MEASUREMENT OF THE TOTAL MUON POLARIZATION IN $K^+ \to \pi^0 + \mu^+ + \nu$ †

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From a measurement of the total polarization of the muon, we have determined the parameter $\xi(q^2)$ in the decay $K^+ \to \pi^0 + \mu^+ + \nu$.

The decay $K^+ \to \pi^0 + \mu^+ + \nu$ ($K_{\mu 3}$) is a particularly interesting subject for study because it is an example of a high-energy weak interaction that can be experimentally investigated in detail. Furthermore, the theoretical description of this process is relatively uncomplicated. By applying the usual ideas of the $V-A$ theory of weak interactions, one is led to the following matrix element for the decay:

$$\sum_{\lambda=1}^{4} f_+(q^2)[(p_k+p_\pi)_\lambda + \xi(q^2)(p_k-p_\pi)_\lambda] \times \bar{u}_\nu \gamma_\lambda (1+\gamma_5) u_\mu.$$

In this expresstion $f_+(q^2)$ and $\xi(q^2)$ are form factors that describe the effects of the strong interactions of the K^+ and the π^0 mesons on the weak-interaction process. These form factors may depend on q^2, the square of the invariant mass of the lepton pair. p_k and p_π are four-momenta.

In this Letter we shall describe an experiment which we have made to determine $\xi(q^2)$. This parameter is important for many reasons. The principle of time-reversal invariance can be checked by determining the phase of ξ. This principle requires that the phase of ξ be 0° or 180° for all values of q^2. It is possible to test the prediction of the $|\Delta T| = \frac{1}{2}$ selection rule which requires that the value of ξ in $K_{\mu 3}^+$ decay be equal to the value of ξ in $K_{\mu 3}^0$ decay. A comparison of the $K_{\mu 3}^+$ and the K_{e3}^+ decays can be used to test the principle of muon-electron universality of weak coupling strength if the value of ξ is known. There are also a variety of predictions made with the use of dispersion theory that can be checked by measuring ξ and its q^2 dependence. Recently there have been predictions made through the use of current-algebra–soft-pion theoretical techniques which relate the $K_{\mu 3}^+$ form factors to other K^+ decay form factors.

Many of the determinations of ξ that have been reported are based on measurements that can be interpreted only if the answers to many of the questions discussed above are known. For example, one frequently employed method of determining ξ has been the measurement of the ratio of the K^+ decay rates into the $K_{\mu 3}^+$ and the K_{e3}^+ channels,

$$R = \frac{K^+ \to \pi^0 + \mu^+ + \nu}{K^+ \to \pi^0 + e^+ + \nu}.$$

This ratio depends on ξ because the term in the matrix element which multiplies ξ is proportional to the lepton mass. In this method of determining ξ from a measurement of R it is necessary to assume that the form factors f_+ and ξ are constant, that ξ is real, and that the principle of muon-electron universality is applicable. There have also been many experiments reported in which ξ is determined from measurements of the π^0 or μ^+ energy spectra, or from the muon longitudinal polarization. The interpretation of these experiments is independent of the assumption of muon-electron universality, but the other assumptions mentioned are necessary in the analysis of practical experiments. As it is of considerable interest to determine ξ in a manner that is independent of all of the assumptions —universality, time-reversal invariance, and independence of ξ on q^2—we have undertaken an experiment suggested in a theoretical paper by Cabibbo and Maksymowicz.[1]

These authors have shown that the muon in $K_{\mu 3}$ decay is 100% polarized at each point in the Dalitz plot. The direction in which the muon is polarized depends only on the value of ξ at the point in the Dalitz plot which characterizes the event in question. To illustrate this effect we shall consider the components of the muon polar-

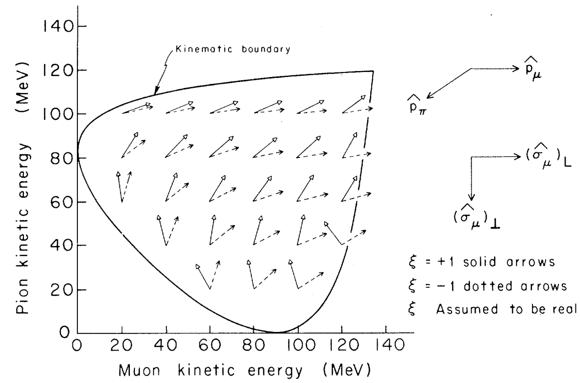

FIG. 1. Total muon polarization in $K^+ \rightarrow \pi^0 + \mu^+ + \nu$ for the cases $\mathrm{Im}\xi = 0$, $\mathrm{Re}\xi = \pm 1$. In these cases there is no component of the polarization out of the plane containing $\vec{p}_\pi$ and $\vec{p}_\mu$. The coordinate system is shown on the right-hand side of the figure.

ization along three axes:

$$\epsilon_L = \vec{p}_\mu / |\vec{p}_\mu|, \quad \epsilon_T = \vec{p}_\pi \times \vec{p}_\mu / |\vec{p}_\pi \times \vec{p}_\mu|,$$

$$\epsilon_\perp = \vec{p}_\mu \times \vec{\epsilon}_T / |\vec{p}_\mu \times \vec{\epsilon}_T|.$$

In Fig. 1 we have shown the muon polarization for the particular cases $\mathrm{Re}\xi = \pm 1, \mathrm{Im}\xi = 0$. In these cases there is no component of σ_μ along ϵ_T as ξ is real. It is apparent from the figure that the direction of the polarization vector is most affected by changes in ξ for the events in which the π^0 energy is small.

In order to determine ξ by observing the muon polarization, we have made use of the apparatus shown in Fig. 2. K^+ mesons from a 500-MeV/c Bevatron secondary beam were brought to rest in a carbon-dust stopper. A counter telescope and several spark chambers, not shown in the figure, were placed in the stopping beam. To determine the point in the Dalitz plot characterizing a given $K_{\mu 3}^+$ decay, we measured the direction and range of the muon in spark chambers $SC1$, $SC2$, and $SC3$, and the aluminum-plate range chamber. We detected muons having a kinetic energy between 55 and 90 MeV. We determined the location of the K^+ decay by projecting the muon and kaon tracks into the carbon stopper. We measured the conversion points of both gamma rays from the π^0 decay in lead-plate spark chambers surrounding the stopper.

The measurements described thus far are sufficient to limit the location of a given event to two positions in the Dalitz plot corresponding to two different π^0 directions. By counting sparks in the lead-plate spark chambers, we were able

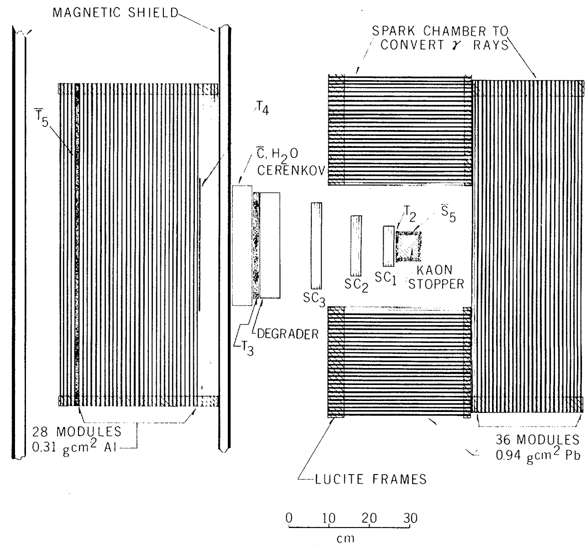

FIG. 2. Apparatus used in this experiment. K^+ mesons entered the apparatus in a direction perpendicular to the plane of the drawing. The muon polarization was determined from the angular distribution of the $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ positrons which we observed in the magnetically shielded aluminum-plate spark chamber on the left-hand side of the apparatus.

to determine the energy of the converted gamma rays with sufficient precision to distinguish between the two positions in those cases where they were not near to each other. When the two positions are near each other, an error in assignment is unimportant. To substantiate this statement we made a study of our gamma-ray energy resolution by detecting gamma rays of known energy from the $K^+ \rightarrow \pi^+ + \pi^0$ decay. A Monte Carlo calculation taking into account this experimentally determined energy resolution has shown that the error introduced into the measurement of ξ by our uncertainties in the location of events in the Dalitz plot was not significant.

We determined the direction in which the muons were polarized by observing the angular distribution of the positrons in the $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ decays in a magnetically shielded aluminum-plate spark chamber. The magnetic field inside this chamber was less than 0.2 G. In order to determine if there was any muon depolarization, we stopped polarized muons from the $K^+ \rightarrow \mu^+ + \nu$ decay. An analysis of 3000 events gave no indication of depolarization or systematic errors in the measurement of the positron angular distribution. By assuming that there was no depolarization, we found that the longitudinal polarization of these $K_{\mu 2}$ muons was -1.0 ± 0.1 in agreement with the expected value of −1.0.

In the course of our experiment we were able to accumulate 3133 events which we established to be $K_{\mu 3}^+$ decays. We have separated these events into four categories according to the value of q^2. To determine $\xi(q^2)$ we have then taken the events in each category and have made a maximum-likelihood analysis, fitting the events to the distributions theoretically predicted for various trial values of complex ξ. The four sets of likelihood contours we have obtained do not show any significant dependence of ξ on q^2. In a later paper we shall give a complete description of these results. When we fit all of our events, regardless of q^2, to the predicted distributions, we find that the most likely value of complex ξ is $\mathrm{Re}\xi = -0.9$, $\mathrm{Im}\xi = -0.3$. The locus of points in the complex ξ plane where the likelihood has dropped to $e^{-1.1}$ of its maximum value is roughly circular with a radius of 0.5 around the most likely value of complex ξ. There is a 67% probability that the true value of ξ lies within this circle. This result is therefore consistent with the principle of time-reversal invariance as 180° is a reasonably probable value for the phase of ξ. In order to verify the prediction that the muon is 100% polarized, we determined the component of the polarization of the muons along the axis theoretically predicted for the most likely value of complex ξ. The component of polarization along this axis was $+0.9 \pm 0.1$.

The results of an analysis we have made by fitting our data to the distributions of Cabibbo and Maksymowicz for real values of ξ are shown in Table I. The polarization of the muon along the predicted direction for the most likely value of ξ in each q^2 category is shown in the column labeled P_{total}. Our data are consistent with the assumption that ξ is constant. If we do not require that ξ be real, the values of $\mathrm{Re}\xi$ in Table I are essentially unchanged. The errors, however, are approximately 1.5 times as large as those shown in the table.

In order to check the principle of μ-e universality it is necessary to combine our result for ξ with measurements of the ratio R described previously. The measurements of R that have been reported[2] are mutually inconsistent and, therefore, conclusions about the question of universality must await the resolution of this discrepancy. The muon polarization measurements[2] of ξ in K^0 decay are consistent with the assumption that ξ is the same in $K_{\mu 3}^0$ and $K_{\mu 3}^+$ decay. This is in agreement with the prediction of the $|\Delta T| = \frac{1}{2}$ rule. Our value of ξ is consistent with the qualitative features of the dispersion-theory calculation.[3] Our value of ξ does not agree with the zero-pion-mass current-algebra prediction made by Callan and Treiman,[4] if we assume that this result can be extrapolated to the physical region. Our results are consistent with the theoretical work of Berman and Roy[5] who suggest that ξ is constant throughout the physical region and equal to −1.

Table I. The results of an analysis made with the assumption that ξ is real. P_{total} is the component of the muon polarization along the theoretically predicted axis for the value of $\mathrm{Re}\xi$ shown in the table. The expected value of this component is +1.0.

q^2 (MeV^2)	T_π (MeV)	Events	$\mathrm{Re}\xi$	P_{total}
$(359)^2$–$(317)^2$	0.0–28.5	122	-0.2 ± 0.7	$+1.2 \pm 0.4$
$(317)^2$–$(288)^2$	28.5–46.5	265	-0.7 ± 0.7	$+1.1 \pm 0.3$
$(288)^2$–$(249)^2$	46.5–67.5	488	-1.9 ± 0.5	$+0.8 \pm 0.2$
$<(249)^2$	>67.5	2258	-0.9 ± 0.6	$+0.9 \pm 0.1$
All events	···	3133	-0.95 ± 0.3	$+0.9 \pm 0.1$

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Field Theory 1 (W. A. Benjamin, Inc., New York, 1963), who concluded that ξ should not depend strongly on q^2 and that $-2<\xi<+2$.

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GHOST-ELIMINATING MECHANISM AND TRAJECTORY PARAMETER OF A_2 BY PHOTOPRODUCTION SUM RULE*

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The A_2 trajectory and residue functions are obtained and are found to change sign around $t=-0.5$ GeV^2. This strongly suggests the Chew or the no-compensation mechanism of ghost elimination. The trajectory parameter $\alpha(t)$ also agrees well with existing estimates.

Finite-energy sum rules have been used by Igi and Matsuda[1] and by Dolen, Horn, and Schmid[2] to obtain some useful information about the ρ Regge parameter from πp scattering data in the intermediate energy range. More recently, Matsuda and Igi[3] have tried to clarify the ghost-eliminating mechanism for A_2 by using the finite-energy sum rule for a combination of KN and $\bar{K}N$ scattering amplitudes. Unfortunately, this program has not been successful because of the large ambiguities in the various hyperon coupling parameters. In this note we obtain the ghost-eliminating mechanism for A_2 by studying the sum rule S_0 for a suitably chosen photoproduction amplitude[4] that couples only to A_2. Thus we are able to dispense with the saturation hypothesis and work directly in terms of the multipoles, which are obtained from photoproduction data. We also estimate the A_2 trajectory parameter by using the second-moment sum rule S_2, in addition to S_0.

Using the notation of Zweig,[5] we pick up the combination of invariant amplitudes $A_1^{(-)}-2MA_4^{(-)}$, where the superscript $(-)$ refers to the t-channel isospin 1 and G-parity negative (isovector photon). This is related to the definite spin-parity helicity amplitude $\tilde{f}^t$ by[6]

$$A_1^{(-)}-2MA_4^{(-)}=-2\sqrt{2}M\tilde{f}^t, \tag{1}$$

where M is the nucleon mass. $\tilde{f}^t$ in this case couples to the nucleon-antinucleon triplet state $|\frac{1}{2}\frac{1}{2}\rangle+|-\frac{1}{2}-\frac{1}{2}\rangle$, which has $C=P=(-1)^J$. This, together with the isospin and G-parity requirement, allows only A_2 exchange. Then, on absorbing certain innocent factors into the reduced residue function $\gamma(t)$, we get the usual Regge contribution

$$A_1^{(-)}-2MA_4^{(-)}=\alpha\left(\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\gamma(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha-1}, \tag{2}$$

where

$$\nu=\frac{s-u}{4M}=k_L+\frac{t-\mu^2}{4M}, \tag{3}$$

μ is the pion mass, k_L is the lab photon momentum, and ν_0 is a scale factor which we choose for convenience to be 1 GeV. Now, using the odd crossing property of our amplitude[5] under $\nu\to-\nu$, we get the following sum rule:

$$S_0=(g/4M)[e+2M(\mu_p-\mu_n)]-\pi^{-1}\int_{\nu_{\rm th}}^{N}\mathrm{Im}(A_1^{(-)}-2MA_4^{(-)})d\nu=\pi^{-1}\gamma(t)N^{\alpha(t)}. \tag{4}$$

The second-moment sum rule gives

$$S_2=\frac{1}{N^2}\left\{\frac{g\nu_{\rm B}^2}{4M}[e+2M(\mu_p-\mu_n)]-\frac{1}{\pi}\int_{\nu_{\rm th}}^{N}\nu^2\,\mathrm{Im}(A_2^{(-)}-2MA_4^{(-)})d\nu\right\}=\frac{1}{\pi}\gamma(t)\frac{\alpha(t)}{\alpha(t)+2}N^{\alpha(t)}. \tag{5}$$

The scale factor ν_0 has been dropped since we work in GeV units. The terms in front of the integrals in Eqs. (4) and (5) are the nucleon Born terms; μ_p and μ_n denote the anomalous magnetic moments of