CHIRAL SYMMETRY BREAKING IN K_{l3} DECAYS*

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A current-algebra calculation of the K_{l3} form factors is made under the assumption that the chiral SU(3) symmetry is broken by an interaction which transforms under the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$. Quantitative results on the weak decay parameters of the mesons, the κ-meson mass, and the form factor $f_+(0)$ are obtained. The factor ξ is found to be very nearly zero.

Over the past few years, considerations based on the chiral SU(3) algebra generated by the weak currents have proved to be highly fruitful. In particular, Callan and Treiman[1] have applied such considerations to K_{l3} decays, and obtained a nontrivial relation between the two decay form factors. This relation is, however, valid strictly at an unphysical point, and it is not clear that the physical form factors satisfy it at all. More precisely, the gradient-coupled term,[2] which is absent at the unphysical point, may be rather important for the actual decay process. We report here a way to estimate this term. The kinematics will be $K(p)\rightarrow\pi(q)+l\bar{l}(k)$.

The precise way in which this estimate is carried out follows the technique of Schnitzer and Weinberg.[3] The three-point functions considered here involve two axial currents carrying strangeness zero and one, labeled π and K, respectively, and a vector current of strangeness one, labeled κ. However, in carrying out the derivation of the Ward identities, one encounters the equal-time commutators involving currents and their divergences. Although the corresponding commutators in Ref. 3 do not contribute due to isospin conservation, they are of importance here, and are related to the particular form of chiral SU(3) symmetry breaking. We shall assume the symmetry-breaking term to transform under the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of the chiral SU(3) group,[4] so that the commutators are, explicitly,

$$[\int J_0^{\ a}(x,t)d^3x,\partial_\mu J_\mu^{\ b}(y,t)]$$

$$=\frac{m_b^{\ 2}F_bZ_b^{\ -1/2}}{m_c^{\ 2}F_cZ_c^{\ -1/2}}\epsilon(a,b,c)\partial_\lambda J_\lambda^{\ c}(y,t), \quad (1)$$

where $\{a,b,c\}=\{K,\pi,\kappa\}$ are particle labels, and $\epsilon(a,b,c)=+1$ except for $\epsilon(K,\kappa,\pi)=\epsilon(\kappa,\pi,K)=-1$. The quantities $m_a^{\ 2}$, F_a, and $Z_a^{\ -1/2}$ denote the mass squared and the weak-decay and renormalization constants of the spin-zero particle, which in this study is assumed to dominate $\partial_\mu J_\mu^{\ a}$ and be responsible for the nonconservation of the current (the Goldstone theorem).[5]

Equation (1) places restrictions on the spectral functions of these particles, which upon saturation by single-particle states yield

$$F_\pi Z_\pi^{\ 1/2}=F_KZ_K^{\ 1/2}+F_\kappa Z_\kappa^{\ 1/2}. \quad (2)$$

Furthermore, to be consistent with the chiral SU(3) current algebra, one has in addition

$$m_\pi^{\ 2}F_\pi Z_\pi^{\ -1/2}=m_K^{\ 2}F_KZ_K^{\ -1/2}+m_\kappa^{\ 2}F_\kappa Z_\kappa^{\ -1/2}. \quad (3)$$

Equations (2) and (3) have been derived in Ref. 4 in the context of a Lagrangian theory.

Following Ref. 3, we say that the currents $J_\mu^{\ K}$, $J_\nu^{\ \pi}$, and $J_\lambda^{\ \kappa}$ are dominated by the spin-one mesons KA, A_1, and K^*, and the spin-zero mesons K, π, κ, and express the notion of single-meson dominance by requiring that all the vertices, like the KA-A_1-K^* vertex $\Gamma_{\mu\nu\lambda}$, KA-A_1-κ vertex $\Gamma_{\mu\nu}^{\ KA\,A_1}$, K-π-K^* vertex $\Gamma_\lambda^{\ K^*}$, and K-π-κ vertex $G_{K\pi\kappa}$, to have the weakest possible momentum dependence consistent with the Ward identities. That is, in the language of dispersion relations, we assume that the contributions from the unitarity cuts beyond the single-particle poles in the three-point functions are rather weak in each of the channels, so that if we restrict to $k^2<1$ BeV^2, they contribute negligibly.[6]

Our first weak momentum dependence or smoothness assumption is that $\Gamma_{\mu\nu\lambda}$ be linear in fourmomenta:

$$\Gamma_{\mu\nu\lambda} \simeq \Gamma_1 g_{\mu\nu}(p+q)_\lambda + \Gamma_2(g_{\mu\lambda}k_\nu - g_{\nu\lambda}k_\mu) + \Gamma_3(g_{\mu\lambda}p_\nu + g_{\nu\lambda}q_\mu) + \Gamma_4 g_{\mu\nu}k_\lambda + \Gamma_5(g_{\mu\lambda}k_\nu + g_{\nu\lambda}k_\mu) + \Gamma_6(g_{\mu\lambda}p_\nu - g_{\nu\lambda}q_\mu), \tag{4}$$

where Γ_1 to Γ_6 are constants independent of p^2, q^2, and k^2. All other vertices can now be expressed in terms of Γ_1 to Γ_6. The ones relevant to K_{l3} decays are

$$\begin{aligned} 2F_K F_\pi F_\kappa G_{K\pi\kappa} = {} & 2g_{KA}g_{A_1}g_{K^*}(m_{KA}{}^2 m_{A_1}{}^2 m_{K^*}{}^2)^{-1}\{(p\cdot q)(p^2-q^2)(\Gamma_1+\Gamma_3)+k^2(\Gamma_4+\Gamma_6) \\ & +2\Gamma_5(k\cdot q)(k\cdot p)+2[(p\cdot q)^2+q^2p^2]\Gamma_6\}+(p^2-q^2)[F_K{}^2+F_\pi{}^2-F_\kappa{}^2] \\ & +k^2[F_K{}^2+F_\kappa{}^2-F_\pi{}^2+2F_K F_\kappa(Z_K/Z_\kappa)^{1/2}] \\ & +2F_K F_\pi[(q^2+m_\pi{}^2)(Z_K/Z_\pi)^{1/2}-(p^2+m_K{}^2)(Z_\pi/Z_K)^{1/2}], \end{aligned} \tag{5}$$

$$\begin{aligned} 2F_K F_\pi \Gamma_\lambda{}^{K^*} = {} & (p+q)_\lambda\{g_{A_1}g_{KA}(m_{A_1}{}^2 m_{KA}{}^2)^{-1}[2(p\cdot q)(\Gamma_1+\Gamma_3)+k^2(\Gamma_2+\Gamma_3) \\ & +(p^2-q^2)(\Gamma_5+\Gamma_6)]+g_{K^*}{}^{-1}(k^2+m_{K^*}{}^2)(F_K{}^2-g_{A_1}{}^2 m_{A_1}{}^{-2})+g_{K^*}\} \\ & +k_\lambda\{g_{A_1}g_{KA}(m_{A_1}{}^2 m_{KA}{}^2)^{-1}[-2(p\cdot q)(\Gamma_4-\Gamma_6)-k^2(\Gamma_5+\Gamma_6) \\ & -(p^2-q^2)(\Gamma_2+\Gamma_3)]-g_{K^*}{}^{-1}(p^2-q^2)(F_K{}^2-g_{A_1}{}^2 m_{A_1}{}^{-2}) \\ & +g_{K^*}{}^{-1}m_{K^*}{}^2[F_K{}^2+g_{A_1}{}^2 m_{A_1}{}^{-2}-g_{K^*}{}^2 m_{K^*}{}^{-2}+2F_K F_\kappa(Z_K/Z_\kappa)^{\frac{1}{2}}]\}. \end{aligned} \tag{6}$$

The second smoothness assumption is that $G_{K\pi\kappa}$ should be only quadratic in momenta. Further, we shall assume that κ is a πK S-wave resonance, and say that πK scattering in the region of interest can be approximated by this resonance. This means that the coefficient of k^2 in $G_{K\pi\kappa}$ can be neglected. We then have

$$\Gamma_1+\Gamma_3=0, \quad \Gamma_4-\Gamma_6=0, \quad \Gamma_5+\Gamma_6=0, \tag{7}$$

$$F_K{}^2+F_\kappa{}^2+2F_K F_\kappa(Z_K/Z_\kappa)^{1/2}=F_\pi{}^2. \tag{8}$$

The K_{l3} form factors, defined to be

$$\langle\pi(q)|J_\lambda{}^\kappa|K(p)\rangle=(p+q)_\lambda f_+(k^2)+k_\lambda f_-(k^2), \tag{9}$$

can be evaluated from $\Gamma_\lambda{}^{K^*}$ and $G_{K\pi\kappa}$, and at $k^2=0$,

$$f_+(0)=(F_K{}^2+F_\pi{}^2-F_\kappa{}^2)/2F_K F_\pi. \tag{10}$$

This expression for $f_+(0)$ is in accordance with the Ademollo-Gatto theorem[7] in that $f_+(0)-1$ is second order in the SU(3)-breaking parameters (F_K-F_π) and F_κ. Equation (10) has also been obtained in Ref. 4.

From Eqs. (3), (8), and (10), an expression for $m_\kappa{}^2$ can be derived:

$$m_\kappa{}^2=m_K{}^2\{1-(\tan\theta_A/\tan\theta_V)_{\text{eff}}{}^{-1}\}^{-1} \times\{1-(m_\pi{}^2/m_K{}^2)(F_\pi/F_K)(Z_K/Z_\pi)^{1/2}\}, \tag{11}$$

where $\theta_{A,V}$ are the axial and vector Cabibbo angles, and

$$(\tan\theta_A/\tan\theta_V)_{\text{eff}}=1.28=|F_K\{F_\pi f_+(0)\}^{-1}|, \tag{12}$$

is determined by the Cabibbo theory[8] from the measured amplitudes for K_{e3}, π_{e3}, $K_{\mu2}$, and $\pi_{\mu2}$ decays.

The above equations are as yet insufficient to determine all physical quantities, but we are in a position to place reasonable numerical bounds on them. Thus we note that the last term in Eq. (11) may be neglected because of the factor $(m_\pi{}^2/m_K{}^2)$, so that $(m_\kappa{}^2/m_K{}^2)\simeq 4.5$. The mass of κ estimated here is approximately 1050 MeV,[9] which puts the κ in an octet with the $\pi_V(1016)$ and the $\eta_V(1070)$.[10]

Table I. The values for the physically observed quantities relevant to the K_{l3} decays and to the KA, A_1, K^*, and κ mesons, as determined from the equations in the text, plus Weinberg's first sum rules (Ref. 3): $g_{KA}^2/m_{KA}^2 + F_K^2 = g_{A_1}^2/m_{A_1}^2 + F_\pi^2 = g_{K^*}^2/m_{K^*}^2 + F_\kappa^2 = 2F_\pi^2$. The values for F_K/F_π are input bounds, and the physical quantities are to lie strictly within the figures displayed in the first two columns. The quantities λ_+ are defined by $f_\pm(k^2) = f_\pm(0)\{1-\lambda_\pm k^2/m_\pi^2\} + O(k^4)$, and $\xi \equiv f_+(0)/f_-(0)$. The decay widths are in units of MeV.

	1	2	3
F_K/F_π	1.18	1.34	1.24
F_κ/F_π	−0.625	−0.46	−0.565
$g_{K^*}^2/g_{A_1}^2$	1.07	1.2	1.12
$g_{KA}^2/g_{A_1}^2$	0.8	0.26	0.615
$f_+(0)$	0.92	1.05	0.975
$f_-(0)$	−0.0175	−0.0015	−0.01
λ_+	0.018	0.018	0.018
$\xi\lambda_-$	−0.002	−0.002	−0.002
$\Gamma(K^*\to K\pi)$	57	45.5	48.5
$\Gamma(KA\to K^*\pi)$	83.3	82.5	83.3

Furthermore, using Eqs. (2), (8), and (10), and requiring that both (F_K/F_π) and (Z_K/Z_π) be positive, we obtain with m_κ^2 as given above that

$$1.34 > F_K/F_\pi > 1.18. \tag{13}$$

Restricting (F_K/F_π) within this range, $f_+(0)$ is found to lie between 0.92 and 1.05. This result may be interpreted within the context of the Ademollo-Gatto theorem[7] to mean that the "vector charges" for the scalar particles are relatively unaffected by renormalization. Motivated by this interpretation, a similar form factor $f_+{}^*(0)$ may be defined for the KA and A_1 mesons. Taking $f_+{}^*(0) \simeq 1$, we deduce from Eq. (4) that

$$\Gamma_1 \simeq m_{K^*}^2 g_{K^*}^{-1}, \tag{14}$$

which should also be accurate to within a few percent.

Equations (13) and (14) furnish us enough information to construct the first two columns of Table I, with all physical quantities lying in a range shown in the first and second columns. To obtain the vector-meson decay widths, we let $\Gamma_2 = \Gamma_1(2+\delta)$ as in Ref. 3, and, in the spirit of Eq. (14), use the value of δ determined from the A_1-A_1-ρ system, for which the best value of δ is estimated to be $-\frac{2}{3}$.[11]

Equations (5) and (6) allow us to write the following detailed form for $f_\pm(k^2)$:

$$f_+(k^2) = (k^2 + m_{K^*}^2)^{-1}\{m_{K^*}^2 f_+(0) + k^2(2F_K F_\pi)^{-1}[(F_K^2 - F_\pi^2) + (g_{KA}g_{A_1}m_K^2/m_{KA}^2 m_{A_1}^2)(1+\delta)]\}, \tag{15}$$

$$f_-(k^2) = (m_K^2 - m_\pi^2)\{-f_+(k^2)m_{K^*}^{-2} + f_+(0)(k^2 + m_\kappa^2)^{-1} + (F_K^2 - F_\pi^2)(2F_K F_\pi m_{K^*}^2)^{-1} + (1+\delta)g_{KA}g_{A_1}(2F_K F_\pi m_{KA}^2 m_{A_1}^2)^{-1}\}. \tag{16}$$

This is about as far as one can go without further information. Note that our results disagree with those of Ref. 4, where the second sum rule $g_{KA} = g_{K^*}$ is assumed to hold. Their value of m_κ^2 ($\simeq 1.6 m_K^2$) is obtained within our context by requiring $g_{KA}^2/g_{K^*}^2 = Z_K/Z_\kappa = 1$.[12] If we now say that the first equality $g_{KA}^2/g_{K^*}^2 = Z_K/Z_\kappa$ is still valid in our model, we then get a definite set of values for all physical quantities, as listed in the third column of Table I. Apparently the K_{l3} parameters are not critically dependent upon this extra relation; in fact, only g_{KA} appears to depend strongly on it. However, we see that the decay width of $K^* \to K\pi$, which depends on g_{KA}, does come out to be in exceptionally good agreement with the observed width $\Gamma_{\rm exp} = 49.2 \pm 1$ MeV.[10] In view of this, the assumption may not be all that implausible. A more critical check will have to await clarifying measurements on other decay rates, like $KA \to K^*\pi$ and $KA \to K\rho$. The relevance of this assumption to the four-point functions, vector-meson decays and mass relations, and Lagrangian models is now under further study.

Our values for the $f_+(k^2)$ are in agreement with experiment. Our λ_+ agrees with the experimental values of $\lambda_+(K^0) = 0.013 \pm 0.009$ and $\lambda_+(K^+) = 0.023 \pm 0.008$.[13] It is also known that our value of $f_+(0)$, which is close to 1, and the value of λ_+ above give a decay rate for K_{e3} close to the experimental value of $(3.61 \pm 0.20) \times 10^6\ \text{sec}^{-1}$.[14] The experimental situation regarding ξ and λ_- is ambiguous, with most of the determinations depending on the assumption that $f_-(k^2)$ is a constant.[15,16] From our point of view, the ambiguity in ξ and λ_- arises precisely because of this assumption. Our ξ and λ_- certainly agree with those found in Ref. 16, where this assumption is not made, especially for k^2 not near to zero, where their results are most accurate.[16] A better test of the values obtained here, however, will be to use Eqs. (15) and (16) to analyze the energy spectra and polarizations of the muons in future $K_{\mu 3}$ experiments in the manner described in Ref. 15 and by Auerbach et al.[17] Our results are similar to those found by Lee[18] and, as in this paper, one may reasonably expect such an analysis to indicate consistency between ξ and λ_- like those obtained here and experimental data.

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