EVIDENCE FOR ELECTRON-ELECTRON SCATTERING IN THE LOW-TEMPERATURE RESISTIVITY OF SIMPLE METALS*

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Observations of the temperature-dependent resistivity of aluminum, indium, sodium, and potassium have shown a T^2 term to exist in the resistivity of indium and aluminum, but not in sodium or potassium.

It has been suggested for some time that electron-electron scattering should dominate the temperature-dependent resistivity of most metals at sufficiently low temperatures.[1] While this effect, characterized by a T^2 temperature dependence, has frequently been observed in the transition metals and rare earths, it has not—with the possible exception of gallium[2]—previously been seen in simple metals. In this Letter we report the results of precise measurements of the temperature dependence of the electrical resistivity of polycrystalline samples of aluminum, indium, potassium, and sodium below 4.5°K. We have verified the existence of a T^2 component in the electrical resistivity of aluminum and indium, but have not observed such a term in potassium or sodium.

Our measurements were made on annealed wire samples wound into the form of noninductive coils. For aluminum and indium the probes were soldered to the specimens, while in the case of the alkali metals copper wires were pressed into place. We used a standard four-probe dc method consisting of a stable current source, low-noise dc amplifier, and an integrating digital voltmeter. Temperatures were determined by measuring the liquid helium vapor pressure, while short-term temperature fluctuations were monitored with a carbon resistance thermometer. The overall error of our voltage measurements was less than 0.1 %; relative temperatures were easily determined to within a few millidegrees, although absolute temperatures could have been in error by as much as 15 mdeg at the lowest temperatures.

In Fig. 1 are shown the data for a typical indium sample. Points below the superconducting transition temperature ($T_C = 3.4$°K) were obtained by an extrapolation procedure. The magnetic field dependence of the resistivity in the normal state was extrapolated to zero field by comparison with magnetoresistance data taken above T_C and assuming the validity of Kohler's rule. Since the critical field H_C of indium is always less than 300 G, this procedure is sufficiently accurate so that the scatter of the points below T_C is only slightly greater than that observed in the direct measurements above T_C. The data of Fig. 1 follow very well the solid curve which represents an equation of the form

$$\rho(T) = \rho(0) + AT^2 + BT^5.$$

Also shown in the figure are graphs of the individual T^2 and T^5 components, and it can be seen that the T^2 term in the resistivity dominates the

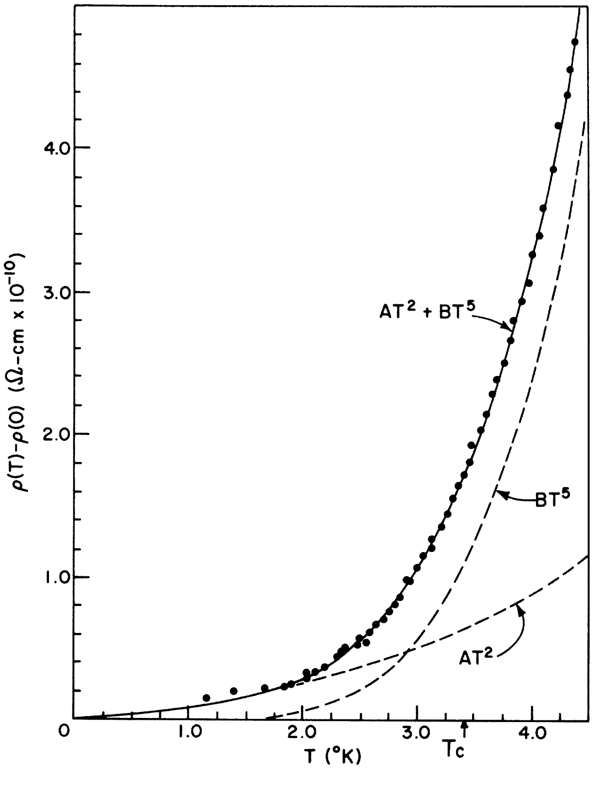


FIG. 1. Temperature dependence of resistivity of indium sample C showing decomposition into T^2 and T^5 components. The solid curve represents an equation of the form $\rho(T)-\rho(0) = AT^2 + BT^5$. The points correspond to the raw data less a large constant term equal to 8.8×10^{-10} Ω cm. The wire diameter of this sample was 0.50 mm.

T^5 term below 3°K. In determining the coefficients A and B we have plotted $[\rho(T)-\rho(0)]/T^2$ as a function of T^3; this results in a straight line whose intercept and slope yield the coefficients A and B directly. The zero-temperature resistivity $\rho(0)$ can be obtained with an experimental error of about 1 % by direct extrapolation of data points to $T=0$. Then small adjustments of $\rho(0)$, of the order of a fraction of a percent, are made to yield best linearity of the curve at low temperatures.

In Fig. 2 we have plotted $[\rho(T)-\rho(0)]/T^2$ as a function of T^3 for each of four samples of comparable purity of aluminum, indium, potassium, and sodium. To within the experimental error, neither of the alkali metals shows any evidence of T^2 behavior. In fact, the total resistance change of the sodium sample was only 0.2 % between 4.2 and 1.2°K. The potassium data are somewhat unusual in that the resistivity increases precisely as T^5 up to about 3.5°K and then appears to increase more rapidly. It is possible that this represents the onset of electron-phonon umklapp processes which are expected to contribute to the resistivity a term of the form $\rho_U \propto \exp(-\Theta_D/\beta T)$, where Θ_D is the Debye temperature and β is a numerical factor approximately equal to 7 for a bcc metal with a spherical Fermi surface.[3]

The resistance of our aluminum sample increased almost exactly as T^2 in this temperature range, the absence of a T^5 term apparently reflecting the relatively large value of Θ_D. Previous measurements of the electrical resistance of aluminum at low temperatures have not been consistent, with temperature dependences between $T^{2.1}$ and $T^{4.6}$ customarily observed.[4-6] However, Aleksandrov and D'yakov[7] and Holwech and Jeppesen[8] have observed a decrease in the power-law exponent with temperature in the range of 10 to 50°K; their results appear to be consistent with our data taken below 4.1°K.

Our measurements on indium indicate a relatively large T^2 component in the resistivity as seen in Fig. 2. In order to ascertain the impuri-

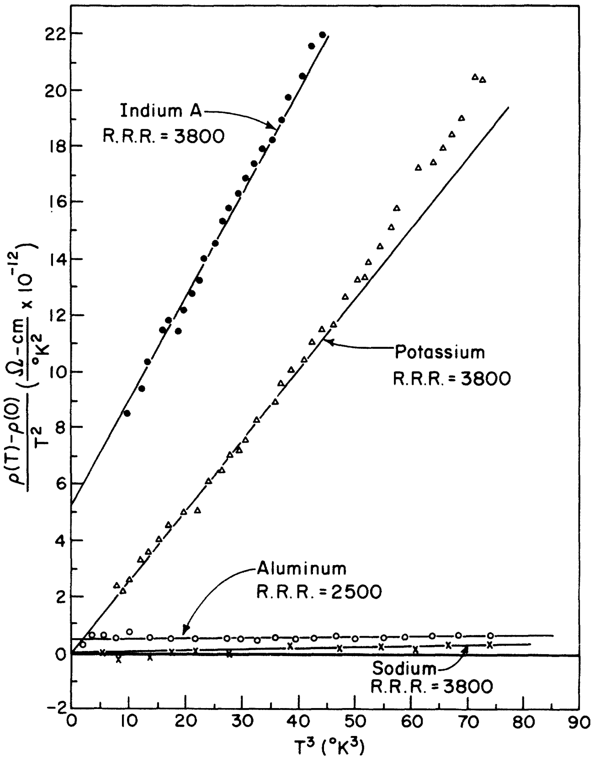
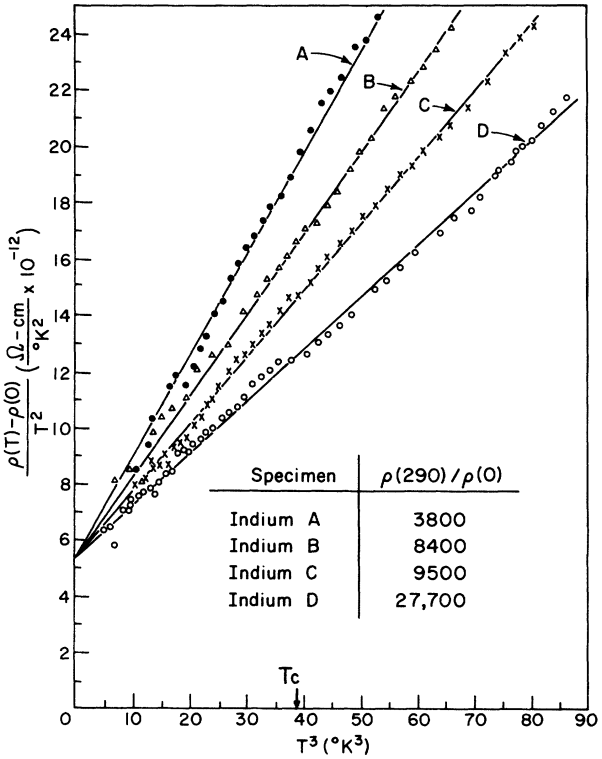


FIG. 2. Temperature-dependent resistivity of indium A, aluminum, potassium, and sodium showing relative magnitudes of T^2 and T^5 components. The residual resistance ratio is shown for each sample. The diameter of these samples varied between 0.75 and 1.05 mm.



Specimen	$\rho(290)/\rho(0)$
Indium A	3800
Indium B	8400
Indium C	9500
Indium D	27,700

FIG. 3. Temperature-dependent resistivity of indium samples A, B, C, and D showing the impurity dependence of the T^2 and T^5 components. The wire diameter for these samples varied between 0.50 and 0.75 mm.

ty dependence of this term, we measured the resistance of a number of indium wires of different purities. As is evident from Fig. 3, the coefficient of the T^2 term does not depend on sample purity, although the coefficient of the T^5 component increases significantly with increasing impurity content. Deviations from Matthiessen's rule of this sort have occasionally been observed in aluminum,[4,8] although recent size-effect measurements by Wyder[9] have not shown the effect in indium. Since our data appear to show an unmistakable deviation from Matthiessen's rule, we are led to conjecture that it may occur when the resistance is impurity limited rather than size limited.

Calculations by Dingle[10] of the effect of surface scattering on the resistivity suggest that the size-enhanced resistivity merely appears as a temperature-independent additive term to the residual resistivity. However, Olsen[11] and Blatt and Satz[12] have proposed a $T^{7/3}$ contribution to the resistivity due to deflection of electrons into the surface by successive small-angle electron-phonon collisions. Although there is a limited amount of evidence that this may occur in gallium[2] and aluminum,[13] no evidence of it was observed by Wyder[9] in indium wires as thin as 0.09 mm. Our measurements on many indium wires whose diameters ranged between 0.5 and 2.0 mm have shown no size dependency in our results. This is a reasonable finding since the ratio of electron mean free path to sample diameter was less than $\frac{1}{2}$ for all of our smaples.

In summary, our measurements show the existence of a quadratic temperature dependence in the low-temperature resistivity of indium and aluminum. In contrast to the ordinary thermal resistivity, the magnitude of this term does not change with impurity concentration. This fact and the absence of an observable T^2 term in sodium and potassium can most directly be explained, we believe, in terms of electron-electron interactions. However we cannot, of course, rule out the possibility of another, more subtle, explanation. Although a calculation by Ziman[3] has suggested that electron-electron umklapp processes could in principle enhance the resistivity of the alkali metals, the effect is expected to be much smaller than in metals with anisotropic Fermi surfaces. Preliminary calculations by Lawrence[14] based on the Kohler variational method and a two-plane-wave approximation have given good agreement with our results for potassium, sodium, and aluminum, but predict a T^2 term for indium which is somewhat smaller than the value observed by us.

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