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FLUIDLIKE ELECTRON AND ION MODES IN INHOMOGENEOUS PLASMAS*

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Collisionless unstable modes, involving the entire velocity space and not due to Landau damping effects, are shown to exist in two-dimensional configurations in which the particle orbits are periodic along the magnetic field lines, where the magnetic intensity varies along the field, and the particle density gradient is in a perpendicular direction. One mode is associated with the velocity dependence of the electron excursion frequency and one with the trapping of ions in the magnetic field wells. Their relevance to laboratory and space plasmas is pointed out.

We consider a general two-dimensional equilibrium configuration for a low-pressure ($\beta \ll 1$) collisionless plasma in which the magnetic field magnitude is varying along the lines of force, and the particle density has a gradient in a perpendicular direction.[1] We assume that the lines of force are closed so that the particle motion along them is periodic. Then we find two types of unstable modes, one associated with the electron periodic motion, and one with that of the ions trapped in the magnetic field wells,[2] which are not due to wave-particle resonance effects but involve the entire velocity space. In this sense they can be classified as macroscopic, and in fact several of their features discussed here make them suitable to explain modes that have been observed experimentally and have been difficult to identify,[3] or, by inferring some of their nonlinear effects, they may explain certain cases of observed anomalous diffusion.

Then if l measures the distance along the lines of force, $\vec{B}=\vec{B}(l)$ indicates the magnetic field, $n=n(r_\perp)$ the particle density so that $\vec{e}_r \perp \vec{e}_\| \equiv \vec{B}/B$, and $\vec{e}_\theta \equiv \vec{e}_\| \times \vec{e}_r$ represents the direction along which the system is homogeneous. We make use of the Vlasov equation

$$\partial f/\partial t + \vec{v}\cdot\nabla f + (e/m)(\vec{\mathcal{E}} + \vec{v}\times\vec{B})\cdot\nabla_v f = 0, \qquad (1)$$

where the notation is standard. The equilibrium distribution is chosen[4] as $f_0 = [n(p_\theta)/(\pi T)^{3/2}]\exp(-E/T)$, where $E = v_\|^2 + v_\perp^2$, measuring the total particle energy, and p_θ, the angular momentum in the direction of symmetry, are constants of the motion. Then we look for electrostatic modes such that $\tilde{\mathcal{E}} = -\nabla\varphi$, $\varphi = \tilde{\varphi}(r_\perp, l)\exp(ik_\theta r_\theta + i\omega t)$, with frequency smaller than the ion cyclotron frequency ($\omega \ll \Omega$). In particular, we consider modes localized around a point $r_\perp = r_0$ such that $k_\theta\tilde{\varphi} \gg \partial\tilde{\varphi}/\partial r_\perp$, $\tilde{\varphi}(r_\perp, l) \approx \tilde{\varphi}(r_0, l)$, and neglect from here on the $r_\perp$ dependence of $\tilde{\varphi}$. Then if we integrate Eq. (1) along particle orbits, we obtain[5] for the perturbed distribution function $\tilde{f}_j$ of the species j

$$\tilde{f}_j = -(e_j/T_j)f_{0j}\{\tilde{\varphi}(l) - i(\omega - \omega_{*j})J_0 \times e^{-i\omega t}\int_{-\infty}^{t} e^{i\omega t'} J_0 \tilde{\varphi}(l')dt'\}, \qquad (2)$$

where the argument of J_0 is $k_\theta v_\perp/\Omega_j = k_\theta(\mu B)^{1/2}/\Omega$, a function of l, μ being the magnetic moment $v_\perp^2/B$ that is invariant along l, $l' = l(t')$ is the guiding center trajectory as a function of t', and $\omega_{*j} = (-k_\theta cT_j/e_jBn)(dn/dr_\perp)_0$ is the "diamagnetic" frequency. Now we consider two limits: one for the electron mode, with $\bar{\omega}_{bi} < \omega < \bar{\omega}_{be}$, and one for the ion mode, with $\omega < \bar{\omega}_{bi} < \bar{\omega}_{be}$, where

$\bar{\omega}_{bi,e}$ are the average orbit frequencies.

Electron mode.—Here we have

$$\tilde{f}_i = -\frac{e}{T_i} f_{0i}\left[1-\left(1-\frac{\omega_{*i}}{\omega}\right)J_0^{\,2}\right]\tilde{\varphi}(l) \qquad (3)$$

for the ions. For the electrons we neglect finite-Larmor-radius effects so that $J_0=1$, and in order to perform the integral in Eq. (2), expand $\tilde{\varphi}$ in harmonics of the orbit periodicity. So

$$\tilde{\varphi}(l)=\sum\Phi^{(n)}(E,\mu)\exp(2\pi int'/\tau),$$

where

$$\Phi^{(n)}(E,\mu)=(1/\tau)\int_0^\tau\tilde{\varphi}[l(t')]e^{-2\pi int'/\tau}dt',$$

$$t'=\int^{l'}dl/v_\parallel,\quad \tau(E,\mu)=\oint dl/v_\parallel,$$

and

$$\omega_b(E,\mu)=2\pi/\tau.$$

Then we have

$$\tilde{f}_e=\frac{e}{T_e}f_{0e}\left\{\tilde{\varphi}(l)-\left(1-\frac{\omega_{*e}}{\omega}\right)\sum_n\Phi^{(n)}\frac{\exp(in\omega_bt)}{1+n\omega_b/\omega}\right\},$$

and for $\omega<\bar{\omega}_{be}$,

$$\tilde{f}_e\approx\frac{e}{T_e}f_{0e}\left\{\tilde{\varphi}(l)-\left(1-\frac{\omega_{*e}}{\omega}\right)\left[\Phi^{(0)}+\sum_{n\neq0}\Phi^{(n)}\exp(in\omega_bt)\frac{\omega}{n\omega_b}\left(1-\frac{\omega}{n\omega_b}\right)\right]\right\}. \qquad (4)$$

To obtain the dispersion relation, we impose the quasineutrality condition

$$\tilde{n}_i=\int\tilde{f}_id^3v=\int\tilde{f}_ed^3v=\tilde{n}_e,$$

and notice that because of the integration over $v_\parallel$, the term linear in ω/ω_b in Eq. (4) does not give a contribution. Then we have

$$\tilde{n}_e=\frac{e}{T_e}\left\{n_e\tilde{\varphi}(l)-\left(1-\frac{\omega_{*e}}{\omega}\right)\int d^3vf_e\left[\Phi^{(0)}(E,\mu)-\frac{\omega^2}{\omega_b^{\,2}}\sum_{n\neq0}\frac{\Phi^{(n)}(E,\mu)}{n^2}\exp(in\omega_bt)\right]\right\}. \qquad (5)$$

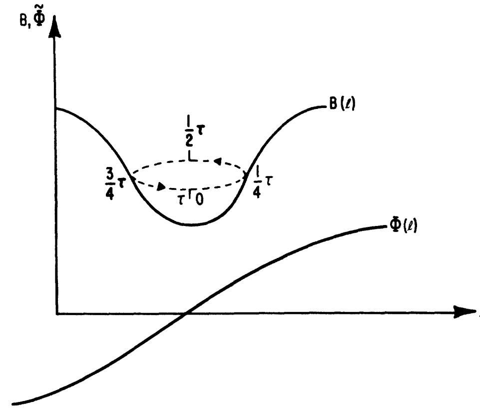

FIG. 1. Magnetic field and electric potential profiles such that $\Phi^{(0)}=0$.

To obtain an effect of the term containing the "bounce" frequency, we are led to consider modes with $\Phi^{(0)}=0$. For this, if the $B(l)$ profile is symmetric around each minimum, we choose $\tilde{\varphi}(l)$ antisymmetric around it. In fact referring to Fig. 1, we compute the integral $\int_0^\tau\tilde{\varphi}[l(t)]dt$ by splitting it into

$$\int_0^{\frac{1}{4}\tau}+\int_{\frac{1}{4}\tau}^{\frac{1}{2}\tau}+\int_{\frac{1}{2}\tau}^{\frac{3}{4}\tau}+\int_{\frac{3}{4}\tau}^{\tau}.$$

So we see that if $\varphi(l)$ is antisymmetric with respect to the minimum of $B(l)$, the above integrals cancel each other. We also have

$$\tilde{n}_i=-\frac{e}{T_i}n_i\tilde{\varphi}(l)\left[1-\left(1-\frac{\omega_{*i}}{\omega}\right)I_0(b_i)\exp(-b_i)\right], \qquad (6)$$

where $b_i=\frac{1}{2}k_\theta^{\,2}a_i^{\,2}=b_0B_0^{\,2}/B^2$, where a_i is the ion Larmor radius and B_0 the average magnetic field. We can see by inspection that, since we look at the limit $\omega<\bar{\omega}_{be}$, we have to consider $b_i>1$. Now it is convenient to write a quadratic form resulting from

$$\int(dl/B)\tilde{\varphi}^*(l)[\tilde{n}_e(l)-\tilde{n}_i(l)]=0$$

and transform the integration on v to one on μ and E, so that

$$\int\tilde{f}d^3v=\tfrac{1}{2}\pi\int\tilde{f}dEd\mu\,B/|v_\parallel|,$$

with a convention that contribution from positive and negative values of $v_\parallel$ are to be added. Then the quadratic form becomes

$$\left(1+\frac{T_e}{T_i}\right)\left\{\oint\frac{dl}{B}|\varphi|^2\right\}+\frac{\omega_{*e}}{\omega}\frac{1}{(2\pi b_0)^{1/2}}\left\{\oint\frac{dl}{B_0}|\varphi|^2\right\}+\frac{\omega_{*e}\omega}{\overline{\omega}_{be}{}^2}\left\{\frac{\pi}{2}\int\frac{f_e}{n_e}|\tau|dEd\mu\frac{\overline{\omega}_{be}{}^2}{\omega_b{}^2}\sum_{n\neq 0}\frac{|\Phi^{(n)}|^2}{n^2}\right\}, \tag{7}$$

where we have considered $\omega<\omega_{*e}$ for consistency with our ordering. If we indicate the integrals within brackets, respectively, by A, C, D, we find

$$\frac{\omega}{\omega_{be}}=\frac{\overline{\omega}_{be}}{\omega_{*e}}\left(\frac{A}{2D}\right)\left(1+\frac{T_e}{T_i}\right)\pm\left\{\left(\frac{\overline{\omega}_{be}}{\omega_{*e}}\right)^2\left(\frac{A}{2D}\right)^2\left(1+\frac{T_e}{T_i}\right)^2-\frac{1}{(2\pi b_0)^{1/2}}\frac{C}{D}\right\}^{1/2}. \tag{8}$$

One root (+ sign) corresponds to the well-known electron drift mode with $b_i>1$; the other root (– sign) corresponds to the new mode. We see that one mode is "macroscopically" unstable when

$$\omega_{*e}>\overline{\omega}_{be}\left(1+\frac{T_e}{T_i}\right)(2\pi b_0)^{1/4}\frac{A}{(CD)^{1/2}}. \tag{9}$$

Notice that this is not a "trapped" particle instability in the sense that it does not rely on $\Phi^{(0)}\neq 0$, as do the modes treated in Refs. 2 and 4 and the ion mode to be discussed. On the other hand, a varying magnetic field is required in order to make ω_b dependent on E and μ in such a way that a Landau type of resonance[1] $\omega=\omega_b$ can be neglected. This, in fact, by involving only particles with given μ or E, gives a smaller contribution[1,5] (typically $\sim\omega^3/\omega_b{}^3$) than in the case of a constant magnetic field where all particles having the same longitudinal velocity equal to the phase velocity of the wave are involved. We also notice that, as long as $\omega_{*e}>\omega$, implying $\omega_{*e}>\overline{\omega}_{be}$ or $k_\theta a_e>r_n/L$, we can follow analytically the instability resulting from Eqs. (7) and (9) through the regime where $\mathrm{Im}\omega\sim\overline{\omega}_b$ and $b_i\lesssim 1$, by making use of the cancellation of odd terms in ω_b when integrating over velocity space. In particular, when $b_i\ll 1$ and $\mathrm{Im}\omega\ll\overline{\omega}_{be}$, the present instability goes over to the known inertial drift instability.[6] Here we have implied that $\overline{\omega}_{be}\approx v_{\mathrm{th}\,e}/2L$, L being a typical distance between a point of minimum B and the nearest of maximum B, and defined $r_n\equiv -n(dn/dr_\perp)^{-1}$. We recall that electron modes that are odd around points of minimum B have been observed on the Princeton linear quadrupole.[7] In Ref. 3 an identification of these observed modes with the "ballooning" type of mode there reported was discussed. However, for the observed relatively large values of b_iT_e/T_i, the theory of that mode indicated that the longest amplitudes should have occurred around the point of minimum field, while the experiment clearly showed maximum amplitude around the point of maximum B. The mode that we have discussed here does have this feature and is, therefore, compatible with the observations.

The ion mode.—This is found in the limit $\omega<\overline{\omega}_{bi}$, so that we obtain $\int dl\,\varphi^*\tilde{n}_e$ directly from Eq. (4), and considering $b_i<1$,

$$\int\frac{dl}{B}\varphi^*\frac{\tilde{n}_i}{n_i}\frac{e}{T_i}\left\{\oint\frac{dl}{B}|\varphi|^2-\left(1-\frac{\omega_{*i}}{\omega}\right)\int\frac{f_i}{n_i}dEd\mu\,|\tau|\,|\Phi^{(0)}|^2-\tfrac{1}{2}[\Phi^{*(0)}(b_i\Phi)^{(0)}+\mathrm{c.c.}]\right\}, \tag{10}$$

where

$$(b_i\Phi)^{(0)}=\frac{1}{\tau}\int_0^\tau b_i\varphi dt'.$$

So the dispersion relation is

$$\left(1+\frac{T_i}{T_e}\right)\left\{\int f_i d\mu dE\,|\tau|\sum_{n\neq 0}|\Phi^{(n)}|^2\right\}-\frac{\omega_{*i}}{\omega}b_0\{\textstyle\int f_i d\mu dE\,|\tau|(2b_0)^{-1}[\Phi^{*(0)}(b_i\Phi)^{(0)}$$

$$+\mathrm{c.c.}]\}=\frac{\omega_{*i}\omega}{\overline{\omega}_{bi}{}^2}\left\{\int f_i dEd\mu\,|\tau|\frac{\overline{\omega}_{bi}{}^2}{\omega_b{}^2}\sum_{n\neq 0}\frac{|\Phi^{(n)}|^2}{n^2}\right\}, \tag{11}$$

or, roughly, if

$$1 > b_0 > (r_n/L)(1+T_i/T_e)X(YZ)^{-1/2}.$$

We notice that this instability, because $\omega < \overline{\omega}_{bi}$ and because it essentially requires $\Phi^{(0)} \neq 0$, is related to the trapped particle modes that are driven by the magnetic field curvature[2,4] which we have neglected here. More precisely, since the unstable modes of the latter type have maximum amplitude around the point of maximum unfavorable curvature, the present modes are directly connected with the stable modes that are due to the presence of favorable magnetic curvature and have their maximum around the point of minimum B. All these modes require that the ions be adiabatic (i.e., conserve μ and the longitudinal invariant J) to lowest order. The electron mode does not have the same requirement, but it tends to exist only for shorter wavelengths relative to the ion Larmor radius or in configurations where the typical periodicity length L is larger. The main geometrical feature of the ion mode is that it is odd around the point of maximum B. Therefore, we can roughly say that the integral X is contributed by the trapped particles while the integral Y is contributed by the circulating ones. Since the electron and ion modes both can have a large growth rate in comparison with the frequency of oscillation, they may be adequate to explain the observation of anomalous transport in the absence of fluctuations as they may give rise to convective patterns. We notice that we can extract from Eq. (13) a stability criterion for the ion mode for large T_i/T_e of the form $r_n/L > b_iT_e/T_i$, where $b_i \lesssim 1$, by considering the conditions in which it can be unstable, while the electron mode from Eq. (9) requires $r_n/L > (mT_e/T_iM)^{1/2}b_i^{1/4}$ for $b_e \lesssim 1$. Additional comparisons are given in the accompanying Table I. As pointed out previously, it is easy to see the relationship of the electron mode with the electron drift mode that is found in a straight magnetic field configuration. On the other hand, the ion mode can be related[8,9] to the impurity ion drift mode,[10] the trapped ions and electrons playing the role of "impurities" in a straight configuration. In fact, let us call n_{eT}, n_{iT} and T_{eT}, T_{iT} the densities and temperatures of the trapped particles and consider modes of the form $\varphi = \tilde{\varphi}\exp(i\omega + ik_\theta r_\theta + ik_\parallel l)$. Then for the untrapped particles we have, for $\omega/k_\parallel < v_{\mathrm{th}\,i} < v_{\mathrm{th}\,e}$, $\tilde{n}_e = e\tilde{\varphi}n_e/T_e$,

$$\tilde{n}_i = (-e\tilde{\varphi}n_i/T_i)[1-(1-\omega_{*i}/\omega)\pi^{1/2}i\omega/k_\parallel v_{\mathrm{th}\,i}],$$

Table I. Growth rates and diffusion estimates for the electron and ion modes. [D_B = approximate Bohm diffusion coefficient ($D_B \equiv a_e v_{\mathrm{th}\,e}$); $a_{e,i}$ = electron (ion) Larmor radius. m = electron mass, M = ion mass, and λ = wave-length.]

Features	Electron Mode	Ion Mode
Typical growth rate	$\gamma \sim \frac{v_{\mathrm{the}}}{2L}\left(\frac{\lambda_\theta}{a_i}\right)^{1/2}$	$\gamma \sim \frac{a_i}{\pi\lambda_\theta}\frac{v_{\mathrm{thi}}}{L}$
"Radial" wavelength	$\lambda_\theta < \lambda_r \lesssim r_n$	$\lambda_\theta < \lambda_r \lesssim r_n$
"Azimuthal" wavelength	$\lambda_\theta \lesssim 2\pi a_i\left(\frac{L}{r_n}\right)^2\left(\frac{m}{M}\frac{T_e}{T_i}\right)$	$\lambda_\theta \lesssim 2\pi a_i\left(\frac{L}{r_n}\right)^{1/2}\left(1+\frac{T_i}{T_e}\right)^{-1/2}$
Symmetry	Odd around points of minimum B	Odd around points of maximum B
Diffusion		
$D_1 \sim \gamma\lambda_\theta\lambda_r$ (most pessimistic estimate)	$D \sim D_B\left(\frac{L}{r}\right)^2\frac{m}{M}\frac{T_e}{T_i}$ $\sim D_B$ if $\left(\frac{L}{r}\right)^2\frac{m}{M}\frac{T_e}{T_i} \gtrsim 1$	$D \sim D_B\frac{r_n}{L}\frac{T_i}{T_e}$
$D_2 \sim \gamma\lambda_\theta^2$ (most optimistic estimate)	$D \approx D_B\left(\frac{L^4 a_i}{r^5}\right)\left(\frac{m}{M}\frac{T_e}{T_i}\right)^2$	$D \sim \left(\frac{a_i}{r_n^{1/2}L^{1/2}}\right)D_B\frac{T_i}{T_e}\left(1+\frac{T_i}{T_e}\right)^{-1/2}$

since they are almost free to relax to a Boltzmann distribution. On the other hand, the mass conservation equation for trapped electrons and ions gives

$$\tilde{n}_{eT} = \frac{e\tilde{\varphi}n_{eT}}{T_{eT}}\frac{\omega_{*eT}}{\omega}$$

and

$$\tilde{n}_{iT} = -\frac{e\tilde{\varphi}n_{iT}}{T_{iT}}\left[\frac{\omega_{*iT}}{\omega}\omega + b_{iT}\left(1-\frac{\omega_{*iT}}{\omega}\right)\right],$$

after neglecting the motion along the magnetic field and including the $\vec{E}\times\vec{B}$, polarization, and finite-Larmor-radius drifts.[3] Then imposing the quasineutrality condition for the equilibrium and the perturbed state we have

$$\left(1+\frac{T_i}{T_e}\right)-\frac{\omega_{*i}}{\omega}\left(b_{iT}-\frac{i(\pi)^{1/2}\omega}{k_\parallel v_{\mathrm{th}\,i}}\right)=0$$

as we could have obtained from Ref. 10 for $\omega/k_\parallel > v_{\mathrm{th}\,eT} > v_{\mathrm{th}\,iT}$. Now we recall that, as shown previously, the effect of a varying magnetic field is to eliminate the first-order term $\omega/k_\parallel v_{\mathrm{th}\,i}$ out of the expansion of the Landau integral[10]

$W(\omega/k_\parallel v_{\mathrm{th}\,i})$ and replace it by $W_{\mathrm{eff}} \approx -1-\alpha\omega^2/\overline{\omega}_b{}^2$. Notice that this result can be obtained by considering a velocity distribution with a "plateau" at small velocities, e.g., $f \propto \exp(-v^4/v_{\mathrm{th}}{}^4)$. Therefore, the equation given above becomes

$$\left(1+\frac{T_i}{T_e}\right)-\frac{\omega_{*i}}{\omega}\left(b_{iT}+\frac{\alpha\omega^2}{\overline{\omega}_{bi}{}^2}\right)=0$$

and reproduces Eq. (11). We also notice that in the limit where we obtain two stable (purely oscillatory) modes, one is of positive energy and one is of negative energy. Therefore, inclusion of Landau damping effects in the theory makes one stable and the other one unstable. A similar situation occurs for the electrons, where, by proper simulation of Eq. (7), two stable modes of opposite energy sign are obtained. Referring again to the simulation of Eq. (11), we can see that the trapped (impurity) electrons, since $\omega_{*Te}/\omega_{*Ti} < 0$, act like impurity ions having a density gradient opposite to that of the "hot" ions. Now the fact that $b_{iT} \neq 0$ does not allow complete cancellation of the contribution of $\tilde{n}_{eT}$. In this sense this instability can be considered same as the one treated in Ref. 8.

A detailed evaluation of the integrals A, C, D and X, Y, Z has been carried out by considering appropriate trial functions, making use of the variational principles that can be extracted, in appropriate limits,[4] from the quadratic forms given above, and has confirmed the order of magnitude evaluations given here.

Finally, we recall that these types of modes have interest in space physics where they may be invoked in mechanisms to explain the dumping[11] of trapped particles when their density gradient, transverse to the magnetic field lines, is greater than a given critical amount. We also point out a similarity of mathematical treatment between these modes and those, relevant to Earth's magnetic tail,[12] occurring around a neutral sheet when $\omega\tau < 1$, τ being the particle orbit periodicity.

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