CRITICAL-REGION SECOND-SOUND DAMPING IN He II

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Second-sound attenuation very near T_λ is found to be proportional to the square of the frequency, and the damping diverges as $(T_\lambda - T)^{-\sigma}$, $\sigma = 0.34 \pm 0.06$, in agreement with dynamic scaling. The coefficient of the damping is consistent with recent estimates of the coherence length.

Recently Ferrell et al.[1] and others[2,3] have predicted an asymptotic power-law divergence of the second-sound damping coefficient $D_2 \sim (T_\lambda - T)^{-1/3}$ in the critical region of He II on the basis of "dynamic scaling." Extending the static scaling ideas[4] involving a single coherence length $\xi(T) \sim (T_\lambda - T)^{-2/3}$, they describe the dynamics near T_λ in terms of a single characteristic frequency $\omega^* = u_2 \xi^{-1}$, where $u_2(T)$ is the second-sound velocity. For frequencies ω such that $\omega < \omega^*$ and $k\xi \lesssim 1$, and for temperatures within the critical region, the above asymptotic form for the second-sound damping $D_2(T)$ results. In order to test the dynamic scaling hypothesis, we have measured the damping of second sound in the macroscopic critical region $k\xi \ll 1$ between 7×10^{-5} and 3×10^{-3} deg below T_λ and for $\omega < \omega^*$. In addition, we report measurements of the frequency dependence of the corresponding anomalous attenuation very near T_λ.

The second-sound wave is generated in a low-power cw resonant cavity which is excited at various harmonics, and the width of the resonance at each harmonic is measured by quadrature phase-sensitive detection. Absolute measurements of $T_\lambda - T$ accurate to $\pm10^{-6}$ deg are obtained from the observed $u_2(T)$ at each point. The cavity is immersed in a superleak-purified He^4 bath which is enclosed within another bath for better temperature regulation and stability. Power input density to the cavity is monitored and kept typically less than 2×10^{-5} W/cm^2 in order to avoid nonlinear effects near T_λ. For several power input levels the resonance widths are time-averaged, and the limiting zero-power resonance width at each harmonic is obtained for various temperatures within the critical region. To test for possible systematic effects, two different cavities are used. The cavities are formed by spacing flat lavite or quartz surfaces with fused quartz spacers, and the temperature wave is generated and detected by thin films of silver and carbon. In addition to bulk attenuation, reflection loss and diffraction (beam spreading) loss give rise to enlarged resonance widths. These other effects are also investigated as a function of power, frequency, and temperature, and corrections made to the observed resonance widths—introducing a corresponding error. The second-sound damping coefficient $D_2(T)$ is defined in terms of the bulk attenuation $\alpha(\omega, T)$:

$$\alpha(\omega, T) = (\omega^2/2u_2^3)D_2(T), \tag{1}$$

where classical superfluid hydrodynamics[5] indicates that D_2 is frequency independent. This is also true of the dynamic scaling theories for $k\xi \ll 1$. The observed resonance width $\Delta\omega_n$ (width at half-amplitude) corresponding to mode number n for a cavity of this type ($\omega_n = n\omega_1$) is given by the sum of three terms:

$$\frac{\Delta\omega_n}{\sqrt{2}} = \frac{\pi^2 n^2}{d^2} D_2 + \frac{2\beta}{\pi}\omega_1 + \frac{4d^2}{\pi n d_0^2}\omega_1, \tag{2}$$

where the first term corresponds to the bulk attenuation; the second and third terms correspond to reflection loss and diffraction loss, respectively. $\beta = -\ln R$ where R is the reflection coefficient, and $\omega_1(T)$ is the fundamental resonant frequency ($n = 1$). The dimensions of the cavities (width d_0, spacing d) were chosen to minimize any corrections due to diffraction. The measured reflection coefficient of the lavite cavity is 0.94 and for the quartz cavity 0.97. This agrees well with other measurements[6] in cw cavities, at lower temperatures. Equation (2) shows that this correction to $\Delta\omega_n$ varies with temperature through $\omega_1(T)$. At $n = 20$, this reflection-loss correction to the observed resonance widths varies from 10% at 10^{-4} deg below T_λ to 50% at 3×10^{-3} deg below T_λ. The power dependence of the width (for 10-30 μW/cm^2) is found to be linear and is apparently singular at T_λ; $\partial\Delta\omega_1/\partial P$ varies between 0.3 Hz $(\mu\text{W/cm}^2)^{-1}$ at 10^{-4} deg below T_λ and 4×10^{-2} Hz $(\mu\text{W/cm}^2)^{-1}$ at 10^{-3} deg below T_λ. This is in good agreement with other measurements[6,7] at higher powers and lower temperatures. $\partial\Delta\omega_n/\partial P$ increases approximately linearly with frequency, and approaches zero for $P < 10\ \mu W/cm^2$ within the critical region. The small error in extrapolating $\Delta\omega_n$ to zero power is used in the determination of total error. Equation (2) indicates that data for $\Delta\omega_n$ over a sufficient range of both n and $\omega_1(T)$ give data for $D_2(T)$. As a check the third term may be calculated, and the resulting values agree with the measured diffraction data to better than 10%. However, this diffraction correction is negligible over the temperature region covered in the $D_2(T)$ data.

The classical ω^2 dependence of the bulk attenuation $\alpha(\omega,T)$ in Eq. (1) may be checked near T_λ by observing $\Delta\omega_n$ for various n near T_λ where $\omega_1(T)$ is small. Figure 1 shows the observed zero-power resonance widths in the quartz cavity, for $T = 1.6\times10^{-4}$ deg below T_λ and $\omega_1/2\pi = 270$ Hz, plotted versus n^2. The n^2 dependence of $\Delta\omega_n$ at large n (where β is wave-number independent) seems to be valid near T_λ, indicating that D_2 is n-independent over this range of frequencies, in agreement with both classical[5] and scaling[1-3] theories. The intercept at $n=1$ of the large-n asymptote gives the reflection loss; here $\beta = 3\%$. The slope of the asymptote is proportional to the damping $D_2(T)$. For each such plot of $\Delta\omega_n$ vs n^2, the corrected value of $\Delta\omega_{20}$ is obtained and the damping D_2 computed. For temperatures farther below T_λ, where $\omega_1(T)$ is large, the percent correction to $\Delta\omega_{20}$ due to reflection loss becomes larger, and the error in $D_2(T)$ becomes relatively large for $(T_\lambda - T) \gtrsim 5\times10^{-3}$ deg K. Diffraction corrections become important only for $(T_\lambda - T)$ $>10^{-2}$ deg K. Crosstalk plus noise limit observation to $(T_\lambda - T) \gtrsim 5\times10^{-5}$ deg K. From a series of graphs like Fig. 1, the damping $D_2(T)$ is obtained as a function of temperature in the critical region.

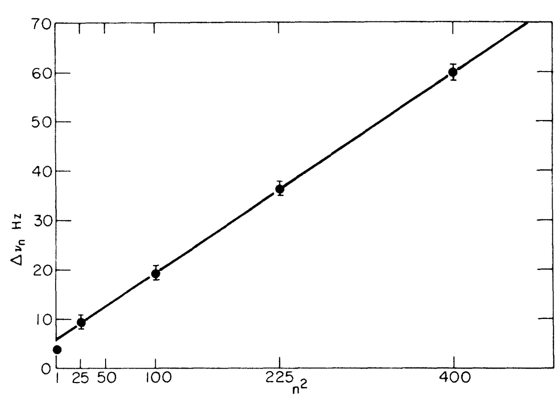

FIG. 1. Plot of the observed resonance width of the nth harmonic at half-amplitude, $\Delta\omega_n/2\pi$, versus the square of the harmonic number n^2 for a single temperature 1.6×10^{-4} deg below T_λ. The ω^2 dependence of $\Delta\omega_n$ is predicted by classical superfluid hydrodynamics and by dynamic scaling for $k\xi \ll 1$. The zero-frequency ($n=0$) intercept is proportional to the reflection loss. Slope is proportional to the damping D_2.

Figure 2 shows the data for the second-sound damping coefficient $D_2(T)$ for several temperatures within the critical region. Also included in Fig. 2 are data of Hanson and Pellam[8] extending out of the critical region which were obtained using a different method, and two points of Notarys[7] from megahertz cw cavity-resonance data. Thus, both relative and absolute damping may be compared. If our data are fitted by a power-law function, we obtain

$$D_2 = D_0(1 - T/T_\lambda)^{-\sigma}, \tag{3}$$

with $\sigma = 0.34 \pm 0.06$ and $D_0 = 1.02\times10^{-4}$ cm^2 sec^{-1}. The extrapolated value of $D_2(T)$ to the edge of the critical region overlaps the absolute attenuation measurements of Hanson and Pellam[8] and the resonance data of Notarys.[7] If these additional data are used in the fit with Eq. (3), reduced error results ($\sigma = 0.34 \pm 0.03$), but this procedure is risky, because of possible differential systematic errors. Outside the critical region, $D_2(T)$ should level out and approach the nonsingular Khalatnikov value[8,9] at lower temperatures. This is apparent in Fig. 2 for $(T_\lambda - T) > 4\times10^{-2}$ deg K. In addition to the linewidth data, measurements of the signal amplitude were taken at each point. These data, when scaled to input power, are consistent with all the resonance-width results.

The experimentally determined exponent $\sigma = 0.34 \pm 0.06$ is not an asymptotic value. It is the apparent exponent for the region between 7×10^{-5}

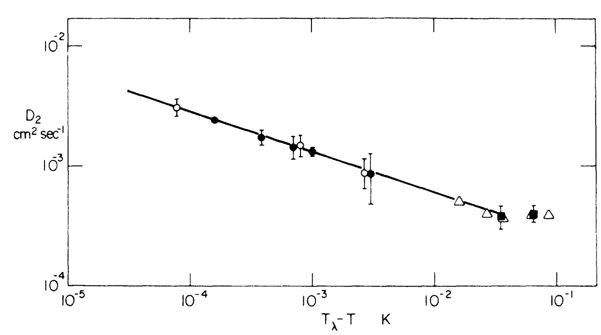

FIG. 2. Data for second-sound damping $D_2(T)$ near T_λ. Our data: solid circles, quartz cavity; open circles, lavite cavity. Data of Hanson and Pellam: open triangles. Notarys' data: solid squares. Outside the critical region, for $(T_\lambda - T) > 4\times10^{-2}$ deg, $D_2(T)$ reaches the Khalatnikov minimum. Solid line has slope $-\frac{1}{3}$.

and 4×10^{-2} deg below T_λ. Although Ferrell et al.[1] predict $D_2(T)\sim\xi^{1/2}$, which $\sim(T_\lambda-T)^{-1/3}$ for all T in the critical region, more recent dynamic scaling theories[2,3] indicate $D_2\sim u_2\xi\sim(T_\lambda-T)^{-\sigma}$ where $\sigma=\sigma(T)$ because of the presence of C_p in the second-sound velocity[10]: $u_2\sim(T_\lambda-T)^{\Phi(T)}$ yields the theoretical prediction $D_2\sim(T_\lambda-T)^{-0.28}$ for $5\times10^{-5}<(T_\lambda-T)<5\times10^{-3}$ deg K. In summary, both dynamic-scaling theoretical estimates for the exponent of second-sound damping ($\sigma=0.33, 0.28$) agree to within experimental error with our result $\sigma=0.34\pm0.06$.

The coefficient D_0 in Eq. 3 is also interesting because, according to dynamic scaling, it contains information about the coherence length coefficient ξ_0 in the relation $\xi(T)=\xi_0(1-T/T_\lambda)^{-2/3}$. Ferrell and co-workers' rough calculation[1] of $D_0\approx10^{-5}$ cm^2 sec^{-1} is far from the observed value. However, they calculate D_0 for the limit $\omega>\omega^*$, where the dispersion relation $\omega_k\sim k^{3/2}$ obtains, and also use a value for ξ_0 (0.27 Å) which is $\frac{1}{5}$ the latest estimate[11,12] $\xi_0=1.2$ Å. We repeat this calculation of D_0 for the conditions $k\xi\ll1$, $\omega<\omega^*$ obtaining in this experiment. Using the same frequency scaling idea[1-3] for the limit $\omega<\omega^*, \omega_k\sim k+\cdots$, a similarly rough calculation using[1] $\langle k\rangle\approx1/\xi$ and $D_2k^2\approx2\omega^*$, with $\omega^*=u_2\xi^{-1}$ gives $D_0\approx10^{-4}$ cm^2 sec^{-1}, closer to the experimental value.

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