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SOLAR NEUTRINOS AND THE SOLAR HELIUM ABUNDANCE*

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The upper limit on the solar neutrino flux set by Davis, Harmer, and Hoffman places an upper limit on the sun's initial helium abundance that is small compared with that estimated for other galactic objects. Adopting current estimates of low-energy nuclear cross-section factors, the upper limit is essentially equal to a lower bound set by demanding that the sum is at least $4\frac{1}{2} \times 10^9$ yr old.

The preliminary upper limit on the solar neutrino flux set recently by Davis, Harmer, and Hoffman[1] is an order of magnitude smaller than the flux that had been expected on the basis of solar model calculations prepared prior to the establishment of this limit. The Davis, Harmer, and Hoffman result has therefore forced a rethinking of the standard assumptions concerning both the input parameters and the input physics that are necessary for the construction of solar models.[2-4]

In an effort to contribute to a better understanding of the implications of the Davis, Harmer, and Hoffman limit, I have prepared an extensive analysis of the relationship between the neutrino flux derived from solar models and several solar input parameters. Many of my results are consistent with those already in the literature.[2-7] However, several new results have emerged and several conclusions are at variance with inferences drawn in two recent papers.[2,8] In this communication, a statement of my basic conclusions will be offered first, followed by a summary of the supporting evidence. A more complete discussion will appear elsewhere.

(1) With the standard choice of solar input parameters, the Davis, Harmer, and Hoffman limit implies an upper limit on the sun's initial helium abundance that is small compared with the helium abundance estimated for other galactic objects. The upper limit on Y (initial He^4 abundance by mass) required for consistency with the Davis, Harmer, and Hoffman limit is $Y_0 \cong 0.16$-0.17. On the other hand, almost every attempt to estimate Y for galactic objects other than the sun has led to values in the range 0.2-0.4, the most probable values clustering about 0.25-0.30. The evidence for a possibly universal, high value for Y has been amply catalogued.[5,9]

Bahcall, Bahcall, and Shaviv[2] claim that a solar $Y = 0.22 \pm 0.03$ (~ 0.22 with standard assumptions) is consistent with the Davis, Harmer, and Hoffman limit. Despite this claim, the quantitative results in the Bahcall, Bahcall, and Shaviv paper clearly indicate that consistency with the Davis, Harmer, and Hoffman upper bound can be achieved only with $Y \leq Y_0 \sim 0.16$ (with standard assumptions), in agreement with the limit presented here.

(2) With the standard assumptions, the upper

limit on Y is essentially equivalent to a lower limit on Y ($Y_{\mathrm{lower}} \sim 0.15$-$0.18$) set by demanding that the sun's age is at least $4\frac{1}{2}\times10^9$ yr. If, therefore, the eventual upper limit on the counting rate determined by the Davis, Harmer, and Hoffman experiment is reduced much below the preliminary limit, a clear internal discrepancy will be established, regardless of outside arguments for a larger helium abundance.

(3) By varying the relevant nuclear cross-section parameters as far as possible within quoted limits in directions most favorable for increasing the upper bound on Y, it is possible to escape comfortably, for the present, the embarrassment of a lower bound that exceeds an upper bound. The resultant upper bound of $Y_0 \cong 0.20$ is still small compared with the most probable Y estimated for other galactic objects. Only by varying several nuclear cross-section factors considerably beyond quoted limits is it possible to obtain a Y_0 on the order of 0.25. It is suggested that such large variations are not out of the question. Experimental cross sections can be measured only at energies large compared with energies relevant in the solar interior; it is quite possible that an extrapolation from known to unknown regions may hold surprises.

(4) Ezer and Cameron[3] have suggested that, because of mixing currents, the helium produced at any point in the solar interior need not remain at the site of formation over the sun's $4\frac{1}{2}\times10^9$-yr lifetime. They estimate that, in the case of complete mixing, the solar neutrino flux is significantly reduced relative to the case of no mixing. A working out of the Ezer and Cameron suggestion reveals that the upper bound on Y increases strongly with the assumed degree of mixing. When complete mixing is permitted, but all other standard assumptions are retained, the upper bound on Y implied by comparison with the Davis, Harmer, and Hoffman limit is $Y_0 \cong 0.24$. A very minor variation of any one of several nuclear cross-section factors (within quoted limits) permits one to achieve a Y_0 in the range 0.25-0.30. Whatever the merits of the mixing assumption may be, this conclusion is in conflict with the conclusion of Bahcall, Bahcall, and Shaviv,[9] who state that "The primordial composition necessary to obtain a solar model $\cdots$ is $\cdots$ almost completely independent of the amount of mixing."

(5) The relationship between calculated neutrino fluxes and the mean interior Y is, to first order, independent of the choice of opacity. It is therefore, to first order, independent of the relationship between Y and the opacity parameter Z. Estimates of mean interior Z, which is approximated roughly by the total abundance of elements heavier than He^4, are normally obtained on the basis of spectroscopic estimates of abundances near the solar surface. Surface abundances, even if they were known exactly, are not necessarily identical to abundances in the deep interior. To exhibit but a few examples, C^{12} has been converted almost completely into N^{14} over the inner half of the sun's mass,[10] while spallation reactions and selective diffusion have possibly affected surface abundances relative to interior abundances. Finally, even if the interior heavy-element abundances were known exactly, there remain many known sources of large errors in the opacity. A Z estimated from spectroscopic data is therefore not necessarily the appropriate choice for the opacity parameter Z.

The value of $Y \cong 0.22$ quoted by Bahcall, Bahcall, and Shaviv[2] is the result of a specific choice of Z and of a particular opacity law. $Y = 0.22$ is not consistent with the Davis, Harmer, and Hoffman limit. An insistence on consistency with this limit, rather than an insistence on a particular choice for Z, leads instead to an upper limit $Y_0 \cong 0.16$-0.17.

(6) The only uncertainty in the equation of state that appears capable of producing a significant increase in derived upper bounds on Y is that associated with the presence or absence of large-scale internal magnetic fields. An average field strength which drops off (according to the two-thirds power of the density) from 10^9 G at the center to 200 G near the surface would lead to an increase of about 0.04 in all upper limits on Y. Whether or not such a field is possible has not been explored.

The supporting evidence for the above conclusions will now be summarized.

Theoretical estimates of $\sum \sigma_i \varphi_i$ (where σ_i is the effective cross section for the absorption of neutrinos that impinge on the earth with a flux φ_i, distributed in an energy spectrum of type i) are obtained by weighting the neutrino fluxes from solar models with theoretically calculated neutrino absorption cross sections.[11]

The canonical choices for the relevant center-of-mass cross-section factors[12] used here are (in keV b) $S_{11}^{\,0} = 3.5\times10^{-22}$ ($p+p \rightarrow d+e^+ +\nu$), $S_{33}^{\,0} = 6.5\times10^3$ ($2\mathrm{He}^3 \rightarrow \mathrm{He}^4 + 2p$), $S_{34}^{\,0} = 0.6$ ($\mathrm{He}^3 + \mathrm{He}^4 \rightarrow \mathrm{Be}^7 + \gamma$), $S_{17}^{\,0} = 0.03$ ($\mathrm{Be}^7 + p \rightarrow \mathrm{B}^8 + \gamma$), $S_{114}^{\,0} = 3$ ($\mathrm{N}^{14} + p \rightarrow \mathrm{O}^{15} + \gamma$). With the exception of $S_{33}^{\,0}$, these cross-section factors are identical to those used by the author in previous investigations.[10]

In Fig. 1, the ratio (call it R) of the counting rate associated with any given solar model to the current preliminary upper limit of Davis, Harmer, and Hoffman ($\sum\sigma_i\varphi_i \leq 3\times10^{-36}$ sec^{-1} per Cl^{37} nucleus) is plotted as a function of assumed initial solar helium abundance Y for several different choices of cross-section factors. Beside each curve is that cross-section factor which differs from the canonical set. For the curve labeled S_{ij}^{1}'s, $S_{11}^{1}=S_{11}^{0}\times(4/3.5)$, $S_{34}^{1}=\frac{1}{2}S_{34}^{0}$, $S_{17}^{1}=\frac{1}{2}S_{17}^{0}$, and all other $S_{ij}^{1}=S_{ij}^{0}$. All curves in Fig. 1 pertain to solar models that have evolved for $4\frac{1}{2}\times10^{9}$ yr with no mixing (the He^4 produced at any point in the interior remains near the site of formation).

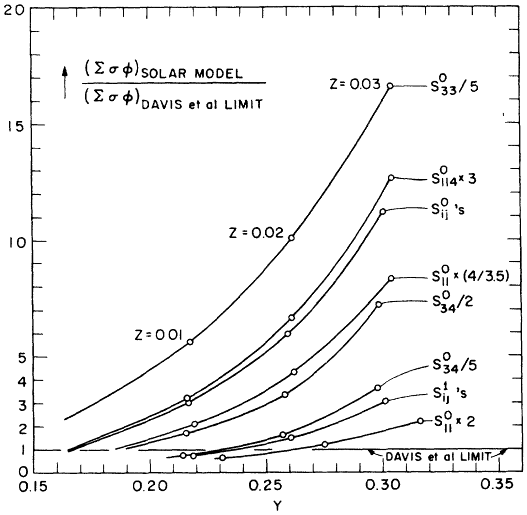

FIG. 1. The ratio of calculated values of $\sum\sigma_i\varphi_i$ to the Davis, Harmer, and Hoffman upper limit.

It is clear that, for any given choice of cross-section factors, the specification of an experimental upper limit on $\sum\sigma_i\varphi_i$ establishes an upper limit on the initial solar Y. The entries in the second column of Table I give this upper limit on Y as a function of cross section factors—<u>if</u> the Davis, Harmer, and Hoffman limit is a hard upper limit, i.e., if $R\leq1$. When the statistics are improved, the Davis, Harmer, and Hoffman limit could either decrease (the preliminary result is consistent with $R=0$) or increase. Anticipating the latter possibility, upper limits on Y have been entered into successive columns of Table I for $R\leq2$, 3, 5, and 10.

For every value of maximum R ($t=4\frac{1}{2}$ and no mixing), a "penultimate" maximum to Y may be defined by varying all cross-section factors to

Table I. Upper and lower limits to the initial solar helium abundance.

S_{ij}'s	R = 1	R = 2	R = 3	R = 5	R = 10	$Y_{\min}$
S_{ij}^{0}'s	0.166*	0.193	0.216	0.249	0.294	0.174
S_{114}^{0} x 3	.165*	.191	.212	.243	.288	.174
S_{114}^{0} x 10	.164*	.190	.210	.239	.283	.175
S_{11}^{0} x (4/3.5)	.185	.217	.241	.271	.317	.177
S_{11}^{0} x 2	.263	.310	.344			.187
$S_{34}^{0}/2$	.189	.226	.252	.280	.316	.172
$S_{34}^{0}/5$	.231	.267	.288	.318		.171
$S_{33}^{0}/5$	.140*	.158	.177*	.209	.261	.175
S_{ij}^{1}'s	.238	.278	.301	.337		.177
$(\frac{1}{u})$ = -0.01	.165*	.191	.213	.244	.288	.167
t = 6, S_{ij}^{0}'s	.148*	.169	.186	.215	.262	.158
t = 3, S_{ij}^{0}'s	.179*	.224	.254	.285	.326	.191
t = 0, S_{ij}^{0}'s	.276	.315	.342	.382		.231
$B^2S(AB)$, t = 4.7	.160*	.185	.205	.236	.282	.163
$B^2S(CDE)$, t = 4.7	0.165	0.206	0.231	0.263	0.310	0.148

the edge of their stated limits in a direction most favorable for increasing Y. Setting $S_{11}=4.1\times10^{-22}$ keV b, $S_{17}=0.03$ keV b, $S_{34}=0.38$ keV b, and $S_{33}=7\times10^3$ keV b, inspection of Table I yields $Y(1)=0.20$ as the maximum value of Y permissible if $R\leqslant1$ and if cross-section factors are held within quoted limits. In a similar fashion, $Y(2)=0.24$ and $Y(3)=0.27$ for $R\leqslant2$ and $R\leqslant3$, respectively.

Cross-section factors stated as $A\pm B$ are not unique interpretations of experimental results. A stated value of A is an extrapolation from cross sections obtained at energies considerably above those of relevance in the solar interior. A re-examination of the experimental data suggests that the following "ultimate" limits may not be out of the question: $S_{17}\geqslant0.015$ keV b, $S_{34}\geqslant0.30$ keV b. An "ultimate" limit of $S_{11}\leqslant4.3\times10^{-22}$ is also not out of the question. Adopting these "ultimate" limits, in addition to $S_{33}=7\times10^3$ keV b, "ultimate" upper limits on Y may be determined from the information in Table I. These limits are $\bar{Y}(1)=0.25$, $\bar{Y}(2)=0.28$, and $\bar{Y}(3)=0.32$ for $R\leqslant1$, 2, and 3, respectively.

The extent to which uncertainties in several input parameters affect the limits on Y is exhibited in Fig. 2. Curves labeled $t=4\frac{1}{2}, 3, 2, 1, 0$ are for different assumed solar ages but all with $S_{ij}=S_{ij}^0$. Partial results of two other investigations[2,5] are also shown. When normalized to the same set of input parameters, the relationships $\sum\sigma_i\varphi_i$ vs Y given by Sears[5] and by Bahcall, Bahcall, and Shaviv[2] are essentially identical to those presented here.

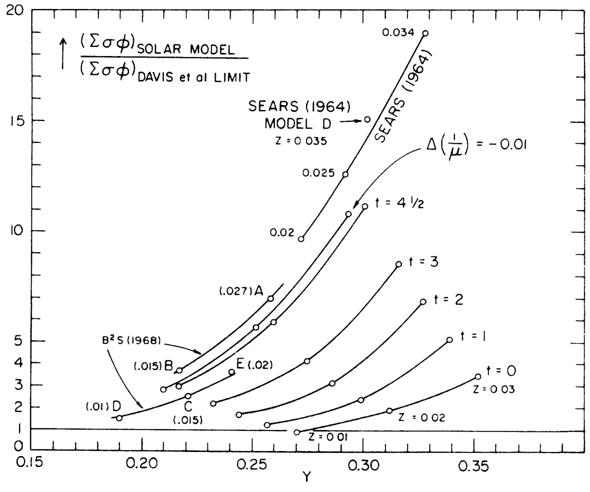

FIG. 2. The dependence of $\sum\sigma_i\varphi_i$ on solar age, opacity, and equation of state. $\mathrm{B^2S}$ = Bahcall, Bahcall, and Shaviv.

Inspection of the Sears result shows that, although the relationship between Y and Z is fairly sensitive to the choice of opacity, the relationship $\sum_i\sigma_i\varphi_i$ vs Y is, to first order, independent of this choice. The Sears model D differs from other Sears models only in the choice of opacity.

The curve in Fig. 2 labeled $\Delta(1/\mu)=-0.01$ and row 11 in Table I exhibit the effect of small variations in the equation of state. Here μ is the average molecular weight (in atomic mass units) and the variation chosen is roughly equivalent to neglecting electron degeneracy in the equation of state. Large-scale magnetic fields in the solar interior would act analogously to an increase in μ^{-1}.

The sun's age is presumably well established at about $4\frac{1}{2}\times10^9$ yr. The relationships $\sum\sigma_i\varphi_i$ vs Y shown in Fig. 2 for considerably more youthful suns are nevertheless of interest in the light of Ezer and Cameron's[3] suggestion that large-scale currents associated with a spin-down mechanism may have maintained chemical homogeneity throughout the sun's interior. The effect of mixing between regions where nuclear transformations are occurring and the rest of the sun is equivalent to choosing a smaller solar age. The limit of complete mixing during an assumed $4\frac{1}{2}\times10^9$-yr solar lifetime is equivalent to the limit of a "zero-age" sun, with a slight adjustment in Y because Y has increased by about 0.035 in the $4\frac{1}{2}\times10^9$-yr old, fully mixed sun. The entries in Table I labeled $t=0$ may, therefore, be interpreted either as upper limits on Y for a "zero-age" sun or, when 0.035 is subtracted, as upper limits on Y for a fully mixed, $4\frac{1}{2}\times10^9$-yr old sun. It is highly unlikely that the sun is younger than $4\frac{1}{2}\times10^9$ yr, but it is not out of the question that a certain amount of mixing may have taken place.

A lower limit on the sun's initial Y is set by the requirement that the solar model reach the sun's present luminosity in $4\frac{1}{2}\times10^9$ yr. The last column in Table I contains extrapolated estimates of this lower limit. Note that, in several instances (distinguished by an asterisk), an extrapolated lower limit exceeds the appropriate upper limit. This means that the parameters describing the spurious upper limit do not form a valid combination. For example, with all $S_{ij}=S_{ij}^0$, there exists no Y such that $R\leqslant1$.

In summary, if one accepts (1) current best estimates for nuclear cross-section factors, (2) a $4\frac{1}{2}\times10^9$-yr old sun, (3) the absence of significant mixing currents and magnetic fields in the sun's interior, and (4) the Davis, Harmer, and Hoffman interpretation that $\sum \sigma_i \varphi_i \leqslant 3\times10^{-36}$ sec^{-1} per Cl^{37} atom, then an upper limit to the initial solar helium abundance lies in the range $Y_0 \cong 0.16\text{-}0.17$. This limit is uncomfortably close to a lower limit of $Y_{\rm lower} \cong 0.15\text{-}0.18$ set by demanding that the solar model have the sun's luminosity after $4\frac{1}{2}\times10^9$ yr. If all cross-section factors are varied in a direction favorable for decreasing $\sum \sigma_i \varphi_i$, but not beyond limits customarily quoted, then a "penultimate" upper limit on Y is $Y(1) \cong 0.20$. Finally, if cross-section factors are varied beyond conventional limits by extrapolating low-energy cross-section measurements differently, an "ultimate" upper limit on Y is $\bar{Y}(1) \cong 0.25$. Thus, if conventional assumptions about the sun are maintained, a close similarity between model-determined values for solar Y and estimates of Y for other galactic objects can be achieved only by adopting cross-section factors outside commonly accepted limits. This conclusion could, of course, be avoided if the Davis, Harmer, and Hoffman limit were an underestimate.

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THE 3π MASS SPECTRUM IN THE REACTION $\pi^- p \to p\pi^+\pi^-\pi^-$ AT 13 AND 20 GeV/c.*†

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In the 3π mass spectrum from the reaction $\pi^- p \to p\pi^+\pi^-\pi^-$ at 13 and 20 GeV/c, we observe an unstructured A enhancement and the production of the $\pi(1640)$ meson. Our data for the $\pi^-\rho^0$ mass are compared with the predictions of the one-pion exchange, the diffraction-dissociation, and the double-Regge-pole models. Only the double-Regge-pole model is able to reproduce the shape of the A enhancement.

The reaction $\pi^- p \to p\pi^+\pi^-\pi^-$ has been studied at incident π^- momenta ranging from 3.2 to 16 GeV/c in recent years.[1] The process has been characterized by strong production of the $\Delta^{++}(1236)$ and ρ^0. Of particular interest in this reaction, however, has been A-meson resonance production in the $\pi^-\rho^0$ mass spectrum between 1.0 and 1.45 GeV. The A_2 meson at a mass of 1305 MeV is the only well established resonance in the A region, while controversy exists as to whether the A_1 effect at a mass of approximately 1070 MeV is a genuine resonance or a Deck[2]-type kinematical enhancement.

We present here our results concerning the $\pi^+\pi^-\pi^-$ system in the process

$$\pi^- p \to p\pi^+\pi^-\pi^- \qquad (1)$$

at incident π^- momenta of 13 and 20 GeV/c. Our data come from 100 000 photographs taken in the Brookhaven National Laboratory 80-in. hydrogen bubble chamber, 50 000 at each of the two incident π^- momenta. We have obtained 1292 events for process (1) at 20 GeV/c from all of the available film at this energy and 1192 events at 13 GeV/c from approximately 70% of the film. The contamination from events with one or more π^0's