STUDY OF THE REACTION $\pi^+n \rightarrow \pi^0\pi^0p$ at 2.15 BeV/c*

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We report on an experimental study of the reaction $\pi^+d \rightarrow \pi^0\pi^0pp$ at 2.15 BeV/c for $|t| < 10\mu^2$, with emphasis on the measurement of the absolute $2\pi^0$ cross section. A direct comparison with $\pi^+\pi^-$ production at the same energy is made. In addition, comparison with the analysis of Malamud and Schlein provides support for the one-pion-exchange model as well as support for a large $T=0$, $l=0$ $\pi\pi$ phase shift in the ρ mass region.

The search for broad boson states is of considerable importance for a complete understanding of the meson spectrum. Narrow states are obviously much easier to identify and study, and, therefore, it seems likely that a sizable fraction of these states have now been observed. On the other hand, no meson states with widths of several hundreds of MeV have been absolutely identified. In analogy to the recent unfolding of the N^* spectrum it appears that broad boson states can only be observed by a phase-shift analysis. Unfortunately, because of the unavailability of meson targets, peripheral interactions must be used with the attendant uncertainties due to absorption and interference between amplitudes for other processes. Given the theoretical uncertainty of the peripheral model, reliable information can only be obtained if several processes are suitably correlated proving internal consistency of the model. In this note we present the essential details of a study of peripheral $2\pi^0$ production by pions. In this study particular care has been taken to obtain a reliable production cross section.

The reaction

$$\pi^+d \rightarrow \pi^0\pi^0p(p) \tag{1}$$

has been studied in a D_2 bubble chamber by converting two of the four $\pi^0\pi^0$ decay photons in Ta plates placed in the bubble chamber. The experiment was carried out using the Argonne National Laboratory 30-in. bubble chamber exposed to a π^+ beam of mean momentum 2.15 BeV/c. All events with one or two final-state protons, identified by ionization, were scanned for. In addition, for each event a search for associated photons converted in the Ta plates was made. An average of approximately 1.5 photons/event was observed. Only events with measurable spectators of momentum less than 300 MeV/c are included in this report.

To associate a gamma with a vertex, the latter had to lie within a cone $\pm 4°$ about the bisector of a symmetric pair of electrons. By considering events with vertices close to the plates, it was determined that a converted gamma has a probability of 0.90 of meeting the above criterion.

A Monte Carlo simulation[1] of the production of $2\pi^0$ events as well as the major background events, such as $3\pi^0$ production, was performed. This study indicated that a clean separation of the $2\pi^0$ sample from the background as well as a reliable cross-section measurement could be obtained by using the sample of events with two associated and converted photons. In addition the Monte Carlo calculations indicate that the detection efficiency for 2γ events, coming from Reaction (1), is issentially independent of the $2\pi^0$ invariant mass in contrast to the detection efficiency for three- and four-converted-gamma events. Henceforth the sample of events with two converted gamma rays will be discussed.

Figure 1 shows that the level of contamination from $3\pi^0$ production is small. The upper curve is a plot of the ratio of the number of $3\gamma+4\gamma$ events to the number of 2γ events obtained by generating a sample of $3\pi^0$ events with a Monte Carlo program. The lower curve shows the same ratio for a sample of peripheral $2\pi^0$ events. The data points are strongly in favor of the dominance of $2\pi^0$ events for this momentum-transfer cut. A partial rescan of the film revealed that the scanning efficiencies for 2γ events and $3\gamma+4\gamma$ events were approximately equal. It is therefore concluded that $3\pi^0$ contamination of the 2γ sample with momentum transfers less than $10\mu^2$ is quite small for $\pi\pi$ invariant mass below 1 BeV.

In the present experiment we are interested in peripheral $2\pi^0$ production events. A potential source of background for such events is due to the sequence

$$\pi^+n \rightarrow \pi^0N^{*+}(1238) \rightarrow \pi^0\pi^0p. \tag{2}$$

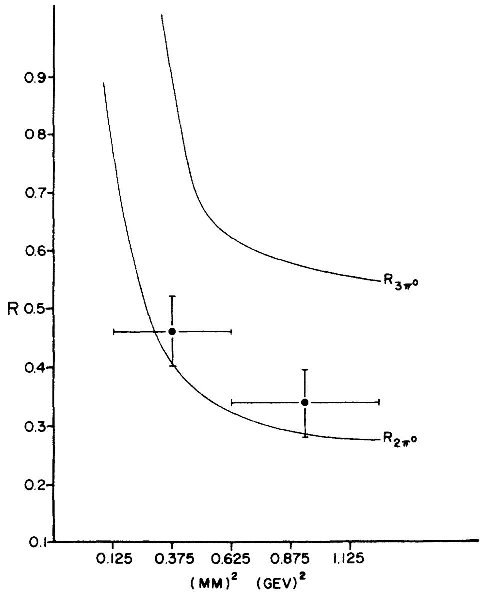

FIG. 1. Ratio of $(3\gamma+4\gamma)$ events to 2γ events versus missing-mass squared. The upper curve represents the ratio for Monte Carlo–generated events for the reaction $\pi^+d\to pp\pi^0\pi^0\pi^0$. Lower curve is ratio for Monte Carlo–generated events for $\pi^+d\to pp\pi^0\pi^0$. Data points are ratios from experimental data. All curves and data points are for momentum transfers less than 15 μ^2.

In order to eliminate these events use was made of the fact that for Reaction (2), when the momentum transfer to the proton is less than $10\mu^2$ ($|t| < 10\mu^2$, μ is pion mass), the π^0 not associated with the N^* will be at high momentum and nearly collinear with the incident beam. The other π^0 will have low momentum and, therefore, the forward π^0 will predominantly contribute to the two-gamma sample. Since the π^0 is fast and forward, the two gammas associated with events of Reaction (2) will have a characteristic opening angle. On the other hand the 2γ opening-angle distribution for Reaction (1) is expected to be much broader going from 0^0 to large angles. A Monte Carlo calculation showed that for events with an opening angle between the two gammas from 8 to 11°, hereafter referred to as angular N^* band, an N^* production event has roughly twice the probability of being detected as a $2\pi^0$ event.

From Fig. 2(a) it is argued that the contamination from N^* production is small. The histogram shows the missing-mass distribution for events inside the angular N^* band. The full curve is a similar plot, but for the two π^0's from N^* production. The dashed curve is $2\pi^0$ phase space. It is qualitatively clear from Fig. 2(a) that N^* production with resultant momentum transfer to the proton of less than $10\mu^2$ is small.

To obtain a sample of $2\pi^0$ events as clean of Reaction (2) as possible, the events inside the angular N^* band were deleted from the 2γ sample with $|t| < 10\mu^2$. A Monte Carlo calculation was used to correct for the resulting loss of $2\pi^0$

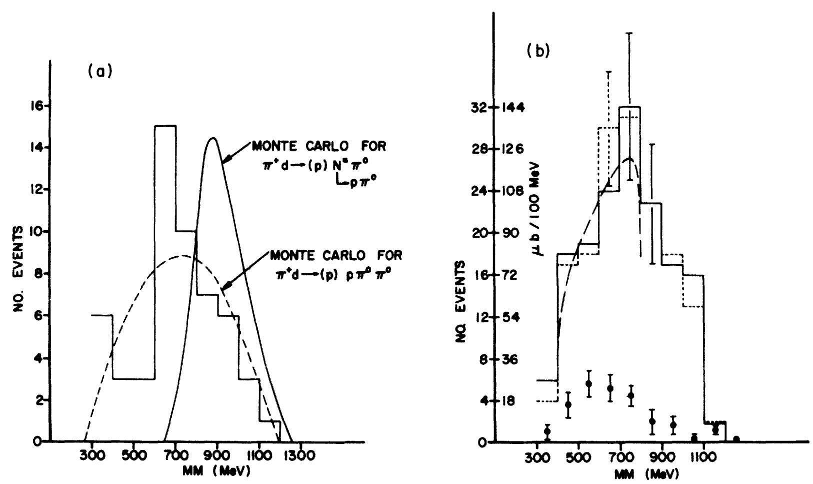

FIG. 2. (a) Missing mass inside N^* band (2γ opening angle between 8 and 11°) for 2γ sample. The histogram represents experimental distribution. The solid curve represents the $2\pi^0$ mass distribution for Monte Carlo–generated N^{*+} events with the momentum transfer calculated to the final proton. The dashed curve is peripheral $2\pi^0$ phase space. All curves are for events with two gammas detected and momentum transfers $\leq 10\mu^2$. (b) Solid histogram: missing-mass distribution for 2γ sample with N^* angular cut applied and corrected for. Dotted histogram: same as above but without N^* cut. Both histograms have $\eta\to\gamma\gamma$ events removed. Dashed curve: prediction of Malamud and Schlein (Ref. 6). Data points: results of Corbett et al., Ref. 4. All plots are for $|t| \leq 10\mu^2$ except data of Corbett et al., which are for $\cos\theta_{\text{c.m.}} \geq 0.8$.

events. The final mass distribution is shown as the solid histogram in Fig. 2(b). To show that the N^* contamination is indeed negligible, the N^* cut was omitted, and the resulting mass distribution is indicated by the dotted histogram in Fig. 2(b).[2]

In order to extract a $2\pi^0$ production cross section from the data, the total neutrals cross section given by previous counter experiment was used to obtain the charge-exchange $(\pi^+n \to \pi^0 p)$ cross section from the mass plot for all events.[3] In order to avoid use of the absolute gamma-ray-conversion probability in the Ta plates, the $2\pi^0$ cross section was then directly related to charge exchange through the relative 2γ detection efficiency for charge exchange to $2\pi^0$ production. The resulting $2\pi^0$ cross section is thereby only weakly dependent on scanning and conversion probabilities. Figure 2(b) (solid histogram) shows the resulting $2\pi^0$ cross section as a function of $\pi\pi$ invariant mass. For comparison the results of a previous experiment at this beam momentum by Corbett et al.,[4] are also shown. Whereas the shape of the invariant-mass plots of the two experiments are not in disagreement, the absolute cross-section measurements appear to differ by approximately a factor of 3. A missing-mass experiment by Smith and Manning[5] is in agreement with our results.

In Fig. 2(b) the dashed curve shows the prediction of Malamud and Schlein[6] for the absolute $2\pi^0$ cross section. In their analysis only $l=0$ angular-momentum states are required. There is striking agreement between the prediction and the data below 800 MeV. Above 800 MeV the theoretical prediction appears to fall off more rapidly than the measured cross section. In part the predicted falloff comes from the increasing value of $t_{\rm min}$ with increasing $M_{\pi\pi}$ where $t_{\rm min}$ is the minimum momentum transfer. There are two possible reasons for the discrepancy: (1) A sizable $l=2$ contribution to the $2\pi^0$ cross section comes in at relatively low invariant mass, perhaps from the tail of the f_0; (2) the Fermi momentum in the deuteron suitably changes $t_{\rm min}$ from what it would be for collisions with free nucleons. At present we cannot decide between these alternatives. At any rate the agreement below 800 MeV supports the peripheral-model analysis of Malamud and Schlein and further suggests that the $T=0$, $l=0$ $\pi\pi$ phase shift is near 90° in the ρ mass region. While similar conclusions have been obtained in other peripheral $\pi\pi$ phase-shift analyses, the prediction of the absolute $2\pi^0$ cross section on the basis of such an analysis lends strong inner consistency to the peripheral model.[7,8] If the $T=0$ phase shift actually goes through 90°, the shape of the $2\pi^0$ invariant-mass spectrum shown in Fig. 2(b) suggests that the resonance (the ϵ^0) is very broad. In fact, the $2\pi^0$ spectrum does not deviate appreciably from phase space and therefore tends to favor the $T=0$ $\pi\pi$ phase-shift solutions which pass through 90° very gradually.[9]

In Fig. 3 the ratio[10] r of cross sections for $\pi^0\pi^0$ production and $\pi^+\pi^-$ production for $|t|<12.5\mu^2$ is shown as a function of $\pi\pi$ invariant mass. This ratio has been plotted because it is relatively insensitive to the increase of the minimum momentum transfer for increasing invariant mass and in addition has certain interesting qualitative features. The $\pi^+\pi^-$ cross section was obtained from experiments with beam momentum of 2.1 BeV/c, very similar to the present experiment.[11] The drastic decrease of this ratio for increasing $\pi\pi$ mass is due to the presence of the ρ in the $\pi^+\pi^-$ channel. Near $\pi\pi$ threshold only $T=0$, $l=0$ and $T=2$, $l=0$ $\pi\pi$ phase shifts are expected to be sizable. Therefore, the absorptive correction for $\pi^0\pi^0$ and $\pi^+\pi^-$ production would be expected to be approximately the same. (Since the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ πN cross sections are equal to within 20%, the isospin dependence of the absorption is ex-

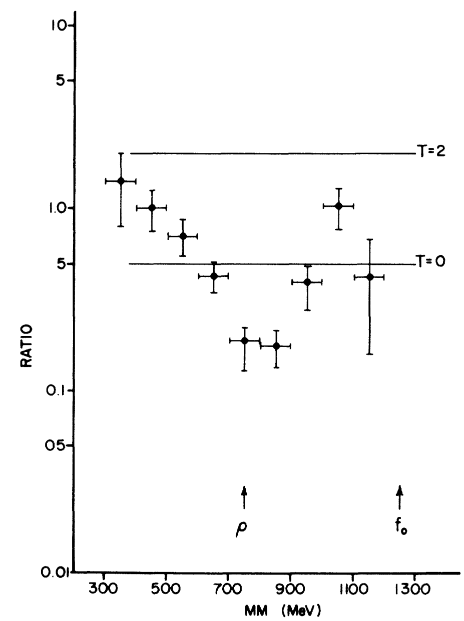

FIG. 3. Ratio of $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-)$ for $|t| \leq 12.5\,\mu^2$, computed from this experiment using 2γ sample and Ref. 10, versus missing mass.

pected to be small.) For $\pi\pi$ scattering the ratio r has the following values for different phase shifts:

$$r=2 \text{ for } \delta_2\neq 0,\ \delta_0=0,\ l=0;$$
$$r=\tfrac{1}{2} \text{ for } \delta_2=0,\ \delta_0\neq 0,\ l=0. \qquad (3)$$

As shown in Fig. 3, between 300 and 500 MeV r appears to be different from either of these values, suggesting that both $T=0$ and $T=2$ $\pi\pi$ scattering amplitudes are important. If it is assumed that the $T=2$ phase shift is small compared with the $T=0$, i.e., $|\delta_0|\gg|\delta_2|$, then a value of $r\simeq 1$ gives the relation $\delta_2\sim-\frac{1}{4}\delta_0$ indicating that the $T=2$ and $T=0$ phase shifts are opposite in sign in this low invariant-mass region. On the other hand, if $|\delta_2|\gg|\delta_0|$, the relation $\delta_2\sim 8\delta_0$ is obtained for $r\simeq 1$. Most peripheral analyses suggest that $\delta_2<0$ at low $\pi\pi$ mass and that δ_0 is larger than δ_2. The results presented here then imply that δ_0 is positive near threshold.[6,8]

Using a dispersion-relation sum rule it is possible to show the following relation between $l=0$ scattering lengths[12,13]:

$$2a_0-5a_2>0,$$

which contradicts the $a_2\sim 8a_0$ solution[14] given above and, provided that $a_0>0$, suggests that $a_2\sim-\frac{1}{4}a_0$. In addition, if the below-threshold $\pi\pi$ amplitude is assumed to be linear in s, t, and u, then the following result is obtained[12]:

$$2a_0-5a_2=(0.72\pm 0.09)\mu^{-1}.$$

Combining this result with the above relation between δ_2 and δ_0 (i.e., between a_2 and a_0) gives $a_0\sim 0.2\mu^{-1}$ and $a_2\sim -0.06\mu^{-1}$. Alternatively, combining a measurement of δ_0 (sign and magnitude) from the study of K_{e4} decay with the value of r for $M_{\pi\pi}$ near threshold would allow a_2 to be determined.[15] Of course, the ratio r should be measured at many different beam momenta to test the assumption that it is approximately independent of absorptive corrections.

It is interesting to note that the value of r shown in Fig. 3 increases again above the ρ region suggesting that $T=0$ again dominates the $\pi\pi$ interaction well below the vicinity of the f_0. The formation of a $T=0$ $\pi\pi$ resonance in the vicinity of 1000-1100 MeV could account for this trend: for example, the formation of $S^*(1068)$ in the $\pi\pi$ channel.[16] A careful study of the $\pi^0\pi^0$ angular distribution would be required to decide between these alternatives.

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[1]G. R. Lynch, Program FAKE, University of California Radiation Laboratory Report No. UCRL-10335 (unpublished). Argonne version was used.

[2]The contribution of reactions like $\pi^+n\to\omega p$, with the ω decaying into $\pi^0\gamma$, with two gammas converted, and with $|t|\leq 10\mu^2$ has been shown to be negligible by Monte Carlo calculation. Also, all η production events with η decaying into two photons were removed. The remaining contribution from the decay $\eta\to 3\pi^0$ was found to be small, and a suitable correction to the data was applied.

[3]M. Feldman, W. Frati, J. Halpern, A. Kanofsky, M. Nussbaum, S. Richert, P. Yamin, A. Choudry, S. Devons, and J. Grunhaus, Nuovo Cimento 50A, 89 (1967); A. E. Brenner, Y. Eisenberg, Y. Goldschmidt-Clermont, I. A. Pless, and A. M. Shapiro, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif., 1967).

[4]I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, W. S. C. Williams, and A. S. Carroll, Phys. Rev. 156, 1451 (1967).

[5]G. A. Smith and R. J. Manning, University of California Radiation Laboratory Report No. UCRL-17917, 1968 (unpublished). See also A. Gezelter et al., to be published.

[6]E. Malamud and P. E. Schlein, Phys. Rev. Letters 19, 1056 (1967), and Bull. Am. Phys. Soc. 13, 31 (1968). We are grateful to the above authors for providing us with their prediction for our beam momentum and momentum-transfer cut. This prediction refers to their up-up solution. The $T=2$ phase shift was taken from J. Baton, G. Laurens, and J. Regnier, Phys. Letters 25, 419 (1967). See also A. B. Clegg, to be published.

[7]W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters 18, 630 (1967).

[8]L. J. Gutay et al., Phys. Rev. Letters 18, 142 (1967). See this paper for earlier references.

[9]For example, see the solid curves in Fig. 2 of Ref. 7.

[10]In the plane-wave approximation, $r=x/2y$, $x=\sin^2\delta_0+\sin^2\delta_2-2\sin\delta_2\sin\delta_0\cos(\delta_2-\delta_0)$, $y=\sin^2\delta_0+\frac{1}{4}\sin^2\delta_2+\sin\delta_2\sin\delta_0\cos(\delta_2-\delta_0)+(27/4)\sin^2\delta_{1,1}$, where δ_T are the $l=0$ phase shifts and $\delta_{1,1}$ is the $T=1, l=1$ phase shift.

[11]E. West, J. H. Boyd, A. R. Erwin, W. D. Walker, Phys. Rev. 149, 1089 (1966).

[12]M. G. Olsson, Phys. Rev. 162, 1338 (1967).

[13]C. J. Goebel and G. Show, "Phenomenological Bounds on $\pi\pi$ Scattering Lengths," to be published.

[14]Near threshold $q\cot\delta_T\simeq 1/a_T$ which implies $\delta_T\propto a_T$ for small δ_T.

[15]R. W. Birge et al., Phys. Rev. 139, B1600 (1955).

[16]A. H. Rosenfeld et al., Rev. Mod. Phys. 77, 40 (1968).