ANISTROPY OF THE CONDUCTION-ELECTRON g FACTOR IN Pt*

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The conduction-electron g factor of the Γ centered surface of Pt is determined from de Haas–van Alphen spin splitting zeros. A theoretical interpretation of the electron g factor in metals is presented.

In this Letter we present direct proof that the conduction-electron g factor in Pt is anisotropic and varies by the same order of magnitude as the cyclotron effective mass. We determine the g factor for the magnetic field in the (100) plane (for the Γ-centered surface). We believe this to be the first determination of the angular dependence of the g factor in a metal. We also suggest a formula which gives a geometrical interpretation of the conduction-electron g factor. Using this formula one may calculate the g factor for a general band structure in the presence of spin-orbit coupling. We also touch on the many-body aspects of the problem.

Consider a set of Landau levels in the absence of spin splitting. The spacing between these levels is $\hbar\omega_c$ (where $\omega_c = eH/m^*c$ is the cyclotron frequency and m^* is the cyclotron effective mass). The inclusion of spin causes each Landau level to be split into two levels separated by an energy

$$\Delta E = 2\mu H = g(\theta, \varphi)e\hbar H/2m_0c, \qquad (1)$$

where $g(\theta, \varphi)$ is the conduction-electron g factor. If $\Delta E = (r+\frac{1}{2})\hbar\omega_c$, where r is an integer, then the spacing between two adjacent levels is $\frac{1}{2}\hbar\omega_c$ and the amplitude of the first harmonic of the de Haas–van Alphen (dHvA) effect will vanish.[1-4] This phenomenon is called a spin-splitting zero. We define a spin mass by the relation $1/m_s(\theta, \varphi) = g(\theta, \varphi)/2m_0$ and the condition for a spin-splitting zero becomes $m^*(\theta, \varphi)/m_s(\theta, \varphi) = r+\frac{1}{2}$. Figure 1 shows the angles (in the basic 1/48th of the unit sphere) at which spin-splitting zeros were observed for the closed Γ-centered sheet of the Fermi surface of Pt. A total of 97 spin-splitting

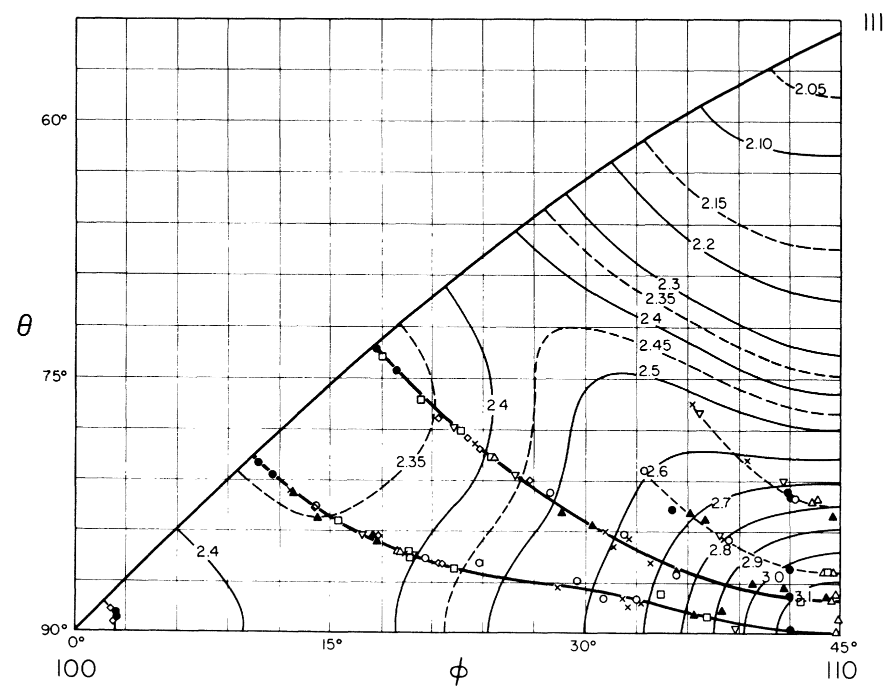

FIG. 1. The basic 1/48 of the unit sphere showing the polar coordinates of the observed spin-splitting zeros for the Γ-centered surface of Pt. Shown also are the contours of constant effective mass which result from a 21-term fit to the effective masses observed in two nonsymmetry planes. The different types of points (triangles, etc.) designate the eight different planes in which spin/splitting zeros were studied.

zeros were observed for this surface, the data points resulting from magnetic field rotations in eight different nonsymmetry planes.[5] The data from these eight planes, when transformed into the basic 1/48th of the unit sphere, form a quite dense net. The planes of rotation of the magnetic field were accurately determined by a technique which uses the cubic symmetry of the data.[6] The lines joining these points are thus the contours on which we expect the amplitude to vanish. Five such contours were observed, three of which are indicated by solid lines (one of which is very close to the [100] axis) and two by dashed lines in Fig. 1. Geometrically, the spin-splitting zero contours can be interpreted as the lines of intersection of a surface with "radius" $m^*(\theta, \varphi)$ and a set of surfaces of "radius" $(r+\frac{1}{2})m_S(\theta, \varphi)$. The locus of the intersection of two such surfaces must be a closed curve. Note that the two dashed contours end abruptly near the center of the triangle. This can happen only if the surfaces $m^*(\theta, \varphi)$ and $(r+\frac{1}{2})m_S(\theta, \varphi)$ (for some r) are tangent to each other (over the distance indicated by the dashed lines) and then separate. Since it would be an unlikely accident for the surfaces to be truly tangent, we interpret this to mean only that the surfaces are extremely close to each other on these lines. Angles at which the signal went to zero due to the sample magnetization being perpendicular to the axis of the pickup coil were calculated so as not to mistake them for spin-splitting zeros. Shown also in Fig. 1 are the contours of constant effective mass. These contours were generated by interpolating (with cubic harmonics[7]) effective-mass data taken in two nonsymmetry planes.[5] The effective masses were measured by observing the temperature dependence of the dHvA amplitude. If the g factor was equal to 2 (or equivalently if the spin mass was 1), then the spin-splitting zero contours would have coincided with the $2.5m_0$ contour of the effective mass (where $r=2$ in this case). As we see, however, the spin-splitting contours and $2.5m_0$ effective-mass contour are quite different. Indeed, two of the spin-splitting contours cut across the effective-mass contours from the region of minimum mass to the region of maximum mass. Thus the g factor is different from 2 and clearly anisotropic. At each angle where a spin-splitting zero is observed we may calculate the effective cyclotron mass $m^*(\theta, \varphi)$ and from this the spin mass $m_S(\theta, \varphi)$ by assuming some value for r. The choice $r=2$ seems most likely since it gives a spin mass closest to one, which is the value in the absence of spin-orbit coupling[8] and many-body effects. We have taken the spin masses calculated in this manner and have fitted them by the method of least squares to cubic harmonics in order that we may interpolate to other angles.[7] Such a procedure is justified only if we assume that the spin mass is more smoothly varying than the cyclotron mass. A 21-term fit was found to give a quite accurate representation ($\pm\frac{1}{2}\%$) of the cyclotron-mass data. After seven terms it was observed that the spin mass near [100] and [110] reproduced the experimental values. Figure 2 shows the interpolated spin mass in the (100) plane using a nine-term fit. An extrapolation throughout the (110) plane is not reliable as no spin-splitting zeros were observed near [111] and an extrapolation in this region would be unreliable.

We now proceed to a theoretical interpretation of the g factor or spin mass. We write the spin-splitting energy μH (of a specfic Landau level n) as an expansion and retain only the leading terms:

$$\Delta E = 2\mu H = \left[\left(\frac{\partial E\uparrow(n,H)}{\partial H}\right)_n - \left(\frac{\partial E\downarrow(n,H)}{\partial H}\right)_n\right]_{H=0} H + O(H^2), \quad (2)$$

where $E\uparrow(n,H)$ is the energy of a level whose spin is largely up and $E\downarrow(n,H)$ is the corresponding level whose spin is largely down. The dHvA-effect experiments are generally carried out for large n. In this quasiclassical limit the motion of an electron may be quantized according to the Bohr-Sommerfeld quantization rule. Onsager[9] has shown that the area A_p swept out by the mo-

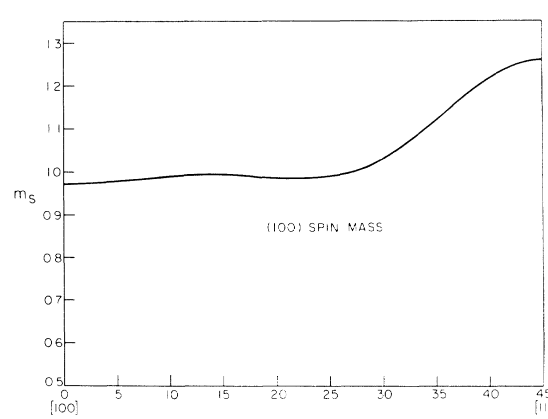

FIG. 2. The interpolated spin mass in the (100) plane (nine-term fit to the data summarized in Fig. 1).

tion of the electron in momentum space is quantized according to the relation $A_p = (n+\gamma)eHh/c$, where γ is an unknown phase. The area of a given orbit also depends on the position of the Fermi level and thus we may write $A_p = A_p(E)$. Equation (2) may then be rewritten (using the chain rule) $\Delta E = (dE/dA)[(\partial A\uparrow/\partial H)_n - (\partial A\downarrow/\partial H)_n]H$, where $A\uparrow$ is the area swept out by an up electron, and $A\downarrow$, by a down electron. Using the Onsager relation and the result $m^* = (2\pi)^{-1}dA(E)/dE$, we have

$$\mu = \frac{g(\theta,\varphi)e\hbar}{4m_0c} = \frac{A_p\uparrow(\theta,\varphi) - A_p\downarrow(\theta,\varphi)}{4\pi m^*(\theta,\varphi)H}. \quad (3)$$

This form is suitable for a calculation. Incidentally, using the Onsager relation the spin mass has the simple interpretation $m^*/m_s = \gamma\uparrow - \gamma\downarrow$ and a spin-splitting zero occurs when $\gamma\uparrow - \gamma\downarrow = r + \frac{1}{2}$. We note that the angle of a spin-splitting zero was independent of magnetic field, so that terminating expression (2) at the leading term is justified.

To calculate the g factor [using Eq. (3)] from a nonrelativistic band structure we would add two terms to the Hamiltonian. The spins couple to the external magnetic field through a term $\sum_i (e\hbar/m_0c)\vec{S}_i \cdot \vec{H}$. In addition a spin-orbit coupling term $\sum_i (\hbar^2/2m_0^2c^2)r^{-1}(\partial V/\partial r)\vec{L}_i \cdot \vec{S}_i$ (in the central-field approximation) must also be added if the g factor is to differ from 2. Strictly speaking, we should also include the magnetic field through the canonical momentum $\vec{p} - (e/c)\vec{A}$, but this results only in area (flux) quantization as mentioned and may be neglected. A calculation would proceed by determining the area of an extremal cross section of the Fermi surface as a function of magnetic field. One would also calculate the effective mass $m^* = (2\pi)^{-1}(\partial A/\partial E)_{\theta,\varphi}$. The g factor would then follow from Eq. (3).

The analysis here has presumed that the g factor is not greatly altered by many-electron effects (since an r value was selected which resulted in a g factor close to 2). This needs theoretical justification.

Since the g factor is related (in our interpretation) to the change in area of the Fermi surface with magnetic field, it should be possible (if this information were available at all angles) to apply the methods of Ketterson, et al.[10] to find the change in radius and on integration the change in volume with magnetic field. Since the change in volume with magnetic field is proportional to the magnetic susceptibility, it may be possible to find the susceptibility of each sheet of the surface. The answer to the question of whether it is the enhanced or unenhanced susceptibility is not known.

Attention should be given to the further development of methods for studying spin mass. Particularly promising are the giant quantum oscillations in the ultrasonic attenuation[11] or the adiabatic quantum oscillations in the temperature.[12,13] The effect of spin splitting is more easily observed in these experiments than in a dHvA experiment.

Absolute amplitude measurements of the dHvA effects (or temperature oscillations) would also in principle allow a determination of the spin mass at all angles. Great difficulties arise in practice however. One would have to measure independently the effective mass and "Dingle temperature" from the temperature and field dependence of the amplitude. The effects of a slight randomness in the orientation of the crystal (due to microstructure or a bent crystal) would also have to be considered.

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