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MASS SPECTRUM FOR $\pi^-\pi^-\pi^+$ PRODUCED IN π^-p AT BeV/c†

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The negative three-pion mass spectrum between 1.0 and 1.4 BeV has been examined in the reactions $\pi^-p \to p(\pi^-\pi^-\pi^+)$, $\pi^-p \to p\pi^0(\pi^-\pi^-\pi^+)$, and $\pi^-p \to n\pi^+(\pi^-\pi^-\pi^+)$ for 5-BeV/c incident π^-. In each reaction peaks are observed near 1.06, 1.17, and 1.30 BeV.

In the charged three-pion mass spectrum produced in $\pi^\pm p$ interactions several peaks have been reported in the region between 1 and 1.4 BeV. Of these the A_2 peak near 1.3 BeV is most certain, with other decay modes such as $K\bar{K}$ and $\eta\pi$ reasonably established.[1] The region below the A_2 is less certain. Although early experiments appeared to resolve an A_1 peak near 1.06 BeV from the A_2, later contributions with higher statistics have made the conclusions uncertain.[1] The largest samples of data have come from the reactions $\pi^\mp p \to (\pi^\mp\pi^-\pi^+)$, where a large background is expected from a Deck-type mechanism process with ρ^0 production.[2] In other reactions such as $\pi^-p \to p\pi^0(\pi^-\pi^-\pi^+)$ and $\pi^-p \to n\pi^+(\pi^-\pi^-\pi^+)$ in which the simple Deck mechanism is not possible, the background should be different and allow an independent examination for resonances below the A_2.

We have studied the reactions

$$\pi^-p \to p(\pi^-\pi^+\pi^-), \tag{1}$$

$$\pi^-p \to p\pi^0(\pi^-\pi^+\pi^-), \tag{2}$$

$$\pi^-p \to n\pi^+(\pi^-\pi^+\pi^-), \tag{3}$$

in π^-p interactions at 5 BeV/c. The results were obtained in an exposure of 150 000 pictures in the Berkeley 72-in. bubble chamber and roughly 55 000 four-prong events have been measured. From these events 6300 fits were obtained in Reactions (1).

For Reactions (2) and (3) we required that the events have a χ^2 confidence level larger than 10%. We also required the square of the missing mass corresponding to the π^0 for Reaction (2) to be between −0.06 and 0.10 BeV^2, and the square of the missing mass corresponding to the neutron for Reaction (2) to be between 0.74 and 1.02 BeV^2. These fits were checked for consistency with ionization. In the case of further ambiguities the best χ^2 fit was used. This left 6822 and 6216 accepted fits for Reactions (2) and (3), respectively. In Reaction (2) there are strong peaks for N^{**++}, ω, and η which are incompatible with $\pi^-\pi^+\pi^-$ resonance formation and we have therefore exclude these events.

The $\pi^-\pi^+\pi^-$, $\rho^0\pi^-$, and $\pi^-\pi^+$ mass spectra for the three reactions are shown in Fig. 1. After the above selections have been made the $\pi^-\pi^+\pi^-$ and $\rho^0\pi^-$ mass spectra between 1.0 and 1.4 BeV are best interpreted in terms of three enhance-

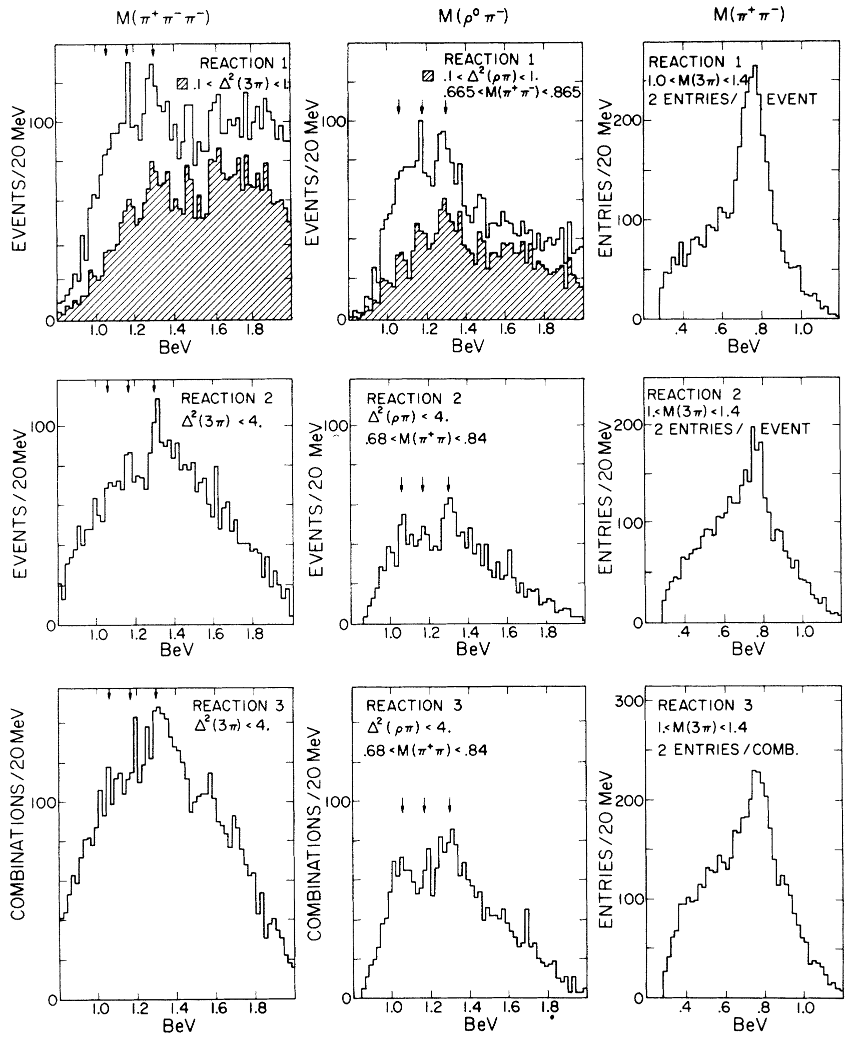

FIG. 1. Mass distributions for Reactions (1)-(3). In Reaction (2), N^{*++}, η, and ω events are excluded. Mass distributions of $\pi^-\pi^-\pi^+$ are in the left-hand column, of $\rho^0\pi^-$ in the middle column, and of $\pi^-\pi^+$ for events with $\pi^-\pi^+\pi^-$ between 1.0 and 1.4 BeV in the right-hand column. Arrows indicate masses of 1.06, 1.17, and 1.30 BeV.

ments, A_1 near 1.06,[3] $A_{1.5}$ near 1.17,[3] and A_2 near 1.3 BeV. We have indicated these masses with arrows in each of our plots.

The mass spectra for Reaction (1) are shown in the first row of Fig. 1. We have not made a cut against the N^{*++} in this reaction. Although a number of events appear with the π^+ in the ρ^0 as well as the N^{*++} region, with the present statistics we have not been able to analyze possible interference effects. In an attempt to reduce Deck background we have cut events with Δ^2 less than 0.1 (BeV/c)^2.[4] Further, since the peaks appear more strongly for low Δ^2, we have also cut down on other background by eliminating events with Δ^2 larger than 1 (BeV/c)^2. In the uncut 3π spectrum the $A_{1.5}$ and A_2 are seen and they remain after the Δ^2 cuts. In the $\rho\pi$ spectrum, where the ρ^0 is chosen between 0.665 and 0.865 BeV, and $A_{1.5}$ and A_2 remain, indicating that the events in the peaks are associated with $\rho\pi$ decay. In the A_1 region, a shoulder is apparent. The Δ^2 cuts reduce the background strongly in the $\rho\pi$ spectrum and perhaps resolve the A_1 from the $A_{1.5}$. Be examining further Δ^2 cuts in the $\rho\pi$ spectra (not shown) it appears that the events in the A_1 region are more peripheral than events in the $A_{1.5}$ region.

The mass spectra for Reactions (2) and (3) are shown in the second and third rows of Fig. 1. For these reactions, the 3π production does not appear to be as peripheral as in Reaction (1) and we have only made a weak cut, requiring Δ^2

<4 (BeV/c)^2. In Reaction (2), the 3π mass spectrum in Fig. 1 shows some peaking in the A_2 region and perhaps a smaller peak near $A_{1.5}$. For Reaction (3) there is an indication of some enhancement in the A_1, $A_{1.5}$, and A_2 regions. These enhancements become more significant when a ρ cut is made. For Reactions (2) and (3) we have plotted $\rho^0\pi^-$ with the ρ^0 between 0.68 and 0.84 BeV. The resulting spectra show enhancements in the locations expected for A_1, $A_{1.5}$, and A_2 and are again consistent with the hypothesis that these decay predominantly by $\rho\pi$. Further cuts in Δ^2 (not shown) do not appreciably change the character of the spectra; the three peaks seem to have similar Δ^2 distributions.

In Fig. 2, we have combined the $\rho\pi$ mass spectra from Reactions (1)-(3), using the shaded events for Reaction (1). The three peaks common to the reactions now stand out. To examine roughly the statistical significance of the A_1 and $A_{1.5}$ we have drawn a smooth curve without assuming enhancements in the A_1 and $A_{1.5}$ in the region from 1.0 to 1.24 BeV. We find a χ^2 corresponding to a 5% confidence level for the 12 bins in this region. We cannot therefore exclude the explanation of the A_1 and $A_{1.5}$ peaks as statistical fluctuations. However, in our experiment, the peaks occur near 1.06 and 1.17 BeV for all three reactions, as seen in the second column of Fig. 1. This we feel argues against their interpretation as a statistical fluctuation.

When we interpret the A_1 and $A_{1.5}$ as resonances and compare the spectra in Fig. 1, column 1 with those in Fig. 1, column 2, we find that the peaks above background are nearly the same before and after the ρ cut, whereas the background is reduced. This suggests some $\rho\pi$ enhancement in the A_1 and $A_{1.5}$ 3π decays, but our data are still too limited to permit us to quote a branching ratio. The masses we estimate as $M=1060$ MeV for the A_1 and $M=1170$ MeV for the $A_{1.5}$. Possible differences between our mass value for $A_{1.5}$ and the values reported[3] from Reaction (1) at 8 and 11 BeV/c can only be resolved with better statistics and by the examination of further reactions at these momenta. The widths of the A_1 and $A_{1.5}$ are difficult to estimate because of the limited statistics. The $A_{1.5}$ appears with roughly the same width in the $\rho\pi$ spectra in all reactions after the Δ^2 cut is made in Reaction (1). We are, therefore, tempted to associate the narrower peak in the uncut spectrum with a statistical fluctuation and we estimate the width of the $A_{1.5}$ to be between 30 and 60 MeV.

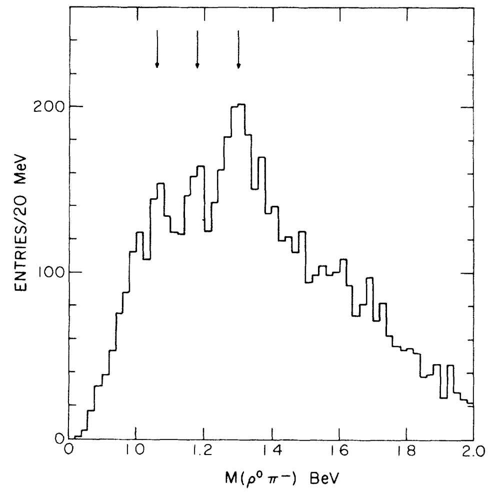

FIG. 2. Mass distributions of $\rho^0\pi^-$ after adding Reactions (1)-(3). For Reaction (1) the events with $0.1<\Delta^2(\rho^0\pi^-)<1$, shaded in Fig. 1, were used. Arrows indicate masses of 1.06, 1.17, and 1.30 BeV.

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CURRENT-ALGEBRA, SUBTRACTED DISPERSION RELATION, AND K_{l3} FORM FACTORS

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A calculation of the K_{l3} form factors is done on the basis of current algebra, partial conservation of axial-vector currents (strangeness conserving), and dispersion relations. Assuming once-subtracted and unsubtracted dispersion relations for $f_+(q^2)+f_-(q^2)$ and $f_+(q^2)-f_-(q^2)$, respectively, and K^* dominance, the K_{l3} decay parameters ξ, λ_+, and λ_- are calculated. All results are consistent with the present experimental indications.

There has been a considerable number of theoretical discussions[1-6] of the K_{l3} form factors on the basis of current algebra and partial conservation of axial-vector currents (PCAC) (or their variants). It seems, however, that the present theoretical status of the K_{l3} form factors is still far from being free from confusion and uncertainty. We shall report in this note a calculation of these form factors from the dispersion point of view, using the PCAC and current-algebra result to fix the subtraction constant in a once-subtracted dispersion relation for the combination $f_+(q^2)+f_-(q^2)$. It is felt that the present calculation is probably less subject to the uncertainties and ambiguities that plagued, to varying degrees, some of the earlier calculations.

The basic relation for the K_{l3} form factors in the approach based on PCAC and current algebra is the Callan-Treiman-Mathur-Okubo-Pandit (CTMOP) relation[1]:

$$f_+(-M_K^2)+f_-(-M_K^2)=F_K/F_\pi, \qquad (1)$$

which holds at the unphysical point $q^2=-M_K^2$, where q^2 is the momentum-transfer variable and $f_\pm(q^2)$ are the usual K_{l3} form factors (to be defined below). In order to derive reliable information concerning the physical form factors, one must have a "suitable" procedure of analytically extrapolating the soft-pion current-algebra result (1) from the unphysical point to the physical region. A natural choice of such an analytic procedure is provided by the dispersion approach. In particular, we shall adopt in our present calculation the point of view forwarded by Okubo and his collaborators,[7] and in a slightly different context by Fubini and Furlan.[8] The basic point is that the soft-pion current-algebra result provides the subtraction constant, if a once-subtracted dispersion relation is assumed for the appropriate amplitude. As we shall see, we can in our calculation always keep the kaon momentum on the mass shell and thus avoid the potentially unreliable large mass extrapolation inherent in the use of the PCAC for the strangeness-changing axial-vector current.

Traditionally,[9] unsubtracted dispersion relations are assumed for the K_{l3} form factors (and for other form factors, such as the π_{l3} form factor and the pion electromagnetic form factor, etc.). However, it has recently been realized[10] that the assumption of unsubtracted dispersion relations may be too restrictive, and in a few instances leads to paradoxical results. We do not know whether a subtracted dispersion relation is necessary in the case of K_{l3} form factors. Notwithstanding, the use of subtracted dispersion relations, provided a knowledge of the subtraction constants is available, definitely offers better hope for a reliable calculation, since practically in every calculation use has to be made, in one way or another, of the assumption of dominance by the low-lying states. In a calculation based on unsubtracted dispersion relations, assuming their validity, the effects of the continuum and the high-lying excitations are hard to estimate, although they may in fact be important. If subtracted dispersion relations are used, most of these effects are presumably effectively represented by the subtraction constants, and the contributions to the dispersion integral from the high-lying states are suppressed. This makes the dominance of the dispersion integral by low-lying states a better approximation.