In conclusion, our results as a whole indicate also that the rf-saturated spin resonance of the electrons in the cloud may be detectable by means of "assisted" Majorana flops connecting it with the cyclotron motion, the latter being strongly connected to the temperature-monitored z motion. Experiments towards this goal are in progress in our laboratory.

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EMISSION OF PULSE TRAINS BY Q-SWITCHED LASERS*

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Maxwell's equations and the two-level density-matrix equations are integrated numerically under conditions which simulate the operation of a Q-spoiled laser. The final output intensity pattern takes the form of a train of sharp pulses separated by the round-trip time.

The phenomenon of laser mode locking which results in periodic short-pulse emission has been demonstrated in a variety of experiments.[1] Particularly dramatic results have been obtained with lasers which are Q switched by means of bleachable dyes.[2] Recently, it has been demonstrated by the use of two-photon fluorescence techniques that short-pulse emission can result when Q switching is accomplished by other methods such as a rotating prism.[3] The possibility that such short pulses could arise as fluctuations of a Gaussian random process has been examined.[4] There is, on the other hand, considerable laboratory experience to indicate that pulsed lasers or lasers Q switched by electro-optic methods can spontaneously emit narrow pulses at round-trip time intervals.[5] In this communication, we report that the emission of such periodic pulse trains from lasers employing Q switches other than saturable absorbers can be predicted theoretically.

We take as the equations which describe the amplification of the laser field in a homogeneously broadened medium[6] the following, which are based on Maxwell's equations and the conventional two-level density-matrix equations:

$$E(z,t)=E^{+}(z,t)e^{i\omega(t-\eta z/c)}+E^{-}(z,t)e^{i\omega(t+\eta z/c)}+\text{c.c.},\tag{1a}$$

$$\rho(z,t)=\rho^{+}(z,t)e^{i\omega(t-\eta z/c)}+\rho^{-}(z,t)e^{i\omega(t+\eta z/c)},\tag{1b}$$

$$\frac{\eta}{c}\frac{\partial E^{\pm}}{\partial t}\pm\frac{\partial E^{\pm}}{\partial z}=-\tfrac{1}{2}\sigma\rho^{\pm},\tag{1c}$$

$$\partial\rho^{\pm}/\partial t+T_2^{-1}\rho^{\pm}=-T_2^{-1}nE^{\pm},\tag{1d}$$

$$\partial n/\partial t+T_1^{-1}(n-n_0)=2\sigma(\rho^{+}E^{+*}+\rho^{-}E^{-*}+\text{c.c.}).\tag{1e}$$

In Eq. (1a) the laser field $E(z,t)$ is expressed as the sum of two running waves moving in opposite directions inside the laser cavity. Both E^{+} and E^{-} are complex. The normalization of E is such that $2|E|^2$ is a measure of photon intensity. The carrier frequency ω is taken for convenience as the laser transition frequency. The variables ρ^{+} and ρ^{-} are proportional to the polarizations induced by the respective E waves, and n is the population difference. Also, η is the index of refraction, σ is the absorption cross section at the line center, T_1 and T_2 are the longitudinal and transverse relaxation times, and n_0 is a constant associated with the pumping rate. In writing Eq. (1e), we have neglected terms of the form $\rho^{+}E^{-*}\times e^{-2ikz}$ and $\rho^{-}E^{+*}e^{2ikz}$. In order to treat such terms, it is necessary to expand ρ^{+}, ρ^{-}, and n in power series in e^{2ikz}. This procedure, involving the retention of first-order terms, has been employed using a rate-equation approximation to study dye-switched lasers.[7] It can be shown, however, that the importance of the resulting terms omitted in Eqs. (1c)-(1e) is minimized if T_1 is long, e.g., the case of ruby, and if the most intense portions of the E^+ and E^- waves do not overlap in the amplifying region at powers capable of saturating gain. Both of these conditions are met in the calculations to be described here.

Equations (1) have been integrated numerically for some typical laser cavities, using reflection boundary conditions at the mirrors. Displayed in Figs. 1 and 2 are results for a cavity with round-trip time $2L/c = 3.3$ nsec and mirror reflectivities of 100 and 80%. The amplifying medium is assumed to be a 7.6-cm ruby rod placed with the end facing the 100% mirror at the center of the cavity and having 1.62×10^{19} active atoms/cc and a cross-section $\sigma = 2.5\times10^{-20}$ cm^{-2}. The only important departure from experimental conditions is that T_2 is taken to be 5.0×10^{-2} nsec. (Experimentally, $T_2 \sim 1.0\times10^{-3}$ nsec.) This choice of T_2 allows an accurate integration of Eqs. (1) with a time step $\Delta t = 0.49\times10^{-2}$ nsec and a division of the cavity into 336 spatial zones. Input noise sources are provided at both ends of the rod to simulate the effect of spontaneous emission into N cavity modes. The source functions take the form

$$E_S^{+}(z_1,t) = \sum_{n=-\frac{1}{2}N}^{\frac{1}{2}N} A_n e^{2\pi int/T} + \text{c.c.} \qquad (2)$$

and a similar expression for $E_S^{-}(z_2,t)$, where $T = 2L/c$ and z_1 and z_2 are the coordinates of the rod ends. The A_n are randomly phased (E_S^+, E_S^-

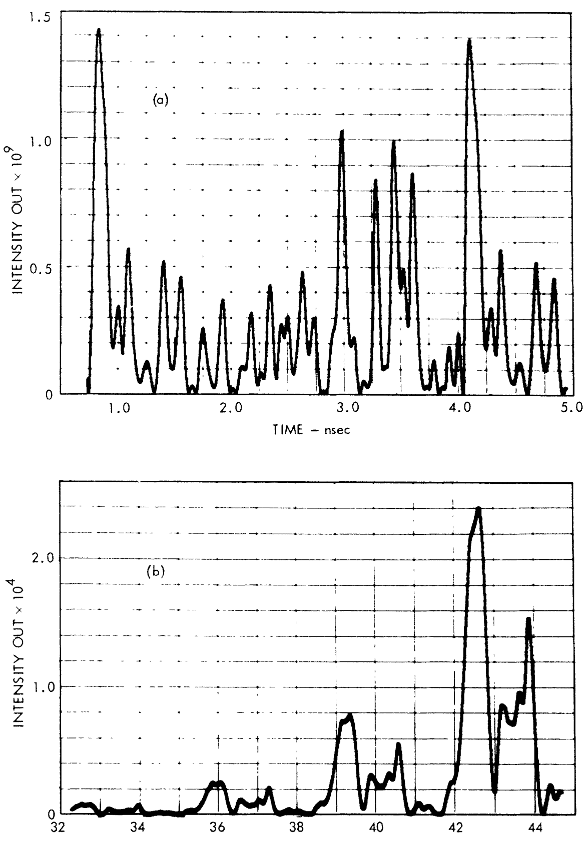

FIG. 1. Evolution of giant pulse from noise. (a) Random noise emission from laser. Mirror reflectivity is gradually switched on starting at time $t = 3.3$ nsec. Intensity is normalized to 1.62×10^{19} photons cm^2 nsec^{-1}. (b) Laser intensity showing the linear amplification of the laser noise over some four round trips. Even though the amplification is still linear, the peaking of the output intensity suggests quasi-mode-locked behavior. Other numerical examples have indicated an even more marked tendency to emit single pulses per round trip at a similar power level.

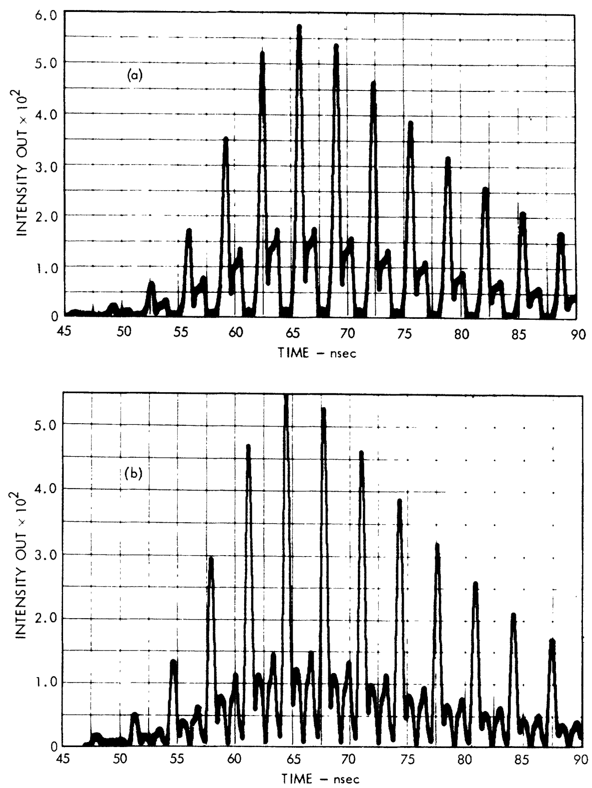

FIG. 2. Evolution of giant pulse from noise. (a) Giant-pulse shape exhibiting saturation and depletion of gain as well as pulse substructure. The sharp peaks occur with the round-trip frequency. (b) Giant-pulse shape for the conditions of Figs. 1(a)-2(a), but employing different random phases. Note how large- and small-amplitude spikes define separate pulse envelopes.

having different phases) and the magnitudes $|A_n|$ are chosen to correspond to a Lorentz spectrum with half-width T_2^{-1}. In generating this source 61 frequencies are employed, encompassing a spread $\Delta\omega = 6T_2^{-1}$. Initially, the reflectivity of the 100% mirror is set equal to 0 and after one round-trip time is "switched on" to increase as $[1-\exp(-t/\tau)]\times 100\%$ with $\tau = 5$ nsec, thus simulating Q switching. Initially $n = 23\%$ of the active atoms leading to a net round-trip gain of 3.2 for radiation at the center frequency ω.

Figure 1(a) shows the emission pattern just before and after the mirror reflectivity is switched on (switching begins at $t = 3.3$ nsec). Figure 1(b) shows the amplification over some four round-trip times in the linear regime of amplification. Figure 2(a) shows the shape of the giant pulse which is modulated by sharp pulses at the round-trip frequency. These pulses have a full width at half-maximum of 0.5 nsec or $10T_2$. From Fig. 1(b), it is evident that the basic output pattern is well established before the intensity is sufficiently high to begin saturating the gain. The problem was rerun with two other different sets of random phases in the noise source. When the principal spikes were emitted, the result was in general features the same: a periodic emission of sharp spikes but with the spike trains translated by different times. The emission pattern between the large spikes also varied as is shown in Fig. 2(b). In a fourth case (not shown), the switching time constant τ was increased to 15 nsec with no significant change in the final emission pattern.

It seems surprising that the linear amplification of random noise could lead to a pattern such as that exhibited in Fig. 1(b), which displays a quasi-mode-locked behavior. It should be borne in mind, however, that the switching on of reflectivity constitutes a loss modulation, albeit a broad-band one, which is capable of introducing phase correlations between the otherwise randomly phased frequency components of the laser radiation. In the case of non-Q-switched pulsed lasers, the switching on and time variation of gain constitutes an analogous gain modulation which could couple the laser modes and lead to an output of pulse trains occurring with the round-trip frequency. An experimental check of the present model could be provided by determining whether the formation of pulse trains occurs at powers substantially below what is required to saturate gain.

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