term, where $t_{20}' = \sum_M \mathfrak{D}_{0M}{}^{2*}(\varphi^Y, \theta^Y, -\varphi^Y) t_{2M}$, with θ^Y and φ^Y giving the Y^* direction in the original coordinate system.

[17] R. H. Dalitz, in Proceedings of the Oxford International Conference on Elementary Particles, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), and in Proceedings of the Thirteenth International Conference on High Energy Physics, 1966 (University of California Press, Berkeley, Calif., 1967); and N. Masuda and S. Mikamo, Phys. Rev. 162, 1517 (1967). These articles indicate a problem, namely, that $\Xi^*(1820) \to \Sigma\bar{K}$ is not observed.

TIME REVERSAL AND THE K^0 MESON DECAYS

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We show that T is not conserved in the CP-nonconserving $K^0 \to 2\pi$ decays by direct data analysis (rather than inferentially via CPT) over a considerable range of values for the Wu-Yang parameters within the span of existent data. In particular, if $|\eta_{00}|/|\eta_{+-}| \leq 1.0$, then T is broken. This result is independent of the final state $\pi\pi$ interaction.

Since the discovery of CP nonconservation in the decay of a neutral kaon into two pions,[1] a concomitant breakdown of time-reversal symmetry has been inferred by application of the fundamental CPT theorem. The experimental validity of this theorem was assumed by Lee, Oehme, and Yang[2] and by Wu and Yang[3] in their analyses of K^0 interference[4] phenomena. Although the possibility of CPT asymmetry has been considered[5] and transitory difficulties in closing the Wu-Yang diagram have occurred,[6] the present K^0 experimental situation is sufficiently fluid that one may reasonably assume that CPT symmetry remains a valid principle.[7] Nevertheless, microscopic time-reversal symmetry and its breaking are of sufficient intrinsic interest that it seems worth analyzing the data on K^0 decay to test the two-pion mode for T nonconservation directly, rather than inferentially via CPT and CP. We report here on the results of such an analysis, where T invariance is assumed at the outset and T nonconservation is established by contradiction over a fairly wide range of values for the relevant parameters within the span of present experimental data.[8]

Consider the amplitude ratios discussed by Wu and Yang[3]:

$$\eta_{+-} \equiv (K_L \to \pi^+\pi^-)/(K_S \to \pi^+\pi^-) \tag{1}$$

$$\eta_{00} \equiv (K_L \to \pi^0\pi^0)/(K_S \to \pi^0\pi^0), \tag{2}$$

where K_S and K_L are the short- and long-lived neutral kaons. The assumption of T invariance implies

$$\eta_{+-} = \bar{\epsilon} + \bar{\epsilon}' \tag{3}$$

$$\eta_{00} = \bar{\epsilon} - 2\bar{\epsilon}', \tag{4}$$

where

$$\theta_{\bar{\epsilon}} = \tan^{-1}[2(m_L - m_S)/(\gamma_S - \gamma_L)] - 90° \tag{5}$$

$$\theta_{\bar{\epsilon}'} = -(\delta_0 - \delta_2) \text{ or } -(\delta_0 - \delta_2) + 180°. \tag{6}$$

In the above $\theta_z \equiv \arg z$, where $z = \bar{\epsilon}$ or $\bar{\epsilon}'$, m_S (m_L) and γ_S (γ_L) are the mass and decay rate of K_S (K_L), and δ_I are the s-wave phase shifts due to the strong final-state pion interactions with isospin $I = 0, 2$. That is, the assumption of T invariance again leads to a Wu-Yang diagram [cf. Eqs. (3) and (4), and the inset of Fig. 1]. The parametrization differs from that when one assumes CPT invariance only in that the T-invariant phase angles $\theta_{\bar{\epsilon}}$ and $\theta_{\bar{\epsilon}'}$ differ by 90° from the corresponding CPT-invariant phases $\theta_{\bar{\epsilon}}$ and $\theta_{\bar{\epsilon}'}$. In obtaining Eq. (5) we have neglected contributions to $\theta_{\bar{\epsilon}}$ from the leptonic and 3π CP-nonconserving terms in the decay matrix as well as terms $O(|A_2/A_0| \times |\bar{\epsilon}'|)$ and $O([A_2/A_0]^2 |\bar{\epsilon}|)$. Here A_I are the $I = 0$ and $I = 2$ standing-wave amplitudes, real by the assumption of T invariance. Our second assumption, consistent with existent data but subject to further experimental test, is that the error $\Delta\theta_{\bar{\epsilon}}$, introduced by the neglect of these terms, satisfies[9]

$$|\Delta\theta_{\bar{\epsilon}}| \leq 10°. \tag{7}$$

Since we make no further approximations and re-

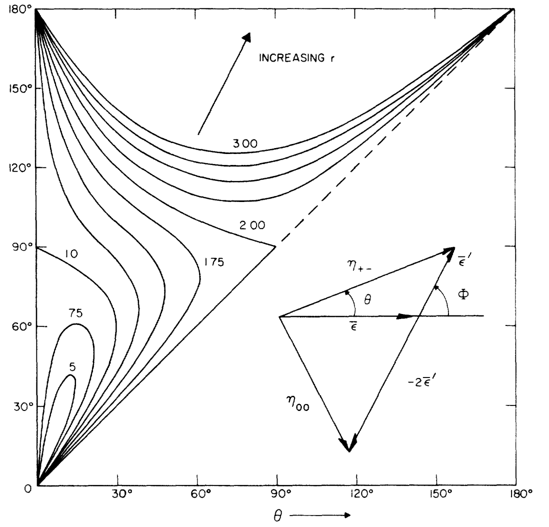

FIG. 1. $\Phi(\theta, r)$ for the Wu-Yang diagram (inset) at values of r from 0.5 to 3.0 at intervals of 0.25. $r \equiv |\eta_{00}|/\eta_{+-}|$. $\Phi(-\theta) = -\Phi(\theta)$.

ject T invariance where data misfits occur, the validity of our assertion of T nonconservation at certain values of the Wu-Yang parameters rests on this second assumption. From Eqs. (5) and (7) and the relevant data (Table I) one obtains

$$\theta_{\bar{\epsilon}} = -46° \pm 10° \text{ or } 134° \pm 10°. \tag{5'}$$

The data on leptonic charge asymmetry[6] do not permit a decision between the two choices in Eq. (5′) unlike the case of the CPT-invariant analysis. Let r denote the ratio

$$r \equiv |\eta_{00}| / |\eta_{+-}|. \tag{8}$$

From Table I, one sees that there is some disagreement concerning the experimentally difficult parameter $|\eta_{00}|$.[7] The following range for r is consistent with all values listed:

$$0 \leqslant r \leqslant 3. \tag{9}$$

We consider values of $\delta_0 - \delta_2$ satisfying

$$-90° \leqslant \delta_0 - \delta_2 \leqslant +90°. \tag{10}$$

Our procedure is to solve Eqs. (3) and (4) simultaneously for $\theta_{+-} \equiv \arg\eta_{+-}$, subject to Eqs. (5′) and (6), for given values of r and $\delta_0 - \delta_2$ in the ranges delineated above, and to compare the solutions with the experimental data for θ_{+-}. In Fig. 1 we exhibit solutions for the universal[10] auxiliary quantity $\Phi(\theta, r)$, where

$$\Phi \equiv \theta_{\bar{\epsilon}'} - \theta_{\bar{\epsilon}}, \tag{11}$$

$$\theta \equiv \theta_{+-} - \theta_{\bar{\epsilon}}, \tag{12}$$

$$\Phi(-\theta) = -\Phi(\theta). \tag{13}$$

Examination of Table I shows that the experimental values for θ_{+-} are essentially confined within the first quadrant. Consider the following ranges for θ_{+-}: (I) $54° \leqslant \theta_{+-} \leqslant 72°$, (II) $93° \leqslant \theta_{+-}$

Table I. Data on K^0 decay.

Quantity	Value	Reference
$\lvert\eta_{+-}\rvert$	$(1.90 \pm 0.06) \times 10^{-3}$	a
$\lvert\eta_{00}\rvert$	$(4.3^{+1.1}_{-0.8}) \times 10^{-3}$	b
	$(4.9 \pm 0.5) \times 10^{-3}$	c
	$(3.9 \pm 0.3) \times 10^{-3}$	d
	$(3.2 \pm 0.6) \times 10^{-3}$	e
	$(-1 \pm 6)^{1/2} \times 10^{-3}$	f
	$< 3 \times 10^{-3}$ (90% confidence)	f
$(m_L - m_S)\gamma_S^{-1}$	0.481 ± 0.018	g
θ_{+-}	45° ± 50°	h
	39° ± 45°	i
	60° ± 17°	j
	70° ± 21°	k
	84° ± 17°	l
	25° ± 25°	m
	60° ± 12°	a
	65° ± 11°	g

[a] Ref. 7.

[b] J.-M. Gaillard et al., Phys. Rev. Letters 18, 20 (1967).

[c] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters 18, 25 (1967).

[d] Cronin, Kunz, Risk, and Wheeler, preliminary result from further analysis of Ref. c, as reported in Ref. g.

[e] S. Parker, Bull. Am. Phys. Soc. 13, 31(T) (1968), as reported in footnote 8 of Ref. 19.

[f] T. Kamae, Bull. Am. Phys. Soc. 13, 31(T) (1968). Also, T. Kamae, Princeton University Elementary Particles Laboratory Report No. 45, 1968 (unpublished.

[g] A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77 (1968).

[h] V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).

[i] A. Firestone et al., Phys. Rev. Letters 16, 556 (1966), and 17, 116 (1966).

[j] D. Bohm et al., in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (North-Holland Publishing Company, Amsterdam, The Netherlands), as reported in Ref. g.

[k] M. Bott-Bodenhausen et al., Phys. Letters 24B, 438 (1967).

[l] C. Rubbia and J. Steinberger, Phys. Letters 24B, 531 (1967); C. Alff-Steinberger et al., Phys. Letters 21, 595 (1966).

[m] R. E. Mischke et al., Phys. Rev. Letters 18, 138 (1967).

$\leq 183°$ and $-87° \leq \theta_{+-} \leq 3°$, (III) $-126° \leq \theta_{+-} \leq -108°$, and (IV) remaining values on the unit circle. Values in range II, essentially the second and fourth quadrants, lie generally outside the data and at least 2.5 standard deviations away from both data compilations for θ_{+-} listed in Table I. Theoretical solutions lying in range II are considered T nonconserving. For fixed values of r and $\delta_0-\delta_2$, and for $\Delta\theta_{\not\epsilon}=0$, there are in general two pairs of solutions for θ_{+-}, the members of a pair being displaced 180° from each other. When, for given values of r and $\delta_0-\delta_2$, but with $\theta_{\not\epsilon}$ allowed to vary over the limits prescribed by Eq. (5′), the entire set of solutions for θ_{+-} lies in range II, we conclude that T is not conserved for those values of r and $\delta_0-\delta_2$. Figure 2 depicts the domain in the $(\delta_0-\delta_2), r$ plane where this condition obtains. There also exists a region in this plane, indicated by shading in Fig. 2, where at least one theoretical solution for θ_{+-} lies in range I, i.e., within 1 standard deviation of both data compilations listed in Table I. For values of r and $\delta_0-\delta_2$ within this region, T-conserving-CPT-nonconserving solutions are possible. However, for each solution in range I there exists a paired solution in range III, 180° out of phase with it. (Whereas the analysis yields a domain of definite T nonconservation, no such region of definite T conservation obtains.)

From examination of Fig. 2 the following features emerge: (a) T is not conserved for $r \leq 1.0$, where r is the ratio of neutral to charged, normalized, CP-nonconserving amplitudes in 2π decay [see Eq. (8)]. This result is wholly independent of the value of $\delta_0-\delta_2$, as may be seen from Fig. 1.[11] (b) T is not conserved over the entire experimentally allowed range for r [cf. Eq. (9)], provided $\delta_0-\delta_2$ satisfies $18° < \delta_0-\delta_2 < 72°$. [The recent result of Kamae, $|\eta_{00}|^2 = (-1 \pm 6) \times 10^{-6}$ or $|\eta_{00}| < 3 \times 10^{-3}$ with 90% confidence,[8] suggests $r < 1.6$, whence T nonconservation occurs at least over the region $-10° < \delta_0-\delta_2 < +90°$.][12] (c) Within the (shaded) region in Fig. 2 where the possibility of a T-conserving-CPT-nonconserving interaction must be considered, $r > 1.5$ and the values of $\delta_0-\delta_2$ are generally negative. Solutions in this region are essentially eliminated by the result of Kamae independently of the value of $\delta_0-\delta_2$. This parameter, as yet not directly measurable, has been the object of extensive study.[13] From analysis of pion production experiments, Walker et al.[14] found $\delta_0-\delta_2 \approx +50°$ at the energy of the kaon mass. Tryon[15] has found essential agreement with their overall results, using dispersion theory and information on scattering lengths from current algebra and the hypothesis of partially conserved axial-vector currents.[16] Other authors have found $\delta_0-\delta_2 \approx +30°$ at the kaon mass.[17] From observation (b) above, each of these results implies T nonconservation. However, there exists controversy even as to the sign of the $I=0$ scattering length,[18,13] so that a definite conclusion cannot be drawn as of now. We remark that if the experimental value of the parameter $|\eta_{00}|$ continues to drop such that $r \leq 1.0$, then by observation (a) above, T is nonconserved independently of the value of $\delta_0-\delta_2$.[19,20]

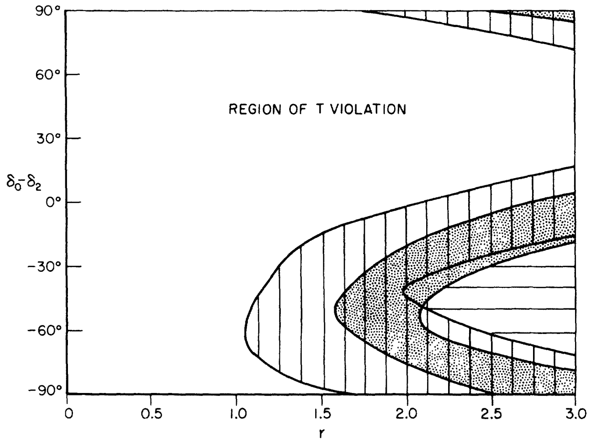

FIG. 2. Region of T nonconservation (so labeled), all solutions in range II. Shaded region, at least one solution in ranges I and III. Vertically lined, at least one solution in and out of II. Horizontally lined, all solutions in ambiguous range IV. $r \equiv |\eta_{00}|/|\eta_{+-}|$. See text for other definitions.

I am indebted to Professor M. Danos, Professor A. Feldman, Professor V. Fitch, Professor S. Meshkov, and Professor G. Snow for helpful comments.

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957).

[3] T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[4] M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955); S. B. Treiman and R. G. Sachs, Phys. Rev. 103, 1545 (1956).

[5] R. G. Sachs, Phys. Rev. 129, 2280 (1963); T. D. Lee and C. S. Wu, Ann. Rev. of Nucl. Sci. 16, 511 (1966); J. Bell and J. Steinberger, in Proceedings of the Oxford International Conference on Elementary Particles, September, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966).

[6] A measurement of the charge asymmetry in leptonic (e^+, e^-) decays by Bennett et al., leads to an inconsis-

tency as pointed out by these authors [Bennett et al., Phys. Rev. Letters 19, 993, 997 (1967)]. No inconsistency resulted from the μ charge-asymmetry data obtained by D. Dorfan et al., Phys. Rev. Letters 19, 987 (1967). The difficulty noted by Bennett et al. was interpreted as arising from a possible failure of the CPT theorem by F. Strocchi, Phys. Rev. Letters 19, 1457 (1967). He assumed that the CPT nonconservation was small enough to be compatible with the small value of the K_S–K_L mass difference. There is, of course, direct evidence which supports the CPT theorem outside the domain of neutral kaon physics, e.g., the equality of the oppositely charge kaon and pion lifetimes within small errors [F. Lobkowicz et al., Phys. Rev. Letters 17, 548 (1966), and D. S. Ayres et al., Phys. Rev. Letters 21, 261 (1968)].

[7] V. L. Fitch, Comments Nucl. Particle Phys. 2, 63 (1968), and private communication.

[8] Pertinent data on K^0 decay are summarized and referenced in Table I of this communication.

[9] A small magnitude for the quantity A_2/A_0 is consistent with (but not necessarily implied by) an analysis comparing the rates of charged and neutral kaon decays into two pions. The situation here is similar to that for the CPT analysis (cf. Ref. 7). Data on the lifetimes and branching ratios for the appropriate 2π, 3π, and leptonic decays that are not listed in Table I are taken from the compilation of A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77 (1968).

[10] "Universal" in the sense that the functional relationship $\Phi(\theta, r)$ does not depend upon whether one assumes T invariance or CPT invariance, provided Φ and θ are suitably redefined for the CPT case.

[11] From Fig. 2 one sees immediately that when $r \leq 1$, T is not conserved for $\delta_0 - \delta_2$ in the range delineated by Eq. (10). From Fig. 1 it is clear that if $r \leq 1$, then $|\theta| \leq 30°$ independently of Φ and hence of $\delta_0 - \delta_2$ [cf. Eqs. (11), (5′), and (6)]. Thus, from Eqs. (12) and (5′) the theoretical values for θ_{+-} lie in region II, Q.E.D.

[12] Prior to Kamae's work, it appeared most probable that $r \approx 2$ (Table I), implying T nonconservation for values of $\delta_0 - \delta_2$ lying roughly in the first quadrant as seen from Fig. 2. This result was the main content of the earlier version of this work.

[13] See the recent review by S. Treiman, Comments Nucl. Particle Phys. 2, 68 (1968)

[14] W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters 18, 630 (1967).

[15] E. P. Tryon, Phys. Rev. Letters 20, 769 (1968).

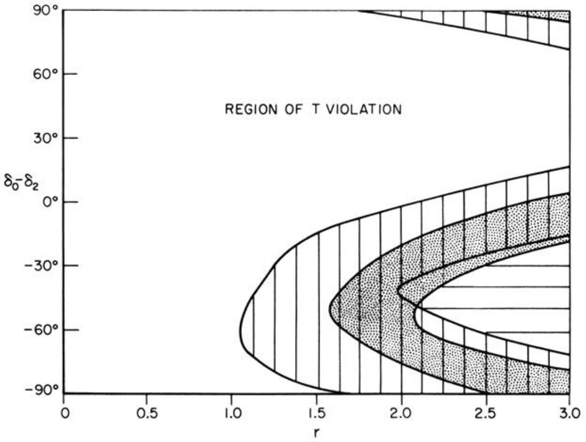
[16] S. Weinberg, Phys. Rev. Letters 17, 616 (1966). Also, via chiral dynamics, by J. Schwinger, Phys. Letters 24B, 473 (1967).

[17] L. S. Brown and R. L. Goble, Phys. Rev. Letters 20, 346 (1968); R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters 20, 475 (1968).

[18] S.-Y. Chu and B. R. Desai, Phys. Rev. Letters 20, 54 (1968).

[19] At completion of this work a new CPT-invariant K^0 analysis has appeared. See C. D. Buchanan and K. Lande, Phys. Rev. Letters 21, 169 (1968).

[20] See, also, P. K. Kabir, Phys. Rev. Letters 21, 314 (1968).



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