[2]D. Frohlich and H. Mahr, Phys. Rev. Letters 16, 895 (1966).
[3]P. J. Regensburger and E. Panizza, Phys. Rev. Letters 18, 113 (1967).
[4]M. W. Dowley, K. B. Eisenthal, and W. L. Peticolas, Phys. Rev. Letters 18, 531 (1967).
[5]To be published elsewhere.
[6]D. S. McClure, J. Chem. Phys. 19, 670 (1951).
[7]D. P. Craig and G. Fischer, Trans. Faraday Soc. 63, 530 (1967).
[8]M. A. El Sayed and T. Pavlopoulos, J. Chem. Phys. 39, 834 (1963).
[9]W. L. Peticolas, R. Norris, and K. E. Rieckhoff, J. Chem. Phys. 42, 4164 (1965).
[10]M. W. Dowley, K. B. Eisenthal, and W. L. Peticolas, J. Chem. Phys. 47, 1609 (1967).
[11]M. W. Windsor and J. R. Novack, private communication of unpublished results.
[12]J. R. Platt, J. Chem. Phys. 17, 484 (1949).

PLASMA WAVE ECHO EXPERIMENT*

J. H. Malmberg, C. B. Wharton, R. W. Gould,† and T. M. O'Neil
General Dynamics, General atomic Division, John Jay Hopkins Laboratory for Pure and Applied Science, San Diego, California
(Received 22 September 1967)

Experimental observation of a new nonlinear plasma phenomenon, the plasma wave echo, is reported.

A recent theory[1-3] predicts that there are echo phenomena associated with electron plasma waves. We report here an experiment which demonstrates existence of second- and higher-order echoes. The echoes appear at the predicted position in the plasma. The frequency of the echo wave and the dependence of its amplitude on the amplitude of the initial waves are correctly predicted by theory. Observation of plasma echoes is of fundamental interest, since it experimentally verifies the reversible nature of Landau damping. In addition, it provides an experimental test of the perturbation method used to calculate the nonlinear behavior of plasmas in a wide variety of other situations.

If an electric field of frequency f_1 is continuously excited at one point in a plasma and an electric field of frequency f_2 is continuously excited at a distance l from this point, and each Landau damps in space as it propagates away from its point of origin, then the theory predicts that a spatial echo of frequency

$$f_3 = mf_2 - nf_1 \quad (1)$$

will appear at a distance

$$l^* = (nf_1/f_3)l \quad (2)$$

from the point where the second field is excited, provided that

$$mf_2 > nf_1. \quad (3)$$

In general echoes will appear on both sides of the pair of transmitting antennas, but the situation is not symmetric: The echoes in opposite directions will differ in frequency, position, and amplitude unless $f_1 = f_2$. The integers m and n are associated with the order of the perturbation theory predicting the particular echo, and for small-amplitude initial waves

$$A_3 \sim A_1^{\,n} A_2^{\,m}, \quad (4)$$

where A_1, A_2, and A_3 are the amplitude of the wave of frequency f_1, the wave of frequency f_2, and of the echo, respectively.

These experiments were performed with an apparatus[4] previously used to test experimentally[5-7] the Landau damping and dispersion theory.[8] These prior experiments guarantee that the plasma meets the assumptions of the echo theory, i.e., that the plasma is "collisionless" in the sense of the theory, that the electron velocity distribution is Maxwellian, that the waves are properly identified, and so on. The experimental geometry is a long column of plasma bounded in the radial direction by a good conductor. For the data reported here, the cylindrical plasma column has a length of 180 cm and a central density of 1.5×10^8 electrons/cm^3, and a temperature of 9.4 eV. It is immersed in a magnetic field of 305 G, which is large enough to be considered "infinite" in the sense of the theory to be tested. The background pressure is 1.5×10^{-5} Torr (mostly H_2).

Three radial probes are used as antennas. Two of the probes are connected by coaxial cable to chopped transmitters which provide an rf signal of variable power and frequency. The other probe is connected to a tuned receiver whose output drives a coherent detector operated at the transmitter chopping frequency. Provision is made to add a reference signal from the transmitter to the receiver rf signal, i.e., we may use the system as an interferometer. Routine Landau-damping measurements on the particular plasma used for the present experiment produced the theoretically predicted dependence on wave phase velocity and rough agreement in absolute magnitude. At the frequencies used for the present experiment the dispersion is dominated by finite temperature effects and we obtain an almost "infinite-plasma" result in agreement with prior experiments and theory.

A plasma wave echo is exhibited in Fig. 1. One transmitter probe is positioned at 0 cm and a 120-MHz signal applied. The second is positioned at 40 cm and a 130-MHz signal applied. The echo frequency, 140 MHz, corresponds to $m=2, n=1$ [Eq. (1)], and is thus a third-order echo. The receiver is tuned successively to 120, 130, and 140 MHz and an appropriate reference signal provided for each case. At 140 MHz the receiver sensitivity is also increased by 20 dB. The three curves are tracings of the interferometer output obtained at each frequency as the receiver probe is moved the length of the plasma. The two primary, Landau-damped waves are launched in both directions. Figure 1 demonstrates our most important result: that plasma wave echoes exist. We have frequency analyzed the echo signal and obtain a full width at half maximum of about 200 kHz. Some frequency spreading is always introduced by fluctuations in the plasma and this result is typical for a primary wave at the same frequency. By scanning the probes radially we measure the radial eigenfunction of the echo. It has a full width at half maximum of 0.9 cm, identical to a primary wave at the same frequency. The wave number of the echo wave is 2.35 cm^{-1}, identical to that of a primary wave of the same frequency. By analogy to the second-order spatial echo theory we expect the wave number to be $(f_3/f_1)k_1$ on the rising part of the echo and k_3 on the decaying side. These numbers are identical within the experimental accuracy.

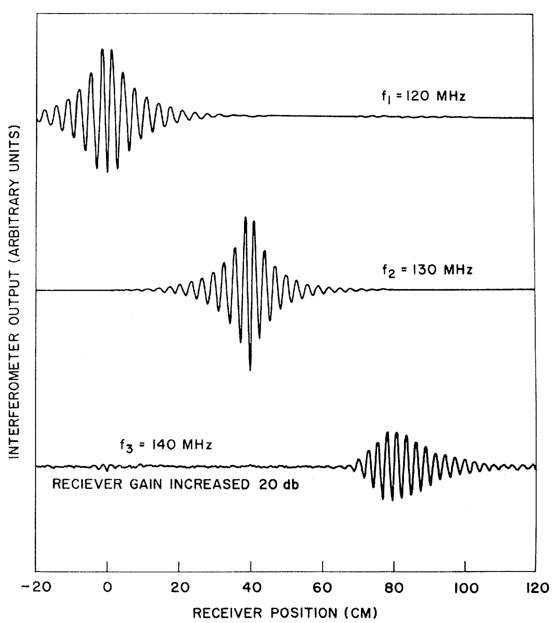

FIG. 1. Third-order echo. The transmitter probes are at 0 and 40 cm. Upper curve, receiver tuned to f_1; second curve, receiver tuned to f_2; third curve, receiver tuned to f_3 and gain increased 20 dB.

We have further tested Eqs. (1)-(3) by searching for echoes arising from various combinations of m and n and using a wide assortment of frequencies. Echoes are routinely found for any set of frequencies in the range of the measured dispersion curve which satisfy Eqs. (1) and (2), typically with signal-to-noise ratios of 20 to 100. We have clearly identified echoes corresponding to second, third, fourth, and fifth order in the perturbation theory (i.e., $m+n=2, 3, 4, 5$). Higher order echoes are not lost in receiver noise. They are confused by other nonlinear effects in the plasma. We have carefully searched for echoes on the "wrong side" of the transmitter pair with negative results: Equation (3) must be satisfied or no echo is observed.

The most distinctive characteristic of plasma wave echoes compared with other nonlinear plasma phenomena is the change in their spatial position as the transmitter separation is varied. A more precise check of Eq. (2) is obtained by shutting off the reference path and measuring the peak of the envelope of the echo power. Using such data, we plot the position of the peak of a third-order echo against the separation of the two transmitters for various values of f_1/f_3. The result is given in Fig. 2. The numbers on the curves are frequencies in MHz given in the order f_1, f_2, f_3. The slopes of the straight lines are computed from Eq. (2). The intercepts of the theoretical curves

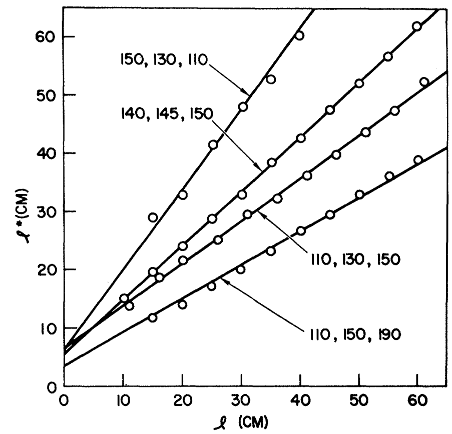

FIG. 2. Echo position versus transmitter separation. The slope of the curves is theory [Eq. (2)]. The numbers on the curves are f_1, f_2, and f_3 in MHz.

have been chosen to fit the data best. Since the theoretical "position" of the antennas is localized no better than a damping length, the nonzero intercepts do not indicate a discrepancy with theory.

The amplitude of each primary wave is proportional to the rf voltage applied to its transmitter probe. This voltage is measured by a rf power meter connected to a detector built into the base of the probes. Using a calibrated receiver we measured the dependence of echo power at the peak on the power of the primary wave for a third-order echo. For small signals $A_3 \sim A_1 A_2^2$ as expected from Eq. (4), since $n = 1$ and $m = 2$ for this case. At large signal levels, the echo amplitude saturates. This is expected since the perturbation treatment of the problem must be carried to higher order when the signals are large.

A third-order "sheath echo" may be obtained with only one transmitter probe if it is near the sheath at the end of the machine. Electrons which pass the probe and are reflected by the sheath have a perturbed velocity distribution identical to that which would be produced by a "virtual transmitter" behind the sheath. The reflected electrons pass the transmitter again and an echo is produced. This kind of echo is also easily observed. It is three times as far from the sheath as the transmitter antenna. Its amplitude is found to be cubic in the primary wave amplitude (below the saturation level), as expected for a third-order echo.

In summary, the existence of various echoes, associated with second-, third-, fourth-, and fifth-order perturbation theory, has been demonstrated experimentally. The echoes appear at the predicted position in the plasma. The frequency and wavelength of the echo wave and the dependence of its amplitude on the amplitude of the initial waves are correctly predicted by theory. The existence of the "sheath echo" is also experimentally established. Further refinement of these experiments could lead to a tool for studying collisional processes in plasmas.

*This research was sponsored by the Defense Atomic Support Agency under Contract No. DA-49-146-XZ-486.

†Consultant. Permanent address: California Institute of Technology, Pasadena, Calif.

[1]R. W. Gould, T. M. O'Neil, and J. H. Malmberg, Phys. Rev. Letters 19, 219 (1967).

[2]R. W. Gould, Phys. Letters 25A, 559 (1967).

[3]T. M. O'Neil and R. W. Gould, to be published.

[4]J. H. Malmberg, N. W. Carlson, C. B. Wharton, and W. E. Drummond, in Proceedings of the Sixth International Conference on Ionization Phenomena in Gases, Paris, 1963, edited by P. Hubert (S.E.R.M.A., Paris, France, 1964), Vol. 4, p. 229.

[5]J. H. Malmberg and C. B. Wharton, Phys. Rev. Letters 6, 184 (1964).

[6]J. H. Malmberg, C. B. Wharton, W. E. Drummond, in Proceedings of a Conference on Plasma Physics and Controlled Nuclear Research, Culham, England, 1965 (International Atomic Energy Agency, Vienna, Austria, 1966), Vol. I, p. 485.

[7]J. H. Malmberg and C. B. Wharton, Phys. Rev. Letters 17, 175 (1966).

[8]L. Landau, J. Phys. (USSR) 10, 45 (1946).