EXPERIMENTAL TEST OF H_β STARK-BROADENING THEORY AT HIGH ELECTRON DENSITIES

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(Received 19 August 1968)

H_β spectral-line profiles produced in atmospheric-pressure high-current arc experiments at the Fowler-Milne normal electron density are compared with theory and found to agree within the approximately 10% uncertainty of the theory under these conditions.

Although the calculations of Griem, Kolb, and Shen[1] for the line profiles of the hydrogen beta line at 4861 Å have been experimentally verified with high precision at electron densities up to 8×10^{16} cm^{-3},[2,3] only limited experimental tests have been performed at higher electron densities.[4] The present Letter and the accompanying one by Morris and Krey[5] describe the results of some experiments suggested by Morris in which H_β profiles were observed at electron densities of 2×10^{17}. The experiments were performed in Maecker-type cascade arcs which were operated near standard atmospheric pressure (1.013×10^5 N/m^2) in argon containing a trace of hydrogen and at sufficiently high currents so as to reach the maximum equilibrium electron number density attainable at this pressure. Since this electron density is calculable, we have the necessary independent measure of the electron density with which to test the hydrogen-line-broadening theory.

In the experiments of Morris and Krey the spectrometric observations were carried out in an end-on view of the column of an arc pulsed momentarily to the necessary high current. In our experiments the spectral observations were made side-on perpendicular to the axis of a steady-state arc of smaller channel diameter (0.32 cm) and hence of lower current than the Morris arcs. Thus our experiments eliminate any uncertainties due to time-resolution requirements and reduce possible self-absorption effects. They suffer, however, a loss of precision arising from the additional step of data reduction required to invert the observed Abelian integrals of intensity to the desired local emission coefficients.

The pressure in the arc observation chamber was adjusted in each experiment to lie between 1.019 and 1.028×10^5 N/m^2 and was maintained constant to 0.3% during the experiment. Spectral intensities were recorded at 29 wavelengths between 4714 Å and 5029 Å chosen so as to avoid any of the numerous Ar II lines in this region. Each measurement was repeated in pure argon in order to obtain by subtraction an approximation to the pure hydrogen contribution to the intensity. The data recording and reduction techniques have been described before.[6] The final Abel-inverted H_β profiles were fitted to theoretical profiles by an interative least-squares fitting routine.[6] The theoretical profiles were obtained from the tabulated profiles of Kepple[7] by interpolation based on the observation that at most points the relative intensities of the reduced theoretical profiles are approximately linear functions of $\ln N$ and $\ln T$, where N and T are the electron density and temperature, respectively.

Averaged electron densities obtained from the H_β profile fitting are shown in Fig. 1 as a function of arc radius for five arc currents. It is clear that a maximum axial electron density is reached at a current of about 120 Å. At higher currents, in spite of the fact that the axial temperature must rise with the increasing power level, the electron density distribution merely broadens, never rising above an indicated value of about 1.78×10^{17} cm^{-3}. The rms deviation of the individual experimental intensities from the theoretical profiles amounted to 5.5% of the pro-

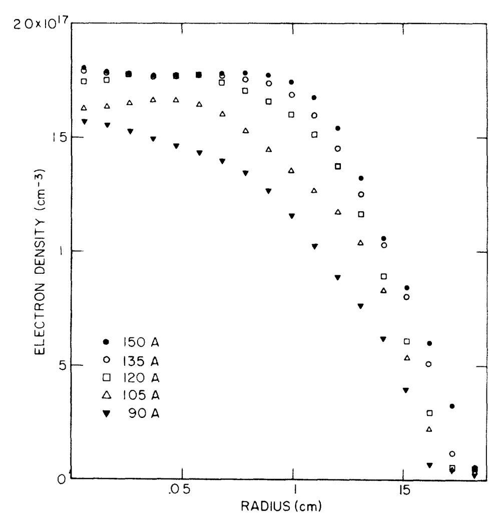

FIG. 1. Distribution of radial electron density as determined from H_β line profiles for several arc currents.

file maximum at the electron density maxima and nearby points. This appears to be caused about equally by random experimental scatter and line asymmetry which is not included in the theoretical profiles. The electron density determination at any one point appears to be repeatable to within about 4.8% near the maximum based upon duplicate experiments and comparison of measurements taken on the left and right sides of the arc. Altogether perhaps 20 experimental points contribute significantly to the determination of the electron density at its maximum, resulting in a relative standard error for this value of the order of 1 or 2%. The 5029-Å continuum intensities shown in Fig. 2 for the same experiments corroborate the observation of a limiting electron density inasmuch as the continuum is approximately proportional to $N^2/\sqrt{T}$.

Under local thermodynamic equilibrium conditions such as are generally believed to prevail in experiments of this kind, the maximum attainable electron density is easily computed for any pressure.[8] The presence of some hydrogen in the argon has little net effect on this calculation because of the counteracting effects of a lower first-ionization potential but an effectively infinite second-ionization potential. At atmospheric pressure the maximum electron density varies from 2.02×10^{17} cm^{-3} for pure argon to 1.99×10^{17} cm^{-3} for a hydrogen-to-argon atomic ratio of 0.15. (The $2\frac{1}{2}$% difference between these calculations for pure argon and the widely used tables of Drellishak, Knopp, and Cambel[9] stems

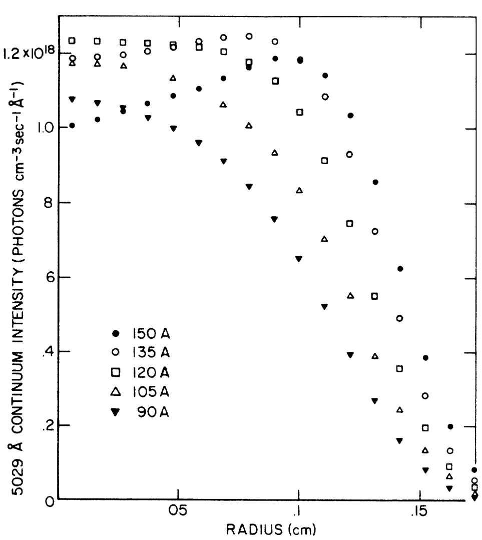

FIG. 2. Radial distribution of 5029-Å continuum intensity for several arc currents.

from our inclusion of the Debye-Hückel pressure correction and from a difference in the value for the partition function cutoff; the calculations performed here employ the expressions recommended by Griem.[8]) Griem's criteria[8] for the validity of local thermodynamic equilibrium in inhomogeneous stationary sources are well fulfilled in these experiments. The pressure uncertainty caused by the magnetic self-compression and the Yankov momentum-transport overpressures would cause the axial electron densities to be an estimated 5% higher in the 150-A arc if the arc were confined axially. However, since in our experiments the plasma can flow freely along the arc axis to unconfined regions of low current density and temperature, it is unlikely that any significant overpressure could be maintained.

We thus conclude that for electron densities near 2×10^{17} the H_β Stark-broadening theory yields electron densities that are too low by about 10%. This means that the theoretical line profiles are too wide by only 7%, which is well within the estimated 10% or so accuracy of the theory under these conditions.[7] The similarity of the results obtained by Morris and Krey in the end-on pulsed-arc experiments lends additional reliability to this conclusion because their experiments can be expected to be affected differently, if at all, by some of the possible sources of systematic error such as axial inhomogeniety, radial gradients, Abel inversion, and magnetic and Yankov overpressures.

If we take the step of scaling the indicated electron densities of Fig. 1 to agree with the local thermodynamic equilibrium calculations, we can then use them together with the measured total H_β intensities to obtain the temperature and hydrogen-argon atomic ratio distributions in the experiments. The axial temperatures range from 18 300°K at 150 A to 14 600°K at 90 A. The temperature at the maximum electron density is about 16 800°K. The hydrogen-to-argon atomic ratio on the arc axis ranges from 0.11 at 150 A to 0.05 at 90 A. The ratio decreases to approximately 0.03 at the edge of the arc as expected for the nominally $1\frac{1}{2}$% hydrogen-gas mixture used. The nearly fourfold enrichment of hydrogen on the axis of 150 A constitutes an impressive example of the arc demixing phenomenon.

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LABORATORY OBSERVATIONS OF PLASMA SATELLITES ON THE 2^1P-4^1D AND 2^3P-4^3D LINES OF HELIUM

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(Received 19 August 1968)

During the implosion phase of a low-density θ pinch in helium preheated to $T_e \approx 1$ eV at $N_e \approx 5\times10^{13}$ cm^{-3}, the He I 2^1P-4^1D and 2^3P-4^3D lines show satellites displaced away from the forbidden $2P$-nF transitions by 0.3–0.6 Å. These satellites are identified with those predicted by Baranger and Mozer as due to the effects of nonthermal longitudinal plasma oscillations on atomic spectra.

The prediction[1] of satellite lines disposed symmetrically in pairs about a forbidden ($\Delta l = 0, \pm 2$) atomic line and separated from it by Ω, the electron plasma frequency, aroused considerable interest because their observation would amount to a measurement of the spectrum of plasma oscillations with a noninterfering probe. Second-order perturbation theory yields[1] for the total intensity of the satellites relative to that of the allowed ($\Delta l = \pm 1$) atomic line

$$S_\pm = \hbar^2 \langle E_p^2\rangle R_{ll'}/6m^2e^2(\Delta \pm \Omega)^2, \qquad (1)$$

where m and e are the mass and charge, respectively, of the electron, Δ is the splitting (in angular-frequency units) between the allowed and forbidden line, and $\langle E_p^2\rangle$ is the time average of the square of the plasma electric field. The quantity $R_{ll'}$ is a dimensionless radial integral[1] (in units of the Bohr radius a_0 squared)

$$R_{ll'} = \frac{1}{a_0^2}\sum_{\alpha, m'} |\langle lm|x^\alpha|l'm'\rangle|^2, \qquad (2)$$

x^α being the α component of the atomic-electron coordinate operator and l, m and l', m' the angular and magnetic quantum numbers of "allowed" and "forbidden" levels. (Perturbations of the lower atomic state are neglected.) To estimate $\langle E_p^2\rangle$ the corresponding energy density may be compared with the thermal energy density $\frac{3}{2}N_e kT_e$ of the electron gas. With the definition

$$\epsilon \equiv \frac{U_{\text{waves}}}{U_{\text{particles}}} = \frac{\langle E_p^2\rangle}{8\pi(\frac{3}{2}N_e kT_e)} = \frac{\langle E_p^2\rangle e^2}{3\Omega^2 mkT}, \qquad (3)$$

the satellite strength can then be written as

$$S_\pm = \epsilon \frac{kT_e \hbar^2}{2me^4}\left[\frac{\Omega}{\Delta \pm \Omega}\right]^2 R_{ll'} = \epsilon \frac{kT_e}{4E_H}\left[\frac{\Omega}{\Delta \pm \Omega}\right]^2 R_{ll'}, \qquad (4)$$

where E_H is the ionization energy of hydrogen. For thermal excitation of plasma waves, which only exist for $k \lesssim 1\rho_D$, the ratio ϵ is estimated by

$$\epsilon_{th} \approx \frac{4\pi k_{max}^3}{3(2\pi)^3 3N_e} \approx \frac{1}{18\pi^2 N_e \rho_D^3} = \frac{2}{27\pi n_D}, \qquad (5)$$

where n_D is the number of electrons in the Debye sphere of radius $\rho_D = (kT_e/4\pi N_e e^2)^{1/2}$. This number is usually large ($n_D \approx 200$, e.g., at $kT_e \approx 1$ eV, $N_e \approx 5\times10^{13}$ cm^{-3}, corresponding to $\epsilon_{th} \approx 10^{-4}$). Furthermore, kT_e/E_H has to be below unity, except in extremely transient situations, to allow the presence of a sufficient number of neutral atoms, and the next factor in Eq. (4) is at best of order 10^{-1} for situations where the satellite displaced toward the allowed line (at $\Delta - \Omega$) is not completely merged into this line. Although