Field Theory 1 (W. A. Benjamin, Inc., New York, 1963), who concluded that ξ should not depend strongly on q^2 and that $-2<\xi<+2$.

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GHOST-ELIMINATING MECHANISM AND TRAJECTORY PARAMETER OF A_2 BY PHOTOPRODUCTION SUM RULE*

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The A_2 trajectory and residue functions are obtained and are found to change sign around $t=-0.5$ GeV^2. This strongly suggests the Chew or the no-compensation mechanism of ghost elimination. The trajectory parameter $\alpha(t)$ also agrees well with existing estimates.

Finite-energy sum rules have been used by Igi and Matsuda[1] and by Dolen, Horn, and Schmid[2] to obtain some useful information about the ρ Regge parameter from πp scattering data in the intermediate energy range. More recently, Matsuda and Igi[3] have tried to clarify the ghost-eliminating mechanism for A_2 by using the finite-energy sum rule for a combination of KN and $\bar{K}N$ scattering amplitudes. Unfortunately, this program has not been successful because of the large ambiguities in the various hyperon coupling parameters. In this note we obtain the ghost-eliminating mechanism for A_2 by studying the sum rule S_0 for a suitably chosen photoproduction amplitude[4] that couples only to A_2. Thus we are able to dispense with the saturation hypothesis and work directly in terms of the multipoles, which are obtained from photoproduction data. We also estimate the A_2 trajectory parameter by using the second-moment sum rule S_2, in addition to S_0.

Using the notation of Zweig,[5] we pick up the combination of invariant amplitudes $A_1^{(-)}-2MA_4^{(-)}$, where the superscript $(-)$ refers to the t-channel isospin 1 and G-parity negative (isovector photon). This is related to the definite spin-parity helicity amplitude $\tilde{f}^t$ by[6]

$$A_1^{(-)}-2MA_4^{(-)}=-2\sqrt{2}M\tilde{f}^t, \tag{1}$$

where M is the nucleon mass. $\tilde{f}^t$ in this case couples to the nucleon-antinucleon triplet state $|\frac{1}{2}\frac{1}{2}\rangle+|-\frac{1}{2}-\frac{1}{2}\rangle$, which has $C=P=(-1)^J$. This, together with the isospin and G-parity requirement, allows only A_2 exchange. Then, on absorbing certain innocent factors into the reduced residue function $\gamma(t)$, we get the usual Regge contribution

$$A_1^{(-)}-2MA_4^{(-)}=\alpha\left(\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\gamma(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha-1}, \tag{2}$$

where

$$\nu=\frac{s-u}{4M}=k_L+\frac{t-\mu^2}{4M}, \tag{3}$$

μ is the pion mass, k_L is the lab photon momentum, and ν_0 is a scale factor which we choose for convenience to be 1 GeV. Now, using the odd crossing property of our amplitude[5] under $\nu\to-\nu$, we get the following sum rule:

$$S_0=(g/4M)[e+2M(\mu_p-\mu_n)]-\pi^{-1}\int_{\nu_{\rm th}}^{N}\mathrm{Im}(A_1^{(-)}-2MA_4^{(-)})d\nu=\pi^{-1}\gamma(t)N^{\alpha(t)}. \tag{4}$$

The second-moment sum rule gives

$$S_2=\frac{1}{N^2}\left\{\frac{g\nu_{\rm B}^2}{4M}[e+2M(\mu_p-\mu_n)]-\frac{1}{\pi}\int_{\nu_{\rm th}}^{N}\nu^2\,\mathrm{Im}(A_2^{(-)}-2MA_4^{(-)})d\nu\right\}=\frac{1}{\pi}\gamma(t)\frac{\alpha(t)}{\alpha(t)+2}N^{\alpha(t)}. \tag{5}$$

The scale factor ν_0 has been dropped since we work in GeV units. The terms in front of the integrals in Eqs. (4) and (5) are the nucleon Born terms; μ_p and μ_n denote the anomalous magnetic moments of

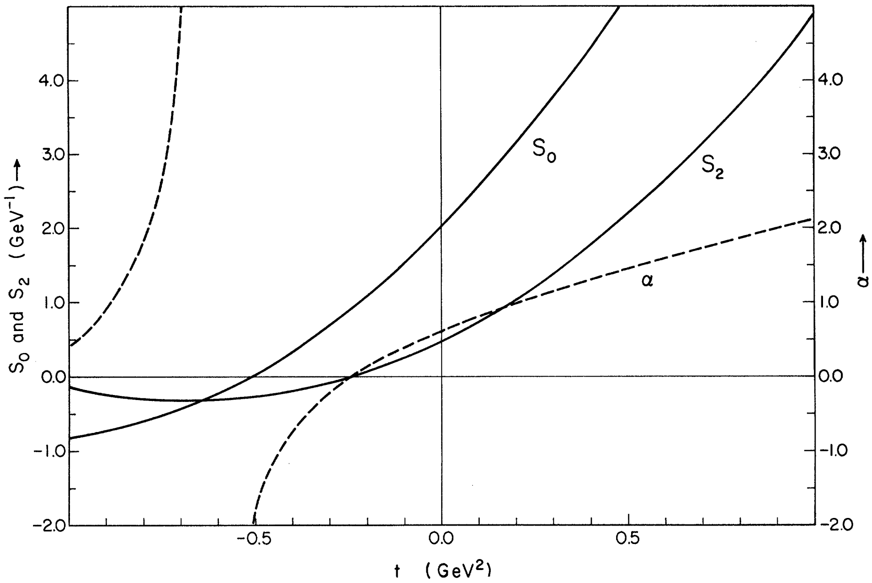

FIG. 1. Sum rules S_0 and S_2 and trajectory parameter α as a function of t.

proton and neutron. We have

$$\mu_p = 1.78e/2M, \quad \mu_n = -1.91e/2M,$$

$$e^2/4\pi = 1/137, \quad g^2/4\pi = 14,$$

and

$$\nu_\mathrm{B} = (t - 4\mu^2)/4M. \tag{6}$$

Finally the trajectory parameter is obtained from Eqs. (4) and (5) as

$$\alpha(t) = 2S_2/(S_0 - S_2). \tag{7}$$

In order to evaluate the continuum contributions to the above sum rules, the invariant amplitudes are expanded in terms of the electric and magnetic multipoles following Ref. 5. These multipoles have been empirically obtained by Walker,[7] in terms of Breit-Wigner poles plus nonresonant parts, up to a lab momentum of 1.2 GeV. The nonresonant parts are indeed very small. We evaluate the continuum contribution using Walker's multipole parameters. The upper limit N of integration corresponds to $k_L = 1.2$ GeV.

In Fig. 1 we have plotted S_0 and S_2 as a function of t, which includes both resonant and nonresonant contributions. Now, if A_2 chooses Gell-Mann's[8] ghost-eliminating mechanism, then S_0 should remain finite, and S_2 should change sign, where $\alpha(t)$ passes through zero. On the other hand, if the ghost-eliminating mechanism is of Chew[9] or no-compensation[10] type (which are indistinguishable for the sense-nonsense amplitude we are considering), then $\gamma(t)$ should have an additional factor $\alpha(t)$. This would imply that S_0 changes sign and S_2 has a vanishing magnitude and slope where $\alpha(t)$ passes through zero. We see from Fig. 1 that our S_0 does change sign and our S_2 strongly suggests a double-zero behavior (even though the overall magnitude is slightly displaced downwards) around $t = -0.5$ GeV^2, where $\alpha(t)$ is usually expected to pass through zero. Our result, therefore, strongly favors the Chew or the no-compensation mechanism over that of Gell-Mann.

We have also plotted, in Fig. 1, $\alpha(t)$ as obtained from Eq. (7). Both the intercept and the slope are in good agreement with existing estimates. Of course, Eq. (7) becomes unstable around $\alpha = 0$, since both the numerator and the denominator vanish. The point $\alpha = 0$ can, however, be estimated by looking at S_0 or S_2 individually.

In order to estimate the sensitivity of our results to the higher resonance parameters and the cutoff, we have shown in Table I individual resonance contributions to S_0 and S_2 at $t = 0$. S_0 is seen to be dominated by the nucleon and $N^*(1236)$ contributions. Even though these two have opposite signs, the remainder is of the same sign as the higher resonance contributions. Thus S_0 should be insensitive to a possible variation in the higher resonance parameter or the cutoff. On the other hand, S_2 is dominated, as expected, by the higher resonance contributions. This, to-

Table I. Resonance contributions to S_0 and S_2 at $t=0$.

Resonances	Multipoles	Contributions (GeV^{-1}) S_0	S_0
$N(940)$	Born term	4.44	0.00
$N^*(1236)$	E_{1+}, M_{1+}	−3.86	−0.37
$N^*(1471)$	E_{1-}, M_{1-}	0.07	0.02
$N^*(1519)$	E_{2-}, M_{2-}	0.55	0.25
$N^*(1561)$	E_{0+}	−0.02	−0.01
$N^*(1652)$	E_{2+}, M_{2+}	0.21	0.13
$N^*(1672)$	E_{3-}, M_{3-}	0.63	0.44

gether with the fact that there is an appreciable cancellation with the $N^*(1236)$ contribution suggests a strong cutoff dependence for S_2 and α. In fact, changing the cutoff to $k_L = 1$ GeV reduces $S_2(0)$ and $\alpha(0)$ by a factor of 2 (which shows that we have cut the integral short of the Regge asymptotic region), but leaves $S_0(t)$ essentially unaltered. Moreover, S_2 retains the general feature of Fig. 1 and is less sensitive to cutoff for negative t.[11] We therefore believe that a more adequate treatment of higher resonance contributions may change our trajectory estimate quantitatively, but not our conclusion about the ghost-eliminating mechanism.

We want to remark, in passing, that our residue has no zero around $t = -0.1\ \mathrm{GeV}^2$. By factorization this should also be true for the sense-sense KN amplitude, which then contradicts the observation of Matsuda and Igi.[3] Putting aside the question of the ambiguity arising from their hyperon coupling parameters, one may interpret their result (see Fig. 1 of Ref. 3) as indicative of a double zero in the sense-sense amplitude, as suggested by the no-compensation mechanism.

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[1]K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967).

[2]R. Dolen, D. Horn, and C. Schmid, to be published.

[3]S. Matsuda and K. Igi, Phys. Rev. Letters 19, 928 (1967).

[4]The present approach follows closely that of our earlier work "Pion Conspiracy and Pion Residue from Photoproduction Sum Rule" (unpublished).

[5]G. Zweig, Nuovo Cimento 32, 689 (1964). The first and the third lines of Eq. (17) should be multiplied by −2 and −1, respectively, on the right-hand side.

[6]S. Frautschi and L. Jones, Phys. Rev. 163, 1820 (1967).

[7]R. L Walker, private communication.

[8]M. Gell-Mann, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 539.

[9]G. F. Chew, Phys. Rev. Letters 16, 60 (1966).

[10]C. B. Chiu, Shu-Yuan Chu, and L. L. Wang, Phys. Rev. 161, 1563 (1967).

[11]This is due to an enhanced contribution from the lower resonances, which go out of the physical region for nonzero t. Thus as suggested in Ref. 2, the higher-moment sum rules may be more reliable for negative t, provided, of course, that the double spectral function does not affect the partial wave expansion seriously.