<\bar{n}$.

The decay probability per unit time in going to an $n$-exciton state may be calculated from perturbation theory:

$$\omega_n = 1/\tau_n = (2\pi/\hbar)|M|^2\rho_n(E), \tag{3}$$

where $\tau_n$ is the lifetime, $|M|^2$ is the square of the transition matrix element, and $\rho_n(E)$ is the density of $n$-exciton states. For a collision between two particles of relative velocity $v$, in the volume $\Omega$, $\omega = v\sigma(v)/\Omega$ where $\sigma(v)$ is the cross section for the interaction, appropriately averaged over all angles.[2] Solving (3) for $|M|^2$ with this assumption yields

$$|M|^2 = \hbar v\sigma(v)/12\pi\rho_n(E)\Omega. \tag{4}$$

The density of states (in energy units) for an exciton state with the restriction that one exciton is in the continuum with channel energy between $\epsilon$ and $\epsilon+d\epsilon$ may be written as

$$\rho_{n-1}(U)\frac{4\pi p^2\Omega}{(2\pi\hbar)^3}\frac{dp}{d\epsilon}d\epsilon. \tag{5}$$

If (5) is substituted into Eq. (3) in place of the total $n$-exciton density of states $\rho_n(E)$, the decay probability to the restricted set of $n$-exciton states represented in (5) may be calculated. If in addition the value for $|M|^2$ calculated in Eq. (4) is substituted into the new equation, the precompound decay probability per unit time for emission of an exciton with channel energy between $\epsilon$ and $\epsilon+d\epsilon$ may be written

$$\omega_{p,n}(\epsilon)d\epsilon = \frac{1}{\tau_n} = \frac{m\epsilon\sigma(v)}{\pi^2\hbar^3}\frac{\rho_{n-1}(U)}{\rho_n(E)}d\epsilon. \tag{6}$$

The total precompound emission probability for an exciton of channel energy $\epsilon$ to $\epsilon+d\epsilon$ may then be obtained by summing Eq. (6) from the initial exciton number to the most probable equilibrium value, $\bar{n}=(gE)^{1/2}$, each $n$ differing by two units. Inclusion of the statistical degeneracy of the emitted particle, $2s+1$, and substitution of Eq. (1) for $\rho_{n-1}(U)$ and $\rho_n(E)$ yields the total precompound emission rate,

$$\omega_p(\epsilon)d\epsilon = \frac{(2s+1)m\epsilon\sigma(v)E}{\pi^2\hbar^3gU^2}\sum_{n\geq 3}^{\bar{n}}\left(\frac{U}{E}\right)^n(n^2-n)d\epsilon. \tag{7}$$

The compound emission probability is computed in the same manner as Eqs. (6) and (7), except that the decay probability of an $n$-exciton state is multiplied by the fraction of all states which may decay to an $n$-exciton state (those of $n$ or $n\pm 2$ excitons) and the sum is extended over all exciton numbers (even or odd), giving the compound emission probability

$$\omega_c(\epsilon)d\epsilon = \frac{(2s+1)m\epsilon\sigma(v)}{\pi^2\hbar^3}\sum_{n\geq 3}\frac{\rho_{n-1}(U)}{\rho_n(E)}\left[\frac{\rho_{n+2}(E)+\rho_n(E)+\rho_{n-2}(E)}{\rho(E)}\right]d\epsilon. \tag{8}$$

Substitution of Eq. (1) followed by evaluation of the resulting power series yields the result

$$\omega_c(\epsilon)d\epsilon = \frac{(2s+1)m\epsilon\sigma(v)\rho(U)}{\pi^2\hbar^3\rho(E)}\left[1+\left(\frac{U}{E}\right)^2+\left(\frac{E}{U}\right)^2\right]d\epsilon, \tag{9}$$

where

$$\rho(E)\propto E^{-1}\exp[2(gE)^{1/2}]. \tag{10}$$

Equation (9) differs from the Weisskopf formulation[3] solely in the quantity in the square brackets. If $E\gg U$, Eq. (9) may be simplified further,

$$\omega_c(\epsilon)d\epsilon = \frac{(2s+1)m\epsilon\sigma(v)U^{-3}\exp[2(gU)^{1/2}]}{\pi^2\hbar^3E^{-3}\exp[2(gE)^{1/2}]}d\epsilon. \tag{11}$$

Equations (7) and (9) are then assumed to sum incoherently to give the total emission probability for channel energy $\epsilon$,

$$\omega(\epsilon) = \alpha\omega_p(\epsilon) + (1-\alpha)\omega_c(\epsilon), \tag{12}$$

where the fraction of precompound decays $\alpha$ is assumed to be small.

The scattering cross section $\sigma(v)$ may probably be approximated by optical-model nonelastic cross sections, with precisely the same uncertainty as arises from use of ground-state rather than excited-state cross sections in the statistical model of Weisskopf. This follows since the process of a projectile entering a nucleus and being absorbed following a two-body collision should be the inverse of the scattering process within the nucleus for the time-reversed process. Further discussion of this point may be found in the work of Lane and Wandel,[4] who calculate the imaginary part of the optical potential from averages over nucleon-nucleon scattering cross sections.

Some of the implications of Griffin's model are illustrated in Figs. 1 and 2. Figure 1 illustrates the relative contributions to precompound emission as a function of exciton number and $U/E$. It may be seen that for low values of $U/E$, precompound emission must take place after very few scattering events, or not at all, whereas for higher $U/E$ (corresponding to emission of the first particle from a compound system at quite high excitation) the maximum in the distribution does not occur until after several scattering events with an extremely slow decrease with increasing exciton number following the maximum. The relative neutron energy spectra for emission from states of several different exciton numbers are shown in Fig. 2, where the curves shown represent the product of $\epsilon$ and the appropriate terms of the sum in Eq. (7). The spectra were calculated for a nucleus initially excited to 20 MeV, with 5-MeV neutron binding energy. These results are independent of mass number and average level spacing of the emitting nucleus. The following

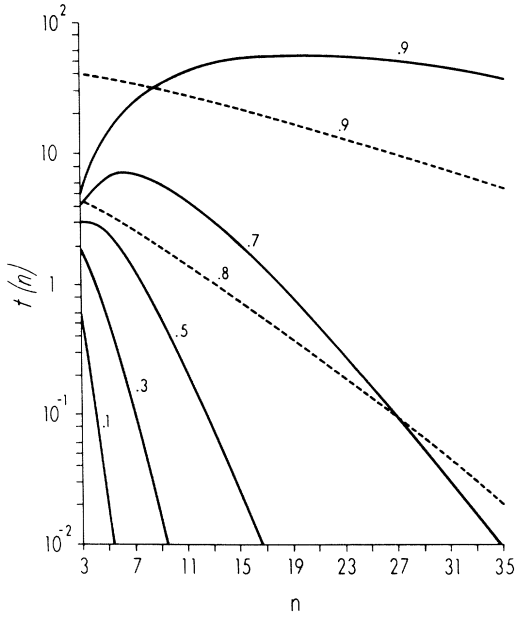


FIG. 1. Relative contribution to precompound emission as a function of exciton number, $n$, and excitation ratio, $U/E$. For the solid curves the ordinate represents relative values of the terms of Eq. (7) for the exciton numbers given by the abscissa; the numbers above the curves are the ratios of excitation of product nucleus to emitting nucleus. The dashed curves have the same significance except that the abscissa represents the difference between the exciton number of the emitting nucleus and the average equilibrium exciton number, $\bar{n}$. These curves would be relevant to situations in which the initial exciton number exceeded the average equilibrium exciton number, and were computed for a nucleus of $g = 10$ MeV$^{-1}$, $E = 50$ MeV.

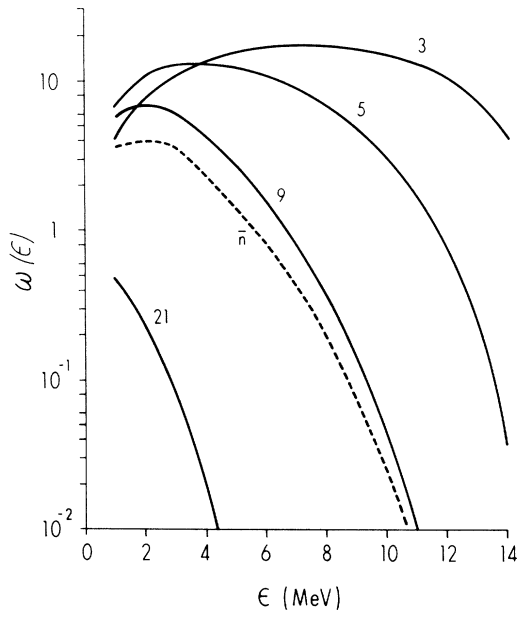


FIG. 2. Relative spectral distributions for neutrons for emission to states of various exciton number. The ordinate represents the emission probabilities calculated with Eq. (7) as a function of neutron kinetic energy. The numbers above the solid curves represent the exciton numbers of the final states involved. The dashed curve with the $\bar{n}$ denotation was calculated with Eq. (11) for an equilibrium distribution. The emitting nucleus parameters are described in the text. The dashed curve has an arbitrary normalization factor.

observations may be made from Figs. 1 and 2: Emission from low exciton number gives spectra with quite high kinetic energies, consistent qualitatively with the high-energy tails observed in neutron spectra and in excitation functions for reactions induced by medium-energy projectiles. The spectra approach the equilibrium distribution as $n \to \bar{n}$. The example of an equilibrium spectrum shown in Fig. 2 was calculated with Eq. (9) for the system described above, with $g = 10$ MeV$^{-1}$ (for which $\bar{n} \simeq 12$).

The spectral distribution is shown in Fig. 2 for $n = 21$, a distribution which gives very much lower kinetic energies than the equilibrium values. Such a situation could arise in heavy-ion reactions, i.e., a situation where the initial exciton number is far in excess of the equilibrium value. For example, in forming $A = 160$ nuclei at 50-MeV excitation via a $Ne^{20}$-induced reaction with $g \simeq 10$ MeV$^{-1}$, $\bar{n} \simeq 22$. If the interaction of the projectile with the nucleus is strictly as in a single-particle model, the initial exciton number could be as high as 60, far in excess of the equilibrium value. The dashed curves of Fig. 1 represent the relative emission probabilities approaching $\bar{n}$ from above [e.g., Eq. (9) evaluated from $n = 60$ to $\bar{n} = 22$], showing a possibility of significant precompound emission. In this case, the precompound spectra may show a considerably lower kinetic energy than the equilibrium value, which is the same qualitative result obtained from the old statistical theory when angular momentum effects are considered to lead to rotational cooling. Thus, this model suggests an alternative explanation for such an effect, rendering certain types of heavy-ion experiments ambiguous in interpretation.

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[1]J. J. Griffin, Phys. Rev. Letters 17, 478 (1966).

[2]P. Morrison, in Experimental Nuclear Physics, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1953), Vol. II, p. 48.

[3]V. F. Weisskopf, Phys. Rev. 52, 295 (1937).

[4]A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524 (1955).

## SCATTERING OF PROTONS BY DEUTERIUM AND HELIUM*

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We present results of detailed spin- and isospin-dependent analyses of $pd$ and $p$-$^4$He elastic-scattering intensities, polarizations, and total cross sections. The basic nucleon-nucleon scattering amplitudes used yield nucleon-nucleon observables in excellent agreement with measurements, including those of the elastic-scattering intensity, polarization, and spin correlation. Contributions of multiple scatterings, including multiple charge-exchange collisions, produce considerable structure in the predicted polarizations.

There has been much theoretical and experimental interest lately in scattering of medium- and high-energy particles by few-nucleon systems.[1-11] Extensive measurements were recently made at the Brookhaven Cosmotron where 1-GeV protons collided with a number of different target nuclei.[10] Intensities for elastic scattering by deuterium and $^4$He were among the observables measured. The most striking and surprising property of these intensities was the virtual absence of a minimum in the $pd$ angular distribution for four-momentum transfers $t \gtrsim -1.4$ (GeV/$c$)$^2$ contrasted to the appearance of a rather deep and sharp minimum in the $p$-$^4$He angular distribution near $t \approx -0.24$ (GeV/$c$)$^2$. There has been no satisfactory explanation of this phenomenon. However, it has been conjectured[6] that the spin dependence of the basic nucleon-nucleon ($NN$) scattering amplitudes might perhaps help solve this puzzling feature. We wish to present calculations which illustrate the influence of that spin dependence upon $pd$ and $p$-$^4$He intensities, polarizations, and total cross sections.

Most of the recent analyses[1-8] of collisions between particles with kinetic energies $\geq 1$ GeV and light nuclei have been made by means of the