conditions the reversal of cyclotron damping is more important in the formation of echoes than that of Landau damping. In particular, numerical estimates show that in our experiment the latter can be orders of magnitude smaller than the former.

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EXPERIMENTAL THERMAL CONDUCTIVITY OF TWO DILUTE SOLUTIONS OF He^3 IN SUPERFLUID He^4 *

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Measurements are presented of the thermal conductivity at saturated vapor pressure of 1.3 and 5.0% dilute solutions of He^3 in superfluid He^4 over the approximate temperature range 0.035-0.5°K and of pure superfluid He^4 over the temperature range 0.04-0.16°K. Heat transport is primarily by He^4 phonons whose paths are limited by the He_3-quasiparticle-He^4-phonon interaction and by boundary scattering, in qualitative agreement with a theory of thermal conduction in dilute solutions by Baym and Ebner. There is, however, quantitative disagreement with their calculation of the thermal conductivity for the above solutions.

Heat is conducted in dilute solutions of He^3 in superfluid He^4 at very low temperatures by He^3 quasiparticles whose free path is determined by their mutual scattering. Experiments demonstrating this aspect of heat conduction in dilute solutions were done by Abel, Johnson, Wheatley, and Zimmermann[1] and relate to the effective-interaction theory of Bardeen, Baym, and Pines.[2] At higher temperatures most of the heat in a thermal-conduction experiment is carried by He^4 phonons rather than by He^3 quasiparticles. Below $\frac{1}{2}$°K the He^4-phonon free path is determined by the He^3-quasiparticle-He^4-phonon interaction and by boundary scattering of the He^4 phonons. A theory of this interaction has been developed by Baym and Ebner[3] and applied by them to thermal conduction in dilute solutions with specific numerical calculations for solutions of nominal concentrations 1.3 and 5.0% He^3. Baym and Ebner's theory makes the interesting qualitative prediction that, aside from boundary scattering, at low temperatures He^4 phonons are absorbed by the He^3 system by the mechanism of viscosity as in the attenuation of ultrasound. At higher temperatures, elastic scattering of the He^4 phonons by He^3 quasiparticles is of primary importance, though according to Baym and Ebner even in this temperature range the absorption mechanism due to viscosity is important. At long He^4-phonon wavelengths the strength of the interactions governing phonon scattering are determined essentially from thermodynamical data,[3] so that one might expect the calculated thermal resistance due to scattering to have not only qualitative but quantitative significance. However, that part of the thermal resistance which is due to phonon absorption is more uncertain to evalute numerically, particularly at high temperatures. The present experiments on He^4-phonon heat conduction in 1.3 and 5.0% dilute solutions of He^3 in He^4 support the qualitative conclusions of the Baym-Ebner theory but are in quantitative disagreement with it, at least at higher temperatures.

Measurements of thermal conductivity were made by determining the ratio of heat flow to resultant temperature difference in a column of helium of known geometry and under conditions such that the temperatures were not changing appreciably with time. Measurements were made for each of two different helium columns contained in reamed nylon tubes sealed into the bottom of a large epoxy cell containing a powdered cerium magnesium nitrate (CMN) thermometer in the form of a right circular cylinder with diameter equal to height. The two helium columns were not interconnected at the bottom, common connection being only through the helium reservoir in the epoxy cell. The smaller helium column had a diameter of 0.250 cm with nylon-wall thickness 0.05 cm; the larger column had diameter of 1.02 cm with nylon-wall thickness 0.125 cm. Speer type 1002, nominal 100_0-Ω, $\frac{1}{4}$-W resistance thermometers were located with their axes perpendicular to the column axes in open spaces at the tops and bottoms of the columns. These resistors were calibrated against the magnetic temperature of the CMN with no applied heat flow in the columns. The heaters were located in open spaces at the bottoms of the columns and below the lower resistance thermometers. At low temperatures the effective ratio of the separation of the resistance thermometers to their column cross-sectional area was 77.8 cm^{-1} for the small column and 4.46 cm^{-1} for the large column. The total volume of solution in the cell was about 12 cm^3. The present geometry is similar to that shown in Fig. 1 of Ref. 1, with the exception that two thermal-conductivity appendices rather than one are used. The change from magnetic to resistance thermometers to indicate the temperature differences was necessary to eliminate unknown temperature drops at higher temperatures which were present in the application of the magnetic thermometers. Cooling of the present device was provided by a continuously operating dilution refrigerator.[4]

The experimental data for the small column are shown in Fig. 1. The magnetic temperature scale is that valid for CMN powder in the form of a right circular cylinder with diameter equal to height. We assume[5,6] that this temperature is no more than a few tenths of a millidegree different from the Kelvin temperature. A suggestion by Abraham and Eckstein[7] that the magnetic temperatures may be less than the Kelvin temperatures by about 1.7 mdeg K is not supported by direct evidence.[6] Heat shunting effects due to the

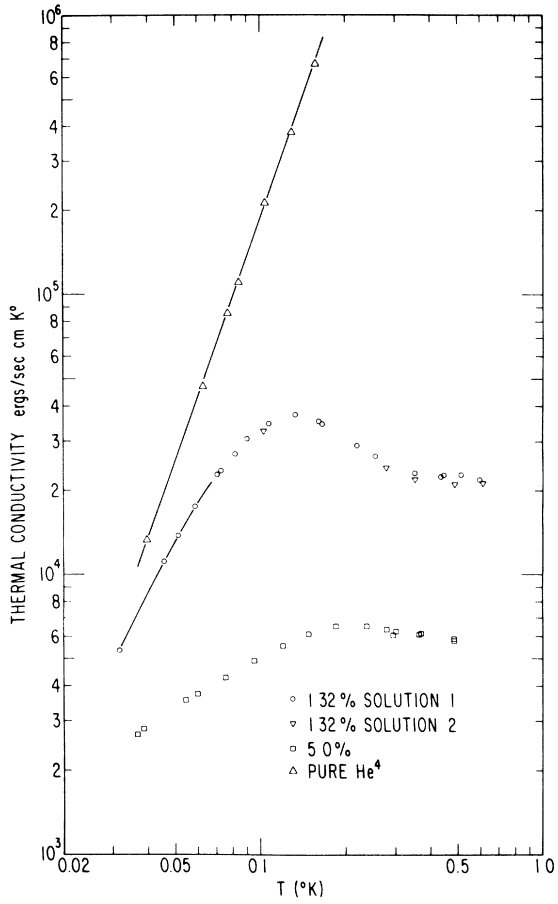

FIG. 1. Thermal conductivity of two dilute solutions of He^3 in He^4 and of pure He^4. These data, obtained with a helium column 2.50 mm in diameter and of effective length 3.82 cm, include the effect of boundary scattering from the nylon walls.

nylon walls have been calculated[8] to be neglibible at all temperatures. The measurements could also yield spuriously high conductivities at a given temperature for the top of the column if the fractional temperature change across the column was too large. This is a consequence of the requirement that in the superfluid the partial chemical potential for He^4 be constant. A gradient in temperature then leads to a gradient in He^3 concentration, the effect being greater at higher temperatures. This effect ("heat flush") was studied experimentally by varying the heat current and, hence, the fractional temperature drop across the column until the derived conductivities were independent of heat current within experimental scatter. The data marked 1.32%, solution 1, and 5.0% refer to the same solutions for which measurements were previously made of spin diffusion coefficient,[9] specific heat,[9] and

thermal conductivity.[1] Above 0.3°K for the 1.3% solution, the thermal conductivity hardly decreases with increasing temperature. Since for the above solutions our cell was filled by an open capillary[10] of 0.25-mm bore, we speculated that some He^3 might be held up in the filling capillary and lead to a spuriously high thermal conductivity at higher temperatures where the cell was warmer than some parts of the dilution refrigerator. We therefore made a new 1.32% solution, indicated as solution 2 in Fig. 1, and repeated the measurements using a filling capillary of 0.10-mm bore. Even if this capillary had been full of pure liquid He^3, the concentration in the cell would have been reduced by less than 1%. The measurements for solution 2 are systematically 3 or 4% below those for solution 1 over the whole temperature range of comparison. It is reasonable to attribute this difference to systematic calibrational errors. Since He^3 held up in the filling capillary cannot affect the results for solution 2, we conclude from the comparison of results for solutions 1 and 2 that the results for solution 1 are trustworthy. The absolute accuracy of the experimental results is difficult to state but is probably estimated by the above 4% discrepancy for the two 1.3% solutions measured, the actual accuracy no doubt being somewhat poorer than this figure. Finally, we note that the results for the 5% solution, for which boundary scattering is relatively less important, extrapolate nicely into those of Ref. 1.

No results for the large column are shown in Fig. 1. These were in agreement within experimental precision with those for the small column for $T > 0.1$°K for the 1.3% solution and for $T > 0.07$°K for the 5% solution. At lower temperatures the thermal conductivity as determined using the large column was systematically greater than that using the small column. However, the design of the apparatus was such that only a certain maximum heat flow was possible at a given temperature. Owing to the larger conductance of the large column, the maximum temperature gradient which could be produced in it was rather small. Hence, the precision of the large-column data at the lower temperatures was substantially less than those for the small column and did not allow us to improve our quantitative understanding of the effect of boundary scattering.

Also shown on Fig. 1 are results using the small column for the thermal conductivity of pure[11] He^4. These measurements were made to study boundary scattering. They may be interpreted in terms of the formula[12,13]

$$\kappa = \tfrac{1}{3} C_{\mathrm{ph}} s d (2-f)/f, \tag{1}$$

where C_{ph} is the phonon heat capacity per unit volume, s is the sound velocity, d is the tube diameter, and f is the fraction of phonons diffusely reflected at the walls. Analysis of the data shows that f increases from 0.44 at 0.04°K to 0.52 at 0.16°K. These values of f for the present reamed nylon tube are reasonable when compared with results of Whitworth[14] obtained at higher temperatures.

As indicated above, at sufficiently high temperatures the thermal conductivity measured for both small and large columns was the same; so one can assume that boundary scattering effects are small in the 5% solution above 0.07°K and in the 1.3% solution above 0.1°K. At low temperatures boundary scattering is important, but for comparison with theory it would be desirable to eliminate its effect. This cannot be accomplished without detailed recomputation using Baym and Ebner's theory, but an estimate of the bulk conductivity κ may be obtained from

$$\kappa^{-1} \simeq \kappa_{\mathrm{meas}}^{-1} - \kappa_4^{-1}, \tag{2}$$

where κ_{meas} is the measured thermal conductivity of a solution in a given column and κ_4 is that for pure He^4 in the same column. The results are shown in Fig. 2 together with the theoretical values for κ computed by Baym and Ebner.[3] The general qualitative features of their theory are confirmed. At the lowest temperatures, the apparent departures from theory indicated in Fig. 2 may not be serious in view of the approximate nature of Eq. (2). For the same reason the present low-temperature results neither confirm nor rule out the recent discrepancies with Baym and Ebner's theory,[3] as applied to ultrasonic attenuation reported by Abraham and co-workers[15] for temperatures below 0.05°K. In this connection, our direct magnetic-thermometry experiments[6] suggest that Abraham et al.[15] incorrectly added 2 mdeg K to their magnetic temperatures to convert them to Kelvin temperatures. Proper adjustment of their temperature scale lessens but does not remove the discrepancy mentioned above.

Particularly at high temperatures Baym and Ebner's calculations predict too high a conductivity, the discrepancy being especially large for the 1.3% solution. It is interesting, though

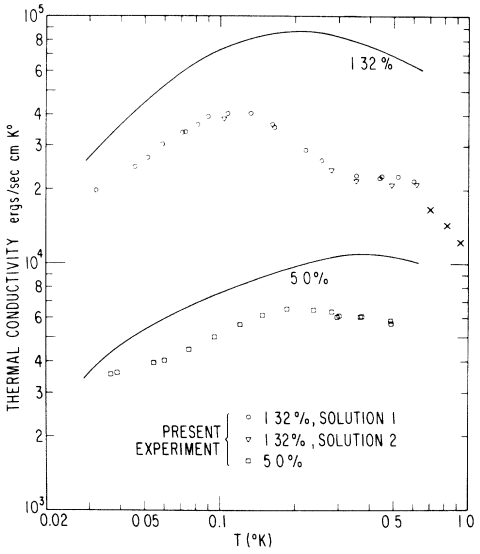

FIG. 2. Thermal conductivity of two dilute solutions of He^3 in He^4. The effect of boundary scattering has been eliminated approximately as explained in the text. The solid lines are the calculation of Baym and Ebner (see Ref. 3). Crosses; 1.36%, measured by Ptukha (see Ref. 16).

perhaps coincidental, that in the region of 0.5°K and somewhat below, the ratio of the thermal conductivity of the 1.3% solution to that of the 5% solution is $3.7_5 \pm 0.2$ (this ratio is 6.4 according to Baym and Ebner). The inverse ratio of concentrations is the same at this temperature within experimental error. It appears that the magnitude of the He^4-phonon–He^3-quasiparticle scattering is substantially greater than that estimated by Baym and Ebner for long-wavelength phonons using essentially thermodynamical data. Numerical adjustments in the absorption calculation will probably also be necessary.

Also shown in Fig. 2 are the results of Ptukha[16] for a 1.36% solution. Our results for 1.3% solutions may not extrapolate into his, even when the latter are adjusted upward slightly to take into account the small difference in concentration. However, the conductivity may actually start to decrease more rapidly above 0.6°K, as suggested by accepting both Ptukha's and the present experiments. Our experimental arrangement unfortunately did not permit us to study this matter.

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