ROLE OF IMPURITIES IN THE PROPERTIES OF NEARLY FERROMAGNETIC SYSTEMS: SPIN-ORBIT INTERACTIONS

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(Received 31 July 1968)

We have investigated the role of spin-independent and spin-dependent impurity scattering in itinerant Fermi systems which have a large susceptibility enhancement and find that at very low temperatures the smearing of the Fermi surface due to spin-independent scattering leads to a stronger temperature dependence of the specific heat than for pure systems, but that at still lower temperatures spin-dependent scattering changes the temperature dependence to the usual power-series expansion.

Since the discovery of the effects of spin fluctuations in the equilibrium[1] and transport[2] properties of alloys, there has arisen the question of how impurity scattering modifies the original theory.[3] Recently, Fulde and Luther[4] carried out a calculation of the effects of spin-independent impurity scattering. We have examined this problem and find that although the calculations they have published are correct, the conclusions they reach differ considerably from ours. We have, in addition, included the effect of spin-dependent (e.g., spin-orbit) impurity scattering and find yet another behavior for the very low temperature susceptibility and specific heat.[5]

We will first address ourselves to the case of spin-independent impurity scattering. Fulde and Luther[4] found that at wavelengths shorter than the mean free path the dynamic susceptibility $\chi(q,\omega)$ was essentially identical to that found originally.[3] However, for wavelengths longer than the mean free path, the low-lying mode of $\chi(q,\omega)$ which went as $\omega=\kappa_0^2 v_F q$ changes into an even lower lying diffusive mode at $\omega=\kappa_0^2 D q^2$, where D is the diffusion constant $D=\frac{1}{3}v_F^2\tau$. (τ is the lifetime, $\kappa_0^2=1-\bar{I}$ is the exchange enhancement of the susceptibility.)

The effect on the specific heat of having a lower lying mode over a small region of phase space is to increase the mass enhancement slightly, and change the temperature dependence of the $T^3\ln T$ term to a more rapidly varying term which goes as $T^{3/2}$ for temperatures much less than $\hbar\kappa_0^2/k_B\tau$. In the microscopic calculation which we will outline later both the change in mass enhancement and the coefficient of the $T^{3/2}$ term go as a power of $1/p_F l$. This coefficient should be much less than 1 if the numerical constants given in microscopic calculations are to be correct. However, we expect on physical grounds that under the conditions $q\ll 1/l\ll p_F$ a diffusive mode will be present, and thus the very low-temperature enhancements mentioned above will also be present. The diagrams left out of the microscopic calculation will serve to renormalize the lifetime appearing in the diffusion constant. It should be emphasized that the lowering of the spin-fluctuation mode does not eliminate the $T^3\ln T$ behavior in favor of a less singular T^3 behavior but rather replaces it by the more singular $T^{3/2}$ dependence. Thus it appears unjustified to conclude[4] from this calculation that the anomaly in the specific heat will be eliminated by normal impurity scattering.

We will now discuss the effects of spin-orbit scattering. One effect is to introduce a new lifetime τ_{SO} which is usually much greater than the lifetime τ introduced above. The presence of spin-orbit scattering will be seen to lead to a new low-temperature regime $T\ll\hbar\kappa_0^2/k_B\tau_{SO}$. In this range the specific heat has the usual low-temperature power-series expansion $C=\gamma T-bT^3+O(T^5)$.[6] This new regime comes about because of the rather radical effect spin-dependent scattering has on the dynamic susceptibility. For $q\ll 1/l$,

$$\chi(q,\omega)=\frac{\chi_S^{\,0}}{\kappa_0^{\,2}}\frac{\kappa_0^{\,2}(1/\tau_{so}+Dq^2)}{[\kappa_0^{\,2}(1/\tau_{so}+Dq^2)-i\omega]}. \qquad (1)$$

The pole in the susceptibility thus no longer approaches the origin as q goes to zero but rather remains at a finite value, $i\kappa_0^2/\tau_{SO}$. Another effect of τ_{SO} is to saturate the mass enhancement due to the low-lying modes. Without spin-orbit scattering the mass enhancement is singular as κ_0^2 goes to zero. However, with spin-orbit scattering the maximum mass enhancement is of the order of τ_{SO}/τ.

For $q\gg 1/l$, the pole structure of χ is just as given in Ref. 3; i.e., there is no effect of impurity scattering.

At this stage we will outline the calculation of the susceptibility. In the contact-interaction model used by Doniach and Engelsberg,[3] the specific heat may be calculated once the dynamic susceptibility is known. The impurity scattering is approximated by a term in the Hamiltonian of the form[7]

$$H' = \sum_{\substack{\vec{R}_i, \vec{k}, \vec{k}' \\ \alpha, \beta}} \exp[i(\vec{k}-\vec{k}')\cdot\vec{R}_i] V_{\alpha\beta}(\vec{k}, \vec{k}') c_{k\alpha}{}^{\dagger} c_{k'\beta},$$

where $V_{\alpha\beta}(\vec{k}, \vec{k}') = a(\vec{k}, \vec{k}') I_{\alpha\beta} + ib(\vec{k}, \vec{k}')(\vec{k}\times\vec{k}')\vec{\sigma}_{\alpha\beta}$, $I_{\alpha\beta}$ is the unit matrix in spin space, and $\vec{\sigma}_{\alpha\beta}$ are the Pauli matrices. Following Abrikosov and Gor'kov,[7] the unenhanced susceptibility is calculated once the vertex function

$$\vec{\Lambda}_{\alpha\beta}(p, q) = \left[\vec{\sigma}_{\alpha\beta} + n_{\mathrm{imp}} \int \frac{d^3p'}{(2\pi)^3} V_{\alpha\gamma}\left(p + \frac{q}{2}, p' + \frac{q}{2}\right) \vec{\Lambda}_{\gamma\mu}(p', q) V_{\mu\beta}\left(p' - \frac{q}{2}, p - \frac{q}{2}\right)\right] G\left(p - \frac{q}{2}\right) G\left(p + \frac{q}{2}\right) \tag{3}$$

is known. Here G is the single-particle propagator modified by the presence of impurities. If the $\vec{q}$ dependence of the scattering matrices V is neglected and the solution for the poles of Λ is required only to order $(q/p_F)^2$, then the solution to (3) is obtained after some algebraic manipulation. The transverse unenhanced dynamic susceptibility is given as

$$\chi_{0\mathrm{imp}}{}^{+-}(q) = i \int \frac{d^4p}{(2\pi)^4} \sigma_{\alpha\beta}{}^{+} \Lambda_{\beta\alpha}{}^{-}(p, q). \tag{4}$$

In the limit $|\vec{q}| \ll 1/l$, the technique of Fulde and Luther[4] may be used to obtain the solution given in (1), where

$$\chi^{+-}(q) = \frac{\chi_{0\mathrm{imp}}{}^{+-}(q)}{1 - I\chi_{0\mathrm{imp}}{}^{+-}(q)},$$

$$\frac{1}{\tau_{\mathrm{so}}} = \frac{4}{3} \frac{n_{\mathrm{imp}}}{2} N(0) \int d\Omega |b(\vec{k}, \vec{k}')|^2 \sin^2\theta_{k,k'}, \tag{5}$$

and

$$\frac{1}{\tau} = \frac{n_{\mathrm{imp}} N(0)}{2} \int d\Omega [|a(\vec{k}, \vec{k}')|^2 + |b(\vec{k}, \vec{k}')|^2 \sin^2\theta_{\vec{k}, \vec{k}'}].$$

Note that within the random phase approximation the effective diffusion constant $(k_0{}^2 D)$ and the effective inverse spin-orbit lifetime $(\kappa_0{}^2/\tau_{\mathrm{so}}{}^{-1})$ go to zero proportional to the inverse static susceptibility as the ferromagnetic limit is approached.

We will now check the consistency of this approximation by use of Fermi-liquid theory. The expression for the collision term in the Landau transport equation given by Heine, Nozières, and Wilkins[9] is easily generalized to include spin-dependent scattering. Making a collision time approximation to this integral we obtain a set of modified Hasegawa equations[8] for the motion of the magnetization,

$$\partial\vec{M}/\partial t = \gamma(\vec{M}\times\vec{H}) - (1/\tau_{\mathrm{so}}{}^{\mathrm{eff}})(\vec{M} - \vec{M}^0) + D_{\mathrm{eff}} \nabla^2(\vec{M} - \vec{M}^0), \tag{6}$$

where $\vec{M}^0 = \chi_s \vec{H}$.

The solution of (6) for the Fourier transform of the transverse susceptibility is identical to (1) if we identify $1/\tau_{\mathrm{so}}{}^{\mathrm{eff}} = \kappa_0{}^2/\tau_{\mathrm{so}}$ and $D_{\mathrm{eff}} = \kappa_0{}^2 D$. However, including the spin-flip scattering vertices and renormalization effects in the effective mass given in the Landau transport equation, we find

$$\frac{1}{\tau_{\mathrm{so}}{}^{\mathrm{eff}}} = \frac{\kappa_0{}^2 \ln\kappa_0{}^2}{\tau_{\mathrm{so}}} \quad \text{and} \quad D_{\mathrm{eff}} = \frac{\kappa_0{}^2 D}{(\ln\kappa_0{}^2)^2} \tag{7}$$

in the limit $\kappa_0^2 \to 0$. We have made the identification[10]

$$\frac{m^*/m}{(1+B_0)} = \frac{1}{\kappa_0^2}$$

and taken the effective mass proportional to $\ln\kappa_0^2$.[3] Thus Fermi-liquid theory also yields an effective lifetime and diffusion rate which goes to zero at the ferromagnetic instability; however the approach to zero is slightly different than that predicted by the random phase approximation. The $\ln\kappa_0^2$ factor enters the inverse spin-orbit lifetime because of the large enhancement of the forward spin-flip scattering amplitude at the ferromagnetic instability, whereas the factor entering the diffusion rate comes from the square of the Fermi velocity, which enters D.

We next consider the change in specific heat of a nearly ferromagnetic system due to impurities by calculating the shift in thermodynamic potential

$$\Delta\Omega = \tfrac{3}{2}\,2\int_0^\infty \frac{d\omega}{\pi} n(\omega) \int_0^{2p_\mathrm{F}} \frac{dq\, q^2}{2\pi^2} \mathrm{Im}[\ln(1 - I\chi_{0\mathrm{imp}}) + I\chi_{0\mathrm{imp}}], \tag{8}$$

where for $q < 1/l$,

$$\chi_{0\mathrm{imp}}(\vec{q}, \omega) = \frac{N(0)(1/\tau_{\mathrm{so}} + Dq^2)}{1/\tau_{\mathrm{so}} + Dq^2 - i\omega} - \alpha' \frac{q^2}{p_\mathrm{F}^2}, \tag{9}$$

where α' is a number $O(1)$ and for $q > 1/l$,

$$\chi_{0\mathrm{imp}}(\vec{q}, \omega) = N(0)\left(1 + \frac{1}{12}\frac{q^2}{p_\mathrm{F}^2} - i\frac{\pi}{4}\frac{\omega}{\epsilon_\mathrm{F}}\frac{p_\mathrm{F}}{q}\right).$$

The result for the mass enhancement is

$$\left(\frac{m^*}{m}\right) - 1 = \frac{3}{2\pi}\left\{\frac{1}{\tau/\tau_{\mathrm{so}} + \frac{1}{3}\kappa_0^2(p_\mathrm{F}l)^2} + 3\pi T^2 \ln\left[\frac{\kappa_0^2 + \frac{1}{3}T}{\kappa_0^2 + T/12(p_\mathrm{F}l)^2}\right]\right\}, \tag{10}$$

where we have set $\alpha' = 1$. The saturation effect mentioned before is apparent in (10): If $1/\tau_{\mathrm{so}} = 0$, the effective mass would increase indefinitely as $\kappa_0^2 \to 0$. However, the spin-orbit scattering limits the mass enhancement to be less than τ_{so}/τ. The next term in the expansion of the low-temperature specific heat depends upon which of the three possible regimes we are in: For region I, $\kappa_\mathrm{B}T \ll \hbar\kappa_0^2/\tau_{\mathrm{so}}$, the next term is an ordinary T^3 term; for region II, $\hbar\kappa_0^2/\tau_{\mathrm{so}} \ll \kappa_\mathrm{B}T \ll \hbar\kappa_0^2/\tau$, $C \propto (\kappa_\mathrm{B}T/\kappa_0^2\tau)^{3/2}$; and for region III, $\hbar\kappa_0^2/\tau \ll \kappa_\mathrm{B}T \ll \kappa_0^2\epsilon_\mathrm{F}$, $C \propto (\kappa_\mathrm{B}T/\kappa_0^2\epsilon_\mathrm{F})^3 \ln(\kappa_\mathrm{B}T/\kappa_0^2\epsilon_\mathrm{F})$; i.e., there are no effects of impurities. This behavior is sketched in Fig. 1.

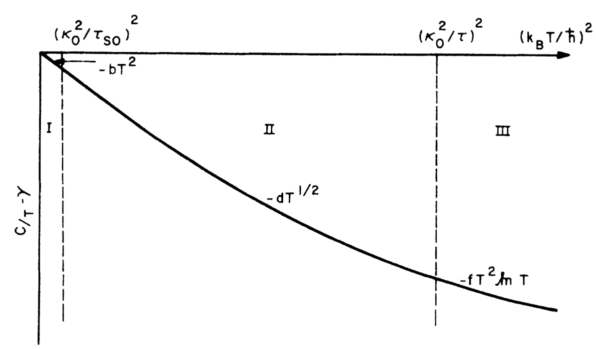

FIG. 1. Influence of impurities which separate the low-temperature behavior of the specific heat into three regions.

We should like to thank P. W. Anderson for discussions of the problem, and P. C. Hohenberg and N. R. Werthamer for reviewing the effects of impurity scattering in superconductors. We are grateful to P. Fulde and A. Luther for sending a preprint of their work.

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TWO-CURRENT CONDUCTION IN NICKEL

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Measurements on the low-temperature electrical resistivity of dilute nickel-based alloys give strong evidence that spin-↑ and spin-↓ electrons carry current in parallel, providing important implications for the interpretation of transport properties of pure as well as alloy ferromagnets.

Electrical-resistance results on Ni[1] have recently been interpreted by Herring[2] by a model in which no account is taken of the ferromagnetic nature of the metal. On the basis of measurements on Fe containing a number of dilute impurities, we have suggested[3] that there is strong evidence that in ferromagnetic metals spin-↑ and spin-↓ electrons carry current in parallel with different conductivities (at least at low temperatures). This model has a direct bearing on the interpretation of transport properties of pure as well as alloy ferromagnets.

We report in this Letter experiments on the electrical resistivity of Ni containing Co, Mn, Cr, and Ti as dilute impurities. The results confirm the two-current model and can lead to a better understanding of the scattering processes in Ni.

The model used can be summarized as follows. It is assumed that the conduction electrons of the two half-bands of opposite spin have different relaxation times $\tau_\uparrow$ and $\tau_\downarrow$. In addition there are spin-flip and electron-electron collision processes characterized by a relaxation time $\tau_{\uparrow\downarrow}$ which couple the Boltzmann equations for the two sorts of conduction electrons; these equations can then be written[4]

$$e\vec{E}\cdot\vec{v}\frac{\partial f_0}{\partial E} = -\frac{f_\uparrow - f_0}{\tau_\uparrow} - \frac{f_\uparrow - f_\downarrow}{\tau_{\uparrow\downarrow}},$$

$$eE\cdot\vec{v}\frac{\partial f_0}{\partial E} = -\frac{f_\downarrow - f_0}{\tau_\downarrow} - \frac{f_\downarrow - f_\uparrow}{\tau_{\uparrow\downarrow}} \qquad (1)$$

in the usual notation.[5] By solving the coupled system of equations, it can be shown[3] that the total resistivity is

$$\rho = \frac{\rho_\uparrow\rho_\downarrow + \rho_{\uparrow\downarrow}(\rho_\uparrow + \rho_\downarrow)}{\rho_\uparrow + \rho_\downarrow + 4\rho_{\uparrow\downarrow}}, \qquad (2)$$

where $\rho_\uparrow = m/ne^2\tau_\uparrow$, $\rho_\downarrow = m/ne^2\tau_\downarrow$, and $\rho_{\uparrow\downarrow} = m/ne^2\tau_{\uparrow\downarrow}$.

We will further assume for $\rho_\uparrow$ and $\rho_\downarrow$ that the effects of the various scattering mechanisms add together by the quasi-Matthiessen rules:

$$\rho_\uparrow(T) = \rho_\uparrow^{\,0} + \rho_\uparrow^{\,i}(T),$$

$$\rho_\downarrow(T) = \rho_\downarrow^{\,0} + \rho_\downarrow^{\,i}(T), \qquad (3)$$

where $\rho_\uparrow^0, \rho_\downarrow^0$ are the temperature-independent scattering components due to the impurities while $\rho_\uparrow^i, \rho_\downarrow^i$ are the temperature-dependent parts of the scattering for the two half-bands. It will also be assumed that the temperature-independent part of $\rho_{\uparrow\downarrow}$ is zero.[6] We will concentrate attention on the low-temperature limit within these restrictions.

When

$$\rho_\uparrow^{\,0}, \rho_\downarrow^{\,0} \gg \rho_\uparrow^{\,i}, \rho_\downarrow^{\,i}, \rho_{\uparrow\downarrow}, \qquad (4)$$

the development of Eq. (2) gives

$$\rho = \rho_0 + \rho_i + \rho_i\frac{(\alpha-\mu)^2}{(1+\alpha)^2\mu} + \rho_{\uparrow\downarrow}\left(\frac{\alpha-1}{\alpha+1}\right)^2, \qquad (5)$$

where $\rho_0 = \rho_\uparrow^0\rho_\downarrow^0/(\rho_\uparrow^0 + \rho_\downarrow^0)$ is the residual resis-