CRITICAL-REGION SECOND-SOUND VELOCITY IN HE II

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We present data for the velocity and dispersion of second sound in the critical region of He II. To good accuracy, the critical-region expressions relating u_2, ρ_S, and C_p are shown to be self-consistent. The anomalous dispersion is consistent with recent estimates of the coherence length.

During the past two years we have been investigating the velocity, dispersion, and attenuation of second sound in the range 10^{-5} to 10^{-1} K below T_λ.[1] We present here measurements of velocity and dispersion near T_λ. Classical superfluid hydrodynamics (which relates the velocity u_2 to the superfluid density ρ_S and the specific heat C_p) is found to be self-consistent in the critical region.

We have also found asymptotic expressions involving u_2 which are completely consistent with those for other quantities in the critical region; and we therefore avoid the difficulties with regard to asymptotic forms encountered by Pearce, Lipa, and Buckingham[2] (PLB). Velocity-dispersion measurements near T_λ yield information regarding the coherence length ξ.

We first report data for the second-sound velocity very near T_λ, which bears on the question of the self-consistency of classical superfluid hydrodynamics in the critical region near T_λ. The interest in second sound near T_λ extends beyond classical theory. The well-documented[3,4] critical power-law dependence of other parameters (ρ_S, C_p) for the superfluid on the temperature difference $T_\lambda - T$ has led to speculation[5,6] that this singular thermodynamic behavior is due to a singularity at T_λ in the order-parameter fluctuation spectrum at low k—a pole in the temperature-dependent coherence length at T_λ. If this is the case, then the critical mode for this system is second sound. As the temperature of the superfluid is raised into the critical region ($T_\lambda - T \lesssim 60$ mK), the growing coherence length $\xi(T)$ becomes macroscopic, giving rise to macroscopic critical fluctuations in the local superfluid velocity, inducing anomalous dispersion and attenuation of the critical mode. (Second-sound critical-attenuation measurements will be reported elsewhere.)

The apparatus[1] consists of several resonant cavities formed by flat lavite surfaces (5 cm^2) spaced with quartz spacers (0.2–1 cm). The cavity surfaces are coated with thin films of carbon and silver, and the cw resonance in each cavity is generated and detected using a phase-lock dual-quadrature system which is reported elsewhere[1,7]. One channel of this dual-quadrature detector is used to regulate the temperature of the liquid-He II bath to an rms accuracy of better than 10^{-7} K. The experimental bath is contained within another isothermal bath.[8] The second-sound velocity $u_2(T)$ data were obtained by measuring the mode spectrum of these cavities as a function of temperature. As the temperature was changed, each resonance was followed to its new frequency, keeping the wavelength constant. If was found that for "normal" power levels, u_2 was power dependent very near T_λ ($T_\lambda - T < 10^{-3}$ K). Thus, if the experiment was performed at constant power, an anomalous negative curvature would be introduced in $u_2(T)$. The criterion which we used for an acceptable upper power level was the presence of a clean second-sound signal at $T_\lambda - T = 10^{-5}$ K, which amounted to a power input density less than 10^{-6} W/cm^2; all measurements are zero-power extrapolations. For these $u_2(T)$ data, the temperature was measured using very low-power carbon-resistance thermometry, and the temperature differences were formed from continuous measurements of the resistance at the λ point.[8]

We present here measurements of second-sound velocity $u_2(T)$, and a test of the self-consistency of the hydrodynamically related parameters C_p, ρ_S, u_2 in the low-k critical region. We may ask if the usual classical relations[9] for the superfluid remain valid in the critical region. For this purpose, the critical region $[\xi \gg \xi(0)]$ may be divided into the high-k microscopic region ($k\xi \gg 1$), and the low-k macroscopic region ($k\xi \ll 1$). We would expect different functional forms for $u_2(T)$ in the two regions. These data cover the low-k macroscopic critical region between 50 mK and 50 μK below T_λ, wave number $3 < k < 470\ \mathrm{cm}^{-1}$, and the frequency region $2.5 \times 10^2 < \omega < 1.3 \times 10^5$. Table I shows the second-sound velocity data obtained by averaging many

Table I. Second-sound velocity versus temperature.

u_2 (cm/sec)	$T_\lambda - T$ (mK)	u_2 (cm/sec)	$T_\lambda - T$ (mK)
1975.9	361.7	396.0	4.41
1515.8	171.4	381.4	4.03
994.96	47.90	279.6	1.79
879.32	34.41	254.5	1.42
704.50	19.21	172.4	0.515
587.16	12.02	104.8 ± 0.8	0.146
548.20	10.10	87.6 ± 2	0.088

observations at various harmonics, for several temperatures within the critical region. Except where noted, errors in velocity are less than 1 part in 10^4. Errors in $T_\lambda - T$ are less than 3 μK. These velocity data join accurately onto all existing data for $T_\lambda - T > 100$ mK. The thermometry is identical to that used in our $\rho_s(T)$ experiment,[3,8] and these data are of comparable accuracy. This allows us to test the self-consistency of the classical[9] relation

$$u_2^2 = \rho_s T S^2 / \rho_n C_p \tag{1}$$

in the critical region. Using our data for $u_2(T)$ together with our data[3,8] for $\rho_s/\rho_n(T)$, we calculate $C_p(T)$ using Eq. (1) with $S = S(C_p)$. This is done on a computer, since a nonlinear integral equation relates $C_p(T)$ to $u_2(T)$. These calculated data for $C_p(T)$ are compared with the measured[4] data [Buckingham, Fairbank, and Kellers (BFK)] in Fig. 1. Comparing these data at large $1 - T/T_\lambda$, where the scatter is small, we find thermodynamic self-consistency to within 2%. Self-consistency of the quantities in Eq. (1) were also obtained by PLB. However, they arbitrarily adjusted the right-hand side of the equation by 14%. Very near T_λ, the specific heats agree within the combined error. Equation (1) seems to be obeyed over all of the presently accessible critical region. The implication is that classical superfluid hydrodynamics is obeyed well into the macroscopic (low-k) critical region near T_λ. However, we should expect large deviations from hydrodynamics at large k in the microscopic critical region, where the critical mode (second sound) ceases to propagate (diffusion regime).

In the absence of a complete microscopic theory, there is no single function describing the observables in both the low-k and high-k limits. However, within the low-k regime we may expect to find asymptotic expressions for the various experimental quantities which may then be

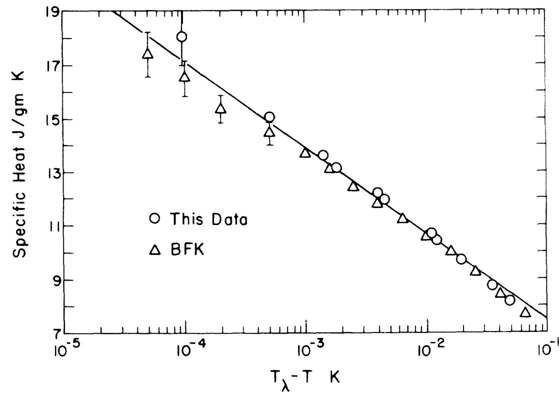

FIG. 1. Specific-heat values calculated from the second-sound velocity and superfluid density data by classical superfluid hydrodynamics, compared with measured specific-heat values.

compared. Although the scaling-law description of critical behavior explicitly assumes power-law functions, we examine other functions as well. The self-consistency of the relations for ρ_s, u_2, and C_p is first checked using power laws. In the region 10^{-4} to 10^{-2} K below T we assume that all quantities, including the specific heat, may be fitted by a power law. The result for the second-sound velocity is $u_2^2 \sim (T_\lambda - T)^{0.773}$, which yields the same exponent obtained by PLB. Fitting the BFK data by a power law yields $C_p \sim (T_\lambda - T)^{-0.107}$, and taking our previous result $\rho_s \sim (T_\lambda - T)^{0.667}$ we see that the exponents on each side of Eq. (1) are equal. We note that the specific heat, which is supposed to diverge logarithmically, does not have zero exponent. This is due to the fact[6] that, functionally, $\ln t$ is never equivalent to t^0 except at $t = 0$; over any finite range of t, the exponent will be nonzero. This is in part the origin of the discrepancies envisioned by Pearce, Lipa, and Buckingham.[2] Fisher[6] has shown that the function $(1/\alpha)(t^{-\alpha} - 1)$ resolves this problem.

Since we have previously measured ρ_s to an accuracy equal to the present u_2 data and with the same thermometry, the experimental quantity $\rho_s/\rho u_2^2$ would be expected to be free from systematic error in temperature. Least-squares fitting our data for $\rho_s/\rho u_2^2$ with the function

$$\frac{\rho_s}{\rho u_2^2} = \frac{A}{\alpha'}\left[(1 - T/T_\lambda)^{-\alpha'} - 1\right] + B, \tag{2}$$

we obtain asymptotically $\alpha' = 0.04 \pm 0.05$, where A and B are constants. The nearly equivalent function[8,10]

$$\rho_S/\rho u_2^{\,2} = A(1-T/T_\lambda)^{-\alpha'}\ln(1-T/T_\lambda)+B \quad (3)$$

results asymptotically in $\alpha' = 0.03\pm0.03$. If Eq. (1) is correct, a purely logarithmic C_p should yield $\alpha'=0$ for these functions. The small positive value of α' in Eqs. (2) and (3) is due in part to a slight temperature dependence of the parameters A and B. Equation (1) suggests a better function: Replace $\rho_S/\rho u_2^2$ in Eqs. (2) and (3) by $\rho_S TS^2/\rho_n u_2^2$. The resulting fit is better, and we obtain $\alpha' = 0.016\pm0.016$. Thus, the asymptotic form of $\rho_S/\rho u_2^2$ is shown to be consistent with the asymptotic form for C_p.

The second-sound data can also be combined with the specific-heat data to show asymptotically a characteristic power-law behavior

$$u_2 C_p^{\frac{1}{2}} = u_{20}(1-T/T_\lambda)^{\frac{1}{2}\zeta'}. \quad (4)$$

By using a least-squares fit, we find asymptotically that $\zeta' = 0.666\pm0.004$, $u_{20} = 3.37\times10^6$, using $C_p = 3.54 - 1.3\ln(1-T/T_\lambda)$ from the specific-heat data; and we see that the asymptotic form of $u_2 C_p^{1/2}$ is consistent with the asymptotic form for ρ_S. Thus, the questions raised by Pearce, Lipa, and Buckingham[2] in the analysis of their recent second-sound data in regard to consistency of asymptotic forms appear to be resolved.

Second-sound velocity dispersion in the macroscopic critical region has been measured by exciting the cavities at two harmonics ω_1 and ω_n simultaneously. The phase difference between the two oscillations is measured[1,7] continuously as a function of $1-T/T_\lambda$. The anomalous dispersion $(\omega_n - n\omega_1)/\omega_n$ is measured to a short-term accuracy of 10 ppm very near T_λ. For a wave number $k = 470\ \text{cm}^{-1}$ and at $T_\lambda - T = 0.15$, 0.45 mK we obtain $(\omega_{30}-30\omega_1)/\omega_{30} = (1.2\pm1.2)\times10^{-4}$, $(1.8\pm1.6)\times10^{-5}$. Fitting with the dispersion relation[1,10] $\omega = u_2 k[1+\lambda\xi^2k^2+\cdots]$, and using the relation[1,3,10] $\xi = \xi_0(1-T/T_\lambda)^{-2/3}$, and assuming $\lambda\simeq1$, we obtain $\xi_0 = 2\pm3$ Å. This value for ξ_0 is consistent with recent calculations[10,11] giving $\xi_0 = 1.2$ Å. We have also examined the possibility of a term linear in the product ξk in the above dispersion relation; if such a term is present, its coefficient must be less than 2×10^{-3}.

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