DOUBLE-REGGE-POLE MODEL ANALYSIS OF $pp \to \Delta^{++}p\pi^-$ AT 6.6 GeV/c†

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Reasonable fits to invariant-mass, momentum-transfer, and Treiman-Yang angle distributions for the reaction $pp \to \Delta^{++}p\pi^-$ at 6.6 GeV/c are obtained from a double-Regge-pole model with pion exchange.

Results obtained from a detailed analysis of the reaction $pp \to \Delta^{++}p\pi^-$ employing the model of double Regge-pole exchange[1,2] are presented in this paper. The data were derived from a sample of four-prong events of the type $pp \to pp\pi^+\pi^-$ produced by 6.6-GeV/c incident protons. Initial discussion of approximately one quarter of the data presented in this paper has been published.[3] The final state $\Delta^{++}p\pi^-$ is produced peripherally, with predominance of small momentum transfers to the final Δ^{++} and p. Moreover, a plot of the invariant mass of the $\Delta^{++}\pi^-$ system evidences a large enhancement in the range 1.38 to 1.58 GeV.

The basic assumption made in this study is that the process $pp \to \Delta^{++}p\pi^-$ proceeds primarily via doubly peripheral collisions of the type diagrammed in Fig. 1, in which the exchanged lines represent Regge poles. We wish to remark that, although this model appears to describe the data adequately, we do not interpret our results as necessarily casting doubt on the validity of certain un-Reggeized one-pion-exchange (OPE) calculations. Rather, we wish to emphasize the application of the Regge-pole model of this paper to inelastic three-body scattering processes.

The requirement that quantum numbers combine appropriately at the three vertices of Fig. 1 limits the possible pairings of Regge-pole exchanges in the diagram. In this analysis, only those diagrams in which a pion Regge pole couples at the $p\Delta^{++}$ vertex are retained. Other diagrams (for example, ρ coupling at the $p\Delta$ vertex) are eliminated because they are not expected to contribute significantly to the cross section for small $\pi\Delta$ mass.[4] However, on the left-hand side of the diagram, all Regge poles which participate in π^-p elastic scattering may be present; and because the c.m. energy of the overall reaction is relatively low, all of these are expected to contribute significantly. In summary, therefore, Fig. 1 is to be understood as representing a pion trajectory coupled at the $p\Delta^{++}$ vertex and then a sum over all allowed trajectories between the pp and middle $\pi\pi$ vertex.

Because the initial particles are identical, we must impose the required antisymmetrization of the reaction amplitude. This is properly accomplished by adding to Fig. 1 a diagram in which the initial particle momenta are interchanged.[5] Because cuts are taken in momentum transfer, the interference term is less than 10% of the contribution from one diagram alone and is therefore ignored altogether. The two diagrams are incorporated by taking twice the magnitude of one.

The differential cross section for the reaction

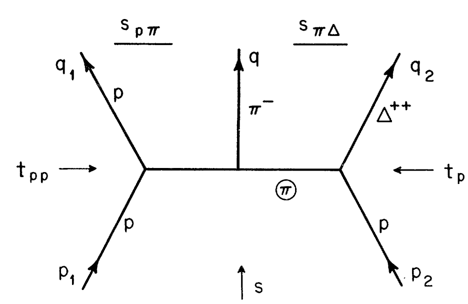

FIG. 1. Basic double-Regge-pole exchange diagram studied here. The p_i and q_j are four momenta; $t_{p\Delta} = (q_2 - p_2)^2$; $s_{\pi\Delta} = (q + q_2)^2$; etc.

$pp \to p\pi^- \Delta^{++}$ is given by

$$d\sigma = (2\pi)^{-5}(4F_I)^{-1}(\sum |M|^2) d\varphi_3, \qquad (1)$$

in which F_I equals the proton mass times the incident proton lab momentum, and $d\varphi_3$ is the differential element of phase space.

The Toller-variable analysis of multi-Regge-exchange processes by Bali, Chew, and Pignotti[1,2] serves as the basis for this study. The double-Regge-pole hypothesis is adopted for the absolute square of the invariant amplitude, M, summed over final spins and averaged over initial spins. After some reduction, this is expressed as

$$\sum |M|^2 = \frac{(\pi\alpha')^2 N}{1-\cos\pi\alpha_\pi}(\sum |M_{\pi^- p}'|^2) \times \{s_0^{-1}[s_{\pi\Delta} - t_{pp} - m_p^2 + \tfrac{1}{2} t_{p\Delta}^{-1}(m_\Delta^2 - m_p^2 - t_{p\Delta}) \times (m_\pi^2 - t_{pp} - t_{p\Delta})]\}^{2\alpha_\pi}. \qquad (2)$$

In Eq. (2), the aforementioned factor of 2 has been incorporated, α_π is the pion Regge trajectory, α' is the slope of that trajectory evaluated at $t_{p\Delta} = m_\pi^2$, and N is a slowly varying function of $t_{p\Delta}$ in which have been combined both the reduced residue function and the usual ratio of gamma functions. The quantity $M_{\pi^- p}'$ is the amplitude describing the full scattering of a Reggeized pion off a proton yielding the final, physical $\pi^- p$ state. In view of the fact that a complete prescription for the coupling of two trajectories to a physical state (the middle vertex) does not yet exist, and because the sum of many trajectories on the left-hand side of the diagram is to be taken, $M_{\pi^- p}'$ is approximated by the on-mass-shell physical $\pi^- p$ elastic-scattering amplitude whose magnitude, squared and appropriately summed, is given in terms of the elastic differential cross section by

$$\sum |M_{\pi^- p}'|^2 = 64\pi^2 s_{\pi^- p}(d\sigma/d\Omega)_{\pi^- p}. \qquad (3)$$

The approximation clearly requires that $t_{p\Delta}$ be small. The insertion of the on-shell distribution appears justified in this calculation because the shape parameters of the angular distribution of the scattered proton p, as measured in the rest frame of the final $p\pi^-$ system, are in good agreement with those for free $p\pi^-$ scattering.[3]

Empirical data[6] were used for $d\sigma/d\Omega$.

The pion Regge trajectory has not yet been emprically determined, as have other trajectories, from two-particle scattering experiments. In this study a linear trajectory was assumed, and its slope at $t_{p\Delta} = m_\pi^2$ was fixed at $\alpha' = 1.0\ \mathrm{GeV}^{-2}$, consistent with the general philosophy of approximately equal slopes for all trajectories and in rough agreement with the value obtained by Arbab and Dash[7] from an analysis of np charge-exchange data.

In order to reduce the number of free parameters, the function $N(t_{p\Delta})$ was chosen to be a constant whose value is determined by requiring that the Regge-model expression [Eq. (2)] be identical to the elementary OPE model expression in the limit that $t_{p\Delta} \to m_\pi^2$. The OPE expression, with Δ^{++} spin factors evaluated at $t_{p\Delta} = m_\pi^2$, is given by

$$\sum |M_{\mathrm{OPE}}|^2 = \frac{64\pi\Gamma m_\Delta^3(\sum |M_{\pi^- p}'|^2)(t_{p\Delta} - m_\pi^2)^{-2}}{\{[(m_\Delta + m_p)^2 - m_\pi^2][(m_\Delta - m_p)^2 - m_\pi^2]\}^{1/2}}. \qquad (4)$$

In Eq. (4), the width of the Δ, $\Gamma = 120$ MeV is expressly incorporated.

With this determination of $N(t_{p\Delta})$, normalization is entirely fixed and there remains only one free parameter in the Regge-model matrix element, the scale constant s_0. Because varying s_0 essentially serves to vary the slope of $d\sigma/dt_{p\Delta}$, s_0 was determined by a fit to that experimental distribution. For various s_0, the differential cross section in $t_{p\Delta}$ was computed by substituting Eq. (2) into Eq. (1) and numerically integrating over allowed phase space, subject to the same cuts taken in the data (the mass of Δ^{++} was fixed at 1.22 GeV). As shown in Fig. 2, the value $s_0 = 0.8\ \mathrm{GeV}^2$ provided the best overall fit. The deviation at the smallest values of $t_{p\Delta}$ may be associated with the fact that a spin-averaged analysis is used here and thus subtleties (as well as extra parameters) associated with the couplings of the various helicity states are washed out. Moreover, the analysis done here assumes no dependence in the Regge-model matrix element on the Toller variable ω associated with the middle vertex function.[1,8] Independent evidence indicates that such an approximation does injustice to behavior at very low momentum transfer.[9]

With the determination of s_0, the Regge ma-

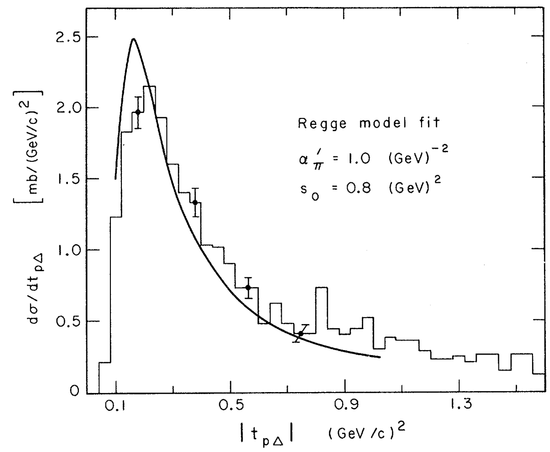

FIG. 2. Distribution in the invariant four-momentum-transfer-squared carried by the pion exchange for the reaction $pp \to \Delta^{++}p\pi^-$. The Δ^{++} is defined as 1.16 <mass $(p\pi^+)$ < 1.28 GeV. In order to eliminate the quasi-two-body $\Delta^{++}\Delta^0$ final state from the sample, only those events with mass $(p\pi^-) > 1.34$ GeV were retained. The plot contains 3915 events, of which 188 are double Δ^{++}'s, counted twice.

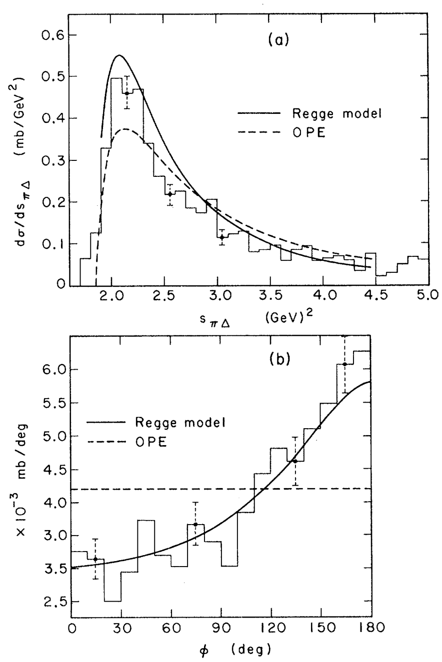

FIG. 3. (a) Distribution in the invariant-mass-squared of the $\pi\Delta$ system. The plot contains 2214 events with $t_{p\Delta} \leq 0.5\ \mathrm{GeV}^2$, mass $(p\pi^-) > 1.34$ GeV. Less than 3% of the events are double Δ^{++}'s. (b) Treiman-Yang angle distribution for 2471 events with $t_{p\Delta} \leq 0.6\ \mathrm{GeV}^2$ and mass $(p\pi^-) \geq 1.34$ GeV.

trix element is entirely fixed. In order to remain within the expected range of validity of the model and approximations, a further cut restricting $|t_{p\Delta}| < 0.5\ \mathrm{GeV}^2$ was taken and the distribution $(d\sigma/ds)_{\pi\Delta}$ was calculated. This is shown in Fig. 3(a). The experimental peak is centered at 1.46-1.48 GeV and has a full width at half-maximum of 200 MeV, whereas the Regge-model curve has a value of 250 MeV. The Regge-model curve is uniformly too high and somewhat too broad at low $s_{\pi\Delta}$ to be called a good fit; however, it reproduces the trend of the data quite well considering the very limited number of parameters involved.

Also presented on Fig. 3(a) is a curve resulting from integrating the OPE matrix element [Eq. (4)] multiplied by a simple exponentially falling form factor in $t_{p\Delta}$, $F(t_{p\Delta}) = \exp[0.8(t_{p\Delta} - m_\pi^2)]$. This form factor was chosen in order that the modified OPE model yield a distribution $(d\sigma/dt)_{p\Delta}$ falling as rapidly in $t_{p\Delta}$ as does the experimental distribution. The OPE curve is clearly a much less satisfactory fit. However, this hardly exhausts the variety of modifications to OPE. Specifically, see the work of Colton et al.[10] The reason for the increased low-mass enhancement of the Regge model in comparison with OPE is discussed in Ref. 2.

The most encouraging support for the Regge-model analysis comes from examining the Treiman-Yang-angle distribution. The angle is defined in the rest frame of the final $p\pi^-$ system by

$$\varphi = \cos^{-1}\left[\frac{\vec{p}_1 \times \vec{q}_1}{|\vec{p}_1 \times \vec{q}_1|} \cdot \frac{\vec{q}_2 \times \vec{p}_2}{|\vec{q}_2 \times \vec{p}_2|}\right].$$

Physically, in the rest frame of definition, φ is a rotation angle about the three-momentum vector $\vec{p}_1$ which, in that frame, is the three-momentum of the exchanged-pion system. The distribution in φ, therefore, should reflect the spin character of the exchange. In Fig. 3(b), the data as well as curves resulting from the Regge and OPE models are shown. The Regge model adequately agrees with the data, yielding both the observed asymmetry about 90° and the peaking towards 180°.

It should be noted, however, that the fits to the $s_{\pi\Delta}$ and φ distributions are not entirely independent arguments in favor of the Regge model. A kinematic relationship connects the two variables, viz.,

$$s_{\pi\Delta} = A(s_{\pi p}, t_{p\Delta}, t_{pp}) + \cos\varphi\, B(s_{\pi p}, t_{p\Delta}, t_{pp}), \qquad (5)$$

in which A and B are positive-valued in the physical region. Thus a matrix element in which there is explicit dependence on $s_{\pi\Delta}$, such as in the Regge model, may in general yield a nonisotropic distribution in φ. On the other hand, the OPE model leads to an enhancement of low values of the $\pi\Delta$ mass but a flat distribution in φ. This is because the enhancement of low $\pi\Delta$ masses is a kinematic reflection of the relative enhancement in that matrix element of small values of the momentum transfers t_{pp} and $t_{p\Delta}$; the matrix element remains independent of φ (unless such dependence is added in an ad hoc fashion, as in Ref. 10).

The agreement between the double-Regge-pole model and the data should not be interpreted as precluding the existence of resonances in the $\pi\Delta$ system. Indeed, recent work on the πN problem strongly suggests that direct channel resonances are already contained in the cross-channel Regge amplitude.[11]

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[1]Multi-Regge-pole exchange models have been studied by various researchers, including N. F. Bali, G. E. Chew, and A. Pignotti, Phys. Rev. Letters 19, 614 (1967), and Phys. Rev. 163, 1572 (1967); H. M. Chan, K. Kajantie, and G. Ranft, Nuovo Cimento 49A, 157 (1967); H. M. Chan, K. Kajantie, G. Ranft, W. Beusch, and E. Flamino, Nuovo Cimento 51A, 696 (1967); F. Zachariasen and G. Zweig, Phys. Rev. 160, 1322, 1326 (1967). See also Ref. 2.

[2]E. L. Berger, Phys. Rev. 166, 1525 (1968).

[3]E. Gellert, G. A. Smith, S. Wojcicki, E. Colton, P. Schlein, and H. Ticho, Phys. Rev. Letters 17, 884 (1966).

[4]For example, as discussed in Refs. 1 and 2, if a ρ trajectory couples to the $p\Delta$ vertex and a pion to the pp vertex, then

$$d\sigma \propto \left(\frac{s_{\pi p}}{s_{\pi\Delta}}\right)^{\alpha_\pi-\alpha_\rho} d\ln\left(\frac{s_{\pi p}}{s_{\pi\Delta}}\right),$$

thus tending to suppress low values of $s_{\pi\Delta}$, since $\alpha_\pi < \alpha_\rho$.

[5]See, for example, J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill Book Company, Inc., New York, 1964), p. 136.

[6]L. Roper, R. Wright, and B. Feld, Phys. Rev. 138, B190 (1965); P. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, J. J. Thresher, H. H. Atkinson, C. R. Cox, and K. S. Heard, Phys. Rev. Letters 15, 468 (1965).

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[8]A physically identical angle is also defined by Chan et al., Ref. 1.

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CORRECTIONS TO THE EXPERIMENTAL VALUE FOR THE ELECTRON g-FACTOR ANOMALY*

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A recalculation of the g-factor anomaly of the free electron, based on the data of Wilkinson and Crane, results in the value $a = 0.001\,159\,557 \pm 30\times10^{-9}$. This is lower than theoretical estimates by more than the quoted experimental error.

The value of the electron g-factor anomaly, as measured by Wilkinson and Crane[1] in an experiment hereafter called g-III (it was the third in a series), was determined to be

$$a_{\rm expt} = 0.001\,159\,622 \pm 27\times10^{-9}.$$

Quantum electrodynamics (QED) theory predicts that $a = 0.5(\alpha/\pi) - 0.328\alpha^2/\pi^2 + \cdots$. If the accepted value[2] of α $[\alpha_A^{-1} = 137.0388(4.5)]$ is substituted we obtain

$$a_{\rm theory}^{A} = 0.001\,159\,617(5).$$

However, this value of α is now in question. Recent work[3,4] suggests an $\alpha_B^{-1} = 137.0359(3)$ which causes the anomaly to be

$$a_{\rm theory}^{B} = 0.001\,159\,614(3).$$