Company, Amsterdam, The Netherlands, 1968).

[4]Throughout this Letter the quantity Δ^2 refers to the square of the four-momentum transfer to the $\pi^-\pi^-\pi^+$ system from the incident π^- meson.

CURRENT-ALGEBRA, SUBTRACTED DISPERSION RELATION, AND K_{l3} FORM FACTORS

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A calculation of the K_{l3} form factors is done on the basis of current algebra, partial conservation of axial-vector currents (strangeness conserving), and dispersion relations. Assuming once-subtracted and unsubtracted dispersion relations for $f_+(q^2)+f_-(q^2)$ and $f_+(q^2)-f_-(q^2)$, respectively, and K^* dominance, the K_{l3} decay parameters ξ, λ_+, and λ_- are calculated. All results are consistent with the present experimental indications.

There has been a considerable number of theoretical discussions[1-6] of the K_{l3} form factors on the basis of current algebra and partial conservation of axial-vector currents (PCAC) (or their variants). It seems, however, that the present theoretical status of the K_{l3} form factors is still far from being free from confusion and uncertainty. We shall report in this note a calculation of these form factors from the dispersion point of view, using the PCAC and current-algebra result to fix the subtraction constant in a once-subtracted dispersion relation for the combination $f_+(q^2)+f_-(q^2)$. It is felt that the present calculation is probably less subject to the uncertainties and ambiguities that plagued, to varying degrees, some of the earlier calculations.

The basic relation for the K_{l3} form factors in the approach based on PCAC and current algebra is the Callan-Treiman-Mathur-Okubo-Pandit (CTMOP) relation[1]:

$$f_+(-M_K^2)+f_-(-M_K^2)=F_K/F_\pi, \tag{1}$$

which holds at the unphysical point $q^2=-M_K^2$, where q^2 is the momentum-transfer variable and $f_\pm(q^2)$ are the usual K_{l3} form factors (to be defined below). In order to derive reliable information concerning the physical form factors, one must have a "suitable" procedure of analytically extrapolating the soft-pion current-algebra result (1) from the unphysical point to the physical region. A natural choice of such an analytic procedure is provided by the dispersion approach. In particular, we shall adopt in our present calculation the point of view forwarded by Okubo and his collaborators,[7] and in a slightly different context by Fubini and Furlan.[8] The basic point is that the soft-pion current-algebra result provides the subtraction constant, if a once-subtracted dispersion relation is assumed for the appropriate amplitude. As we shall see, we can in our calculation always keep the kaon momentum on the mass shell and thus avoid the potentially unreliable large mass extrapolation inherent in the use of the PCAC for the strangeness-changing axial-vector current.

Traditionally,[9] unsubtracted dispersion relations are assumed for the K_{l3} form factors (and for other form factors, such as the π_{l3} form factor and the pion electromagnetic form factor, etc.). However, it has recently been realized[10] that the assumption of unsubtracted dispersion relations may be too restrictive, and in a few instances leads to paradoxical results. We do not know whether a subtracted dispersion relation is necessary in the case of K_{l3} form factors. Notwithstanding, the use of subtracted dispersion relations, provided a knowledge of the subtraction constants is available, definitely offers better hope for a reliable calculation, since practically in every calculation use has to be made, in one way or another, of the assumption of dominance by the low-lying states. In a calculation based on unsubtracted dispersion relations, assuming their validity, the effects of the continuum and the high-lying excitations are hard to estimate, although they may in fact be important. If subtracted dispersion relations are used, most of these effects are presumably effectively represented by the subtraction constants, and the contributions to the dispersion integral from the high-lying states are suppressed. This makes the dominance of the dispersion integral by low-lying states a better approximation.

Denoting the K_{l3} hadronic matrix element by

$$\langle\pi^0(k)|V_\mu^{(K^+)}(0)|K^+(p)\rangle = 2^{-\frac{1}{2}}[f_+(q^2)(p_\mu+k_\mu)+f_-(q^2)(p_\mu-k_\mu)], \tag{2}$$

where $q\equiv p-k$, we shall assume once-subtracted and unsubtracted dispersion relations for

$$f_1(q^2)\equiv f_+(q^2)+f_-(q^2) \tag{3}$$

and

$$f_2(q^2)\equiv f_+(q^2)-f_-(q^2), \tag{4}$$

respectively. The once-subtracted dispersion relation for $f_1(q^2)$ can be obtained following the heuristic argument of Ref. 7. Using PCAC and current algebra, we have, for $p^2=-M_K^2$ and $k^2=0$ (but not necessarily $k\to 0$),

$$\langle\pi^0(k)|V_\mu^{(K^+)}(0)|K^+(p)\rangle = F_\pi^{-1}(F_K p_\mu + k^\nu M_{\mu\nu}), \tag{5}$$

where

$$M_{\mu\nu}=i\int d^4x\, e^{ik\cdot x}\langle 0|\theta(x^0)[A_\nu^{(\pi^0)}(x), V_\mu^{(K^+)}(0)]|K^+(p)\rangle, \tag{6}$$

and F_π and F_K are the pion and kaon decay constants, respectively. By letting $k\to 0$ and $p\to\infty$ such that $p^2=-M_K^2$ and q^2 is finite, we obtain

$$f_1(q^2,k^2=0,p^2=-M_K^2)=\frac{F_K}{F_\pi}+\frac{(q^2+M_K^2)}{\pi}\int\frac{\mathrm{Im}f_1(q'^2,k^2=0,p^2=-M_K^2)}{(q'^2-q^2)(q'^2+M_K^2)}dq'^2. \tag{7}$$

If, in the spirit of PCAC, we assume that the form factor $f_1(q^2,k^2=-M_\pi^2,p^2=-M_K^2)$ is not appreciably different from the off-mass-shell value at $k^2=0$, then (7) can be taken as the once-subtracted dispersion relation for the physical form factor. We note that the CTMOP relation (1) is obtained from (7) by setting $q^2=-M_K^2$.

In addition to the once-subtracted dispersion relation (7) for $f_1(q^2)$, we assume an unsubtracted dispersion relation for $f_2(q^2)$:

$$f_2(q^2)=\frac{1}{\pi}\int\frac{\mathrm{Im}f_2(q'^2)}{q'^2-q^2}dq'^2. \tag{8}$$

We shall assume[11] the dominance of the dominance of the dispersion integrals by the $K^*(890)$. In this approximation, we obtain (neglecting pion mass)

$$f_1(q^2)=\frac{F_K}{F_\pi}-\frac{(q^2+M_K^2)}{(q^2+M_{K^*}^2)}\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}, \tag{9}$$

$$f_2(q^2)=\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}\frac{(M_{K^*}^2+M_K^2)}{(q^2+M_{K^*}^2)}, \tag{10}$$

where G_{K^*} and $G_{K^*K\pi}$ are the coupling constants defined in the usual manner, for $K^*\to 0$ and $K^*\to K+\pi$, respectively. When expansion is made in q^2, (9) and (10) yield

$$f_+(q^2)\simeq f_+(0)(1-\lambda_+q^2/M_\pi^2), \tag{11}$$

$$f_-(q^2)\simeq f_-(0)(1-\lambda_-q^2/M_\pi^2), \tag{12}$$

with

$$f_+(0)\simeq\left(\frac{1}{2}\right)\left(\frac{F_K}{F_\pi}+\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}\right), \tag{13}$$

$$f_-(0)\simeq\left(\frac{1}{2}\right)\left(\frac{F_K}{F_\pi}-\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}\frac{M_{K^*}^2+2M_K^2}{M_{K^*}^2}\right), \tag{14}$$

$$\lambda_+f_+(0)\simeq\left(\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}\right)\left(\frac{M_\pi^2}{M_{K^*}^2}\right), \tag{15}$$

$$\lambda_-f_-(0)\simeq-\left(\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2}\right)\left(\frac{M_K^2}{M_{K^*}^2}\right)\left(\frac{M_\pi^2}{M_{K^*}^2}\right). \tag{16}$$

On the basis of the Ademollo-Gatto theorem,[12] we shall assume

$$f_+(0)=1, \qquad (17)$$

which is believed to be correct to within a few percent.[13] Using (17), we can express the three K_{l3} decay parameters ξ, λ_+, and λ_- in terms of a single parameter, which is the empirically known ratio F_K/F_π:

$$\xi \equiv \frac{f_-(0)}{f_+(0)} = \left(1+\frac{M_K^2}{M_{K^*}^2}\right)\left(\frac{F_K}{F_\pi}\right) - \left(1+2\frac{M_K^2}{M_{K^*}^2}\right), \qquad (18)$$

$$\lambda_+ = \left(2-\frac{F_K}{F_\pi}\right)\left(\frac{M_\pi^2}{M_{K^*}^2}\right), \qquad (19)$$

$$\xi\lambda_- = -\left(2-\frac{F_K}{F_\pi}\right)\left(\frac{M_K^2}{M_{K^*}^2}\right)\left(\frac{M_\pi^2}{M_{K^*}^2}\right). \qquad (20)$$

It is clear from (18) and (20) that ξ and $\xi\lambda_-$ are both very small, and the value of λ_- is extremely sensitive to the ratio F_K/F_π [and to the SU(3) symmetry-breaking effect on $f_+(0)$, in view of our use of (17)]. Taking[14]

$$F_K/F_\pi = 1.28 \text{ (or 1.26)} \qquad (21)$$

we obtain the following numerical values:

$$\xi = 0.06 \text{ (or 0.03)}, \qquad (22)$$

$$\lambda_+ = 0.017 \text{ (or 0.018)}, \qquad (23)$$

$$\lambda_- = -0.09 \text{ (or } -0.18), \qquad (24)$$

$$\xi\lambda_- = -0.005 \text{ (or } -0.005). \qquad (25)$$

Thus, λ_- could well be considerably larger than λ_+, with a distinct possibility of being an order of magnitude larger.

Experimentally, while the averaged K_{l3}^+ and K_{l3}^0 value for λ_+ is given by[15]

$$\lambda_+ = 0.019 \pm 0.006, \qquad (26)$$

in agreement with our calculated result (23), reliable values for ξ and λ_- are still lacking. It seems to be the present consensus that ξ is small, and the analysis of Auerbach et al.[16] indicates the possibility that λ_- could indeed be an order of magnitude larger than λ_+. It is of great interest to have an experimental clarification of these points.

Our results are in qualitative agreement with those of Lee,[5] which are obtained on the basis of broken SU(3) chiral dynamics and the field-current identity, but at variance with the usual pole-dominance calculations[2,17] based on unsubtracted dispersion relations. The difference between the latter calculations and our present one lies in the fact that we may have effectively taken into account the effects due to the high-lying states by using a subtracted dispersion relation. In this regard, it is worth mentioning that the smallness of ξ and consequently the large value for λ_-, as is clear from (14), results from a cancellation[18] between the contribution from the low-lying K^* state and the contribution from the high-lying states, the latter contribution being effectively represented by the subtraction constant, which is learned from current algebra and PCAC.

It is a pleasure to thank Professor B. W. Lee for an illuminating discussion.

Note added in proof.—The problem of extrapolation of the CTMOP relation is discussed in a preprint by Ademollo, Denardo, and Furlan.[19] Subtracted dispersion relations are used to discuss the K_{l3} form factors by N. Fuchs.[20]

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[10]S. G. Brown and G. B. West, to be published, and references therein.

[11]There is no strong evidence for the existence of the $\kappa(725)$. See A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77 (1968). Even if it exists, it would probably be weakly coupled to other systems (according to the present experimental indications) and thus numerically insignificant. It is a reasonable assumption that the κ is related to the divergence of the strangeness-changing vector current and its coupling constants are nonvanishing only to the extent that the SU(3) symmetry is broken. This means that the contribution of κ_{l3} form

factors is of second order in SU(3) symmetry breaking. It is thus expected that the inclusion of κ in our calculations would not significantly change the results.

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EXCHANGE DEGENERACY AND BACKWARD SUPERCONVERGENCE IN $N\bar{N}$ SCATTERING*

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The treatment of a weak u channel by the usual prescription of exchange degeneracy, based on a potential-theory model, is shown to violate crossing badly. On the other hand, a crossing-symmetric treatment of the same situation would lead to backward superconvergence relations, which provide the basis for a new prescription of exchange degeneracy. Several consequences of this crossing-symmetric prescription of exchange degeneracy are discussed, with particular reference to $N\bar{N}$ scattering.

It was pointed out some time back that the low-lying mesons (π, η, ρ, ω, and φ) fail to saturate the superconvergence relations for backward $N\bar{N}$ scattering.[1] In fact, for some of the superconvergent amplitudes all these contributions were shown to be adding up. It was then suggested that one should expect an equally significant coupling from the axial-vector and tensor mesons in order to match the above.[2] In this note we shall invoke the idea of exchange degeneracy in order to suggest that the major cancellation should indeed occur between the exchange-degenerate partners[3] – between π and B, ρ and A_2, ω and f, etc. This will then give a relation for each pair of residues, which is significantly distinct from the degenerate residue functions postulated by Arnold[4] and Ahmadzadeh[5] on the basis of a potential-theory analogy.

To illustrate this we consider a hypothetical situation where there are no low-lying single-particle states in the u channel. It should be noted that, in view of the weak couplings of the deuteron and the virtual singlet state, the physical $N\bar{N}$ scattering is not far from this ideal case. In such an ideal situation, all the u-channel spin-parity amplitudes (and their first few moments) should be superconvergent for fixed $u=0$. Thus one can form a combination which singles out a pair of exchange-degenerate partners in the s channel, as for instance the combination corresponding to the s-channel spin-pairty amplitude $f_2(s,t,u)$, which singles out the ρ-A_2 pair.[6] Then there must be cancellation between these exchange-degenerate partners in order to ensure superconvergence for such amplitudes. In the narrow-resonance approximation this gives a sum rule between their coupling constants, since any representation for writing down the s-channel Regge contributions to the s-channel discontinuity of the scattering amplitude reduces to the Breit-Wigner form in the narrow-resonance approximation. We consider as an example the contribution of ρ and A_2 trajectories to $f_2(s,t,0)$ in the usual Regge representation. We have

$$f_2(s,t,0)=\beta_\rho(s)\frac{[1-\exp(i\pi\alpha_\rho)]}{2\sin\pi\alpha_\rho}P_{\alpha_\rho}(-1)+\beta_{A_2}(s)\times\frac{[1-\exp(i\pi\alpha_{A_2})]}{2\sin\pi\alpha_{A_2}}P_{\alpha_{A_2}}(-1). \quad (1)$$

In the narrow-resonance approximation, this gives an s channel discontinuity

$$\operatorname{Im}f_2(s,t,0)=\beta_\rho(s)P_{\alpha_\rho}(-1)\{[-\pi\alpha_\rho'(s-m_\rho^2)+i\epsilon]^{-1}-[-\pi\alpha_\rho'(s-m_\rho^2)-i\epsilon]^{-1}\}$$
$$+\beta_{A_2}(s)P_{\alpha_{A_2}}(-1)\{[\pi\alpha_{A_2}(s-m_{A_2}^2)+i\epsilon]^{-1}-[\pi\alpha_{A_2}(s-m_{A_2}^2)-i\epsilon]^{-1}\}. \quad (2)$$