THERMAL CONDUCTIVITY OF He I NEAR THE SUPERFLUID TRANSITION

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The thermal conductivity of He I was measured at saturated vapor pressure from 10^{-7} °K to 5×10^{-3} °K above T_λ. The results agree in detail with dynamic scaling predictions and yield a critical exponent of 0.334 ± 0.005. The prediction is $\frac{1}{3}$.

The purpose of this communication is to present high-resolution measurements of the thermal conductivity κ of He I near the superfluid transition temperature T_λ, which agree quantitatively with a recent theoretical prediction[1] based on an extension[1,2] of the Widom-Kadanoff scaling laws[3,4] to dynamic processes. The data cover the temperature range 10^{-7} °K $\leq t \leq 5\times10^{-3}$ °K, where $t=T-T_\lambda$, and constitute a reduction by one order of magnitude of the minimum value of t at which any property of liquid helium has been studied.[5-7]

Accurate experimental information on transport properties near critical points is very limited, and for the helium λ point only qualitative confirmation of dynamic scaling laws exists. Although Ferrell et al.[1] assert that the thermal conductivity measurements of Kerrisk and Keller[8] are "sufficiently good to provide a rigorous test of the theory," the critical exponents derived from those data[8] range from 0.2 to 0.7, whereas the prediction is $\frac{1}{3}$. Therefore the data of Kerrisk and Keller, although they yield the very important experimental information that κ diverges near T_λ, cannot be interpreted as a quantitative confirmation of dynamic scaling. The present data do yield this confirmation and result in a critical exponent of 0.334 ± 0.005.

The measurements were made by the "parallel-plate" method in a cylindrical cell, 0.95 cm long and 0.93 cm in diameter, with 0.015-cm thick stainless steel walls. This cell was attached to the bottom of a liquid-helium reservoir of 80-cm^3 volume. The entire assembly was suspended in virtual thermal isolation in a vacuum inside a liquid-helium bath and could be filled through a capillary. The large reservoir, when filled with helium, served as a heat sink and assured thermal stability. A controlled heat leak from the reservoir to the bath compensated for the power dissipated in the system during measurements ($\approx10^{-6}$ W). The temperature drift of the system was constant to $\pm5\times10^{-8}$ °K/h, and when necessary could be reduced to 10^{-8} °K/h. The thermal gradient across the sample cell was measured by a differential thermometer consisting of two carbon thermometers which served as two arms of an ac bridge. At a thermometer power of 9×10^{-8} W a differential temperature resolution of 5×10^{-8} °K was possible. A second thermometer was used to measure the temperature at the top of the cell. The thermal gradient across the cell was caused primarily by the power dissipated in the member of the differential thermometer located at the bottom of the cell. Comparison of results at two power levels indicated a parasitic heat input to the bottom of the cell of 0.76×10^{-8} W. Measurements using the full length of the cell were made at power densities of 4×10^{-8} and 14×10^{-8} W/cm^2, and cover the range 3×10^{-6} °K $\leq t \leq 4\times10^{-3}$ °K. The smallest accessible value of t is dictated by the thermal gradient which results when the top of the cell is at T_λ. The upper limit for the range of measurements is determined by the density maximum at $t\approx6\times10^{-3}$ °K. Above this temperature, there was heat transfer by convection. Each measured value of κ was assigned to a value of t corresponding to the midpoint between the two end temperatures. Appropriate curvature corrections were made. The cell had been designed deliberately with a large length-to-area ratio so as to minimize end effects due to possible differences in Kapitza resistance between He II and He I and other unknown sources in He I. A discontinuous increase by a factor of 2 at T_λ of the measured Kapitza resistance in He II (2.4°K W^{-1} cm^{-2} at $t=-4\times10^{-6}$ °K) would be a negligible correction to the measurements in He I. The thermal conductivity of the walls can be assumed to be temperature independent over the range involved here, and makes an additive contribution of 7.5×10^{-5} W cm^{-1} °K^{-1} to the measured value of κ. The results are shown as open and solid circles in Fig. 1. The data as shown include any contribution from wall conduction.

The range of the measurements was extended at a power density of 4×10^{-8} W/cm^2 by utilizing the effect of the gravitational field on the superfluid transition.[7] The sample was cooled at constant rates varying for different runs between 8×10^{-7}

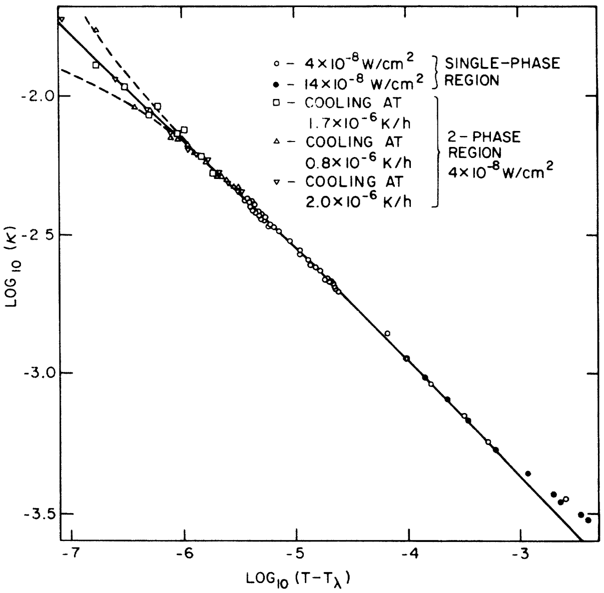

FIG. 1. The logarithm of the thermal conductivity κ as a function of the logarithm of $T-T_\lambda$.

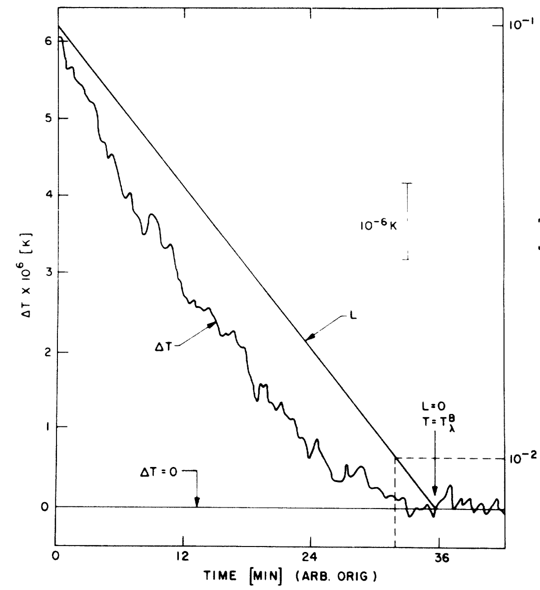

FIG. 2. The temperature difference across He I, and the length of the He I sample (inferred on the basis of a constant cooling rate), in the two-phase region as a function of time. This figure covers about the last $\frac{1}{10}$ of the two-phase region (0.1 cm or 10^{-7}°K). The power density was about 10^{-7} W/cm^2.

°K/h and 2×10^{-6} °K/h, starting at a temperature at which the entire sample was above T_λ. These cooling rates were sufficiently small to maintain essentially an equilibrium temperature gradient in the sample. When the transition temperature at the top of the sample was reached, there was a discontinuous change in the time derivative $d(\Delta T)/d\tau$ of the temperature difference ΔT across the sample because the length L of the He I sample discontinuously started to decrease.[9] When the interface between He II and He I reached the bottom of the sample, $d(\Delta T)/d\tau$ continuously vanished even though $dL/d\tau$ vanished discontinuously. This is shown in Fig. 2. The continuity of $d(\Delta T)/d\tau$ is already indicative of a divergent thermal conductivity in the vicinity of $t=10^{-7}$ °K. Results obtained for κ in the two-phase region, after appropriate curvature correction, are shown also in Fig. 1. The dashed lines over the range $t<10^{-6}$ °K indicate the possible systematic errors due to systematic errors in L.

The specific prediction of dynamic scaling to be tested here was first made by Ferrell et al.,[1] who asserted that the diffusion constant D should diverge at T_λ according to

$$D=\kappa/\rho C_p\sim\xi^{1/2}\sim\rho_s^{-1/2}, \tag{1}$$

where ρ and ρ_s are the total and superfluid densities, C_p is the heat capacity at constant pressure, and ξ is a correlation length for the fluctuations of the order parameter. A more precise expression which follows from the relation between the critical frequencies for ϵ and $-\epsilon$ ($\epsilon=t/T_\lambda\geq 0$) predicted by a dynamic scaling is[2,10]

$$[\kappa(\epsilon)/\rho C_p(\epsilon)]\xi^{-2}(\epsilon)=AC_2(-\epsilon)\xi^{-1}(-\epsilon). \tag{2}$$

Here A is a numerical constant of order unity;

$$C_2(-\epsilon)=\left[\frac{TS^2(-\epsilon)\rho_s(-\epsilon)}{\rho_n(-\epsilon)C_p(-\epsilon)}\right]^{1/2} \tag{3}$$

is the hydrodynamic expression for the second-sound velocity; S is the entropy; ρ_n is the normal-fluid density. The correlation lengths for He II and He I are given by

$$\xi(-\epsilon)=\xi_0'|\epsilon|^{-\nu'};\quad \xi(\epsilon)=\xi_0\epsilon^{-\nu}. \tag{4}$$

On the basis of static scaling, $\nu=\nu'$ and $\rho_s\sim\xi^{-1}(-\epsilon)$. From the measured ρ_s,[11,12] $\nu'=\frac{2}{3}$. Thus, it follows from Eq. (2) that

$$\kappa=B\{S(-\epsilon)C_p(\epsilon)/[\rho_n(-\epsilon)C_p(-\epsilon)]^{\frac{1}{2}}\}\epsilon^{-x}+\kappa', \tag{5}$$

where a constant background term κ' has been added, and where the dynamic-scaling prediction is

$$x = 2\nu - \tfrac{3}{2}\nu', \tag{6}$$

which is equal to $\frac{1}{3}$. The constant B is given by

$$B = (395)(1.43)^{1/2}\rho^{3/2}T^{1/2}T_\lambda^{\ 1/3}A\xi_0^{\ 2}(\xi_0{}')^{-1}$$

$$\cong 50.4A\xi_0^{\ 2}(\xi_0{}')^{-1} \tag{7}$$

when the specific heat and the entropy are in units of J mole^{-1} °K^{-1}.[13]

The data were fitted by a least-squares method with Eq. (5), permitting B, x, and κ' to vary. For this purpose, recent measurements of C_p[6] and ρ_S/ρ[11,12] were used and S was taken as 62.4 cm^3 bar mole^{-1} °K^{-1} at $\epsilon = 0$.[14] Appropriate weights based on the estimated probable error of each point were used, and 7.5×10^{-5} W cm^{-1} °K^{-1} was subtracted to compensate for wall conduction. The analysis was limited to the single-phase data ($t > 3\times10^{-6}$ °K) and to $t < 10^{-3}$ °K. For larger values of t, appreciable deviations from Eq. (3) occur (see Fig. 1). The resulting parameters are

$$\kappa' = (-6\pm2)\times10^{-5}\ \mathrm{W\ cm^{-1}\ °K^{-1}},$$

$$A\xi_0^{\ 2}(\xi_0{}')^{-1} = (0.87\pm0.06)\times10^{-8}\ \mathrm{cm},$$

$$x = 0.334\pm0.005. \tag{8}$$

Here the uncertainties are standard errors. The solid line in Fig. 1 corresponds to Eq. (3) with these parameters. It is evident that the prediction of κ based on dynamic scaling is confirmed over the entire temperature range $10^{-7} \leq t \leq 10^{-3}$ °K.

It is worthwhile to emphasize the importance of the logarithmic terms in Eq. (5) which originate from C_p. Since $\ln\epsilon$ does not vary strongly with ϵ and asymptotically corresponds to a critical exponent equal to zero, there may be a tendency to neglect the temperature dependence due to $\ln\epsilon$ for $\epsilon > 0$. In expressions like Eq. (5) this can lead to noticeable errors in the derived critical exponent. Recently, Pearce, Lipa, and Buckingham[15] elaborated upon this point. To demonstrate the effect and the existence of the logarithmic terms, Eq. (5) was used to calculate an apparent critical exponent given by $d\ln\kappa/d\ln\epsilon$. This apparent critical exponent as a function of $\log_{10}t$ is shown in Fig. 3. Shown for comparison is the apparent critical exponent resulting from a more approximate relation given by Ferrell, et al.[1,13] Apparent critical exponents were computed also from the data by a least-squares

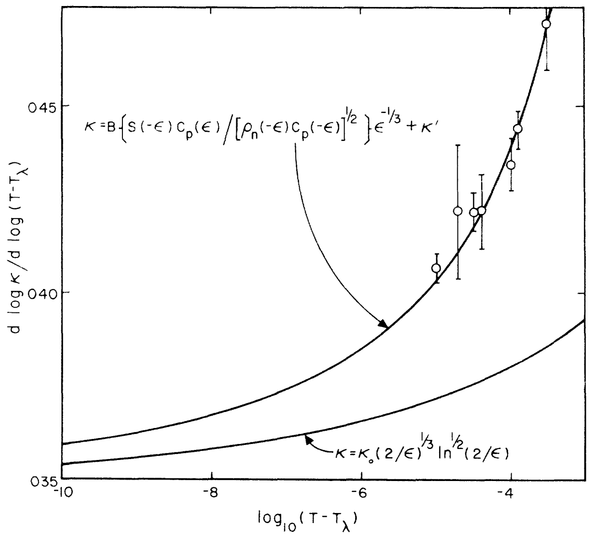

FIG. 3. The apparent critical exponent resulting from neglect of the logarithmic contributions to the thermal conductivity. Circles indicate experimental data.

method, using limited ranges in t for each point. These are shown as well. It can be seen from the temperature dependence of the apparent exponents that the logarithmic terms do indeed exist in the expression for κ, and that their neglect in any accessible temperature range can lead to appreciable errors in the determination of the real exponent. It is clear that Eq. (5) is a better analytic form than Eq. (1) over the experimental temperature range. The accuracy of Eq. (5) is somewhat surprising. Swift and Kadanoff[16] expressed the belief that "scaling arguments are not good enough to give logarithmic terms correctly."

Although dynamic scaling predicts only the temperature dependence of κ, and additional assumptions about $A\xi_0^{\ 2}(\xi_0{}')^{-1}$ are required to estimate theoretically the magnitude of κ, one can attempt to compare $A\xi_0^{\ 2}(\xi_0{}')^{-1}$ with $\xi_0{}'$. Estimates of $\xi_0{}'$ range from 0.3×10^{-8} cm^1 to 1.2 $\times10^{-8}$ cm.[17,18] Thus, $A\xi_0^{\ 2}(\xi_0{}')^{-2}$ is likely to have a value between 0.7 and 3.2, indicating the likelihood that $A \approx 1$ and $\xi_0 \approx \xi_0{}'$. At the smallest value of ϵ accessible to this experiment (5×10^{-8}) the length $A\xi^2(\epsilon)\xi^{-1}(-\epsilon)$ has the rather large value 7×10^{-4} cm.

The validity of Eq. (5) is dependent upon the validity of the hydrodynamic expression for the second-sound velocity, Eq. (3), which has been confirmed experimentally by Tyson[17] for $-t \geq 9\times10^{-5}$ °K, and by Pearce, Lipa, and Buckingham[15] for $-t \geq 2\times10^{-4}$ °K. The present data can be considered as indirect evidence for the validity of Eq. (3) for $-t \geq 10^{-7}$ °K.

It is apparent from Fig. 1 that κ deviates from its limiting behavior [Eq. (5)] for $t \geq 10^{-3}$ °K. The excess thermal conductivity could be associated with higher order singular terms.[19] Also, it is possible that the minimum in κ at $\epsilon \approx 10^{-2}$ which was observed by Kerrisk and Keller[8] is caused by such higher order terms.

I am very grateful to P. C. Hohenberg for calling my attention to the importance of high-resolution measurements of κ in He I and for stimulating discussions throughout the course of this work.

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[19]There are contributions to certain thermodynamic properties whose temperature dependences are ϵ and $\epsilon \ln\epsilon$ (G. Ahlers, to be published).

QUANTUM KINETIC EQUATION FOR ELECTRONS WITH MAGNETIC INTERACTIONS*

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We have derived the quantum kinetic equation appropriate to the Darwin Hamiltonian by a method identical to that given earlier in deriving the classical kinetic equation, with a redefinition of operators. It is conjectured that the transverse interaction term may lead, as in the classical case, to a long-ranged effective interaction.

There has been recent criticism[1] of the use of the Darwin Hamiltonian in the approximate treatment of relativistic effects in statistical mechanics[2-4] as opposed to the use of the Darwin Lagrangian. This question will be examined in more detail in a forthcoming paper.[5] However, we suggest that if fault is to be found (and this may not be so), it is more likely to be found in the ring approximation, and not in the symmetrical treatment of electric and magnetic effects characteristic of the Darwin Hamiltonian. Thus while the electrostatic interactions are well described within the ring approximation, it may not be so for the transverse interactions.

But it is clear that the derivation of a quantum kinetic equation follows from a well-defined Hamiltonian formulation. As Balescu indicates in his book,[6] "···the momenta are natural variables at the molecular level, whereas velocities are more natural macroscopic variables. This is even more true in quantum mechanics; the velocity has no simple meaning, whereas the momentum is a variable which can be readily quantized."