ly of this form, where, however, L_{46} is parity conserving. In (14), $|\bar{n},p\rangle$ are the so-called "tilted" states also occurring in H-atom and hadron calculations (for covariance, the tilting operation is always done before boosting):

$$|\bar{n},p=0\rangle = e^{\frac{1}{2}\theta\gamma_5\gamma_0}|n,p=0\rangle. \quad (15)$$

In such a theory the vertex function is given by

$$\langle\bar{n}'|j_\mu|\bar{n}p\rangle = \langle n'|e^{-\frac{1}{2}\theta\gamma_5\gamma_0}(\alpha_1\gamma_\mu-\alpha_3P_\mu\gamma_5) \times e^{i\vec{\xi}\cdot\vec{M}}e^{\frac{1}{2}\theta\gamma_5\gamma_0}|n\rangle.$$

The mass spectrum can be obtained either from Eq. (14), or from the current conservation requirement (9), first by operating with $e^{-i\vec{\xi}\cdot\vec{M}}$ and then with the inverse of (15), $e^{-\frac{1}{2}\theta\gamma_5\gamma_0}$. It is interesting that one gets now two mass values; for example, for $\beta=0$ in (14) one finds

$$m^2=(2\alpha_3^2)^{-1}[\alpha_1^2\pm(\alpha_1^4-4\gamma^2\alpha_3^2)^{1/2}]. \quad (16)$$

I have profited from discussions with D. Corrigan, C. Iddings, S. Malin, K. Mahanthappa, and J. R. Taylor.

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[8]Actually only the diagonal elements of $\mathcal{V}_{nn'}$ are physical because of the conservation of total charge in the S matrix. This can be taken into account by requiring the invariance of the S matrix under the compact subgroup O(2), rotation in the 56 plane, analog to the compact subgroup O(3) for angular momentum.

[9]Note that Eq. (5) is not the Hilbert-space norm of the states, but the scalar $\bar{u}(0)u(0)$. Because the representation in spin space is not unitary, $D^{-1}\neq D^\dagger$; hence these two things are different.

[10]See Ref. 8 for charge conservation.

[11]If we write an equivalent Dirac equation, it will have the form $[\eta\gamma_\mu p^\mu - M_0]\psi=0$, i.e., the charged quantum number occurring in the free-particle equation. This is appropriate because it is an internal quantum number of the free system in the present interpretation.

AN EVALUATION OF SEARCHES FOR *C* NONCONSERVATION IN ETA DECAY

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Since the suggestion was made[1] that C invariance might not hold for the electromagnetic interaction, a number of attempts[2-13] have been made to find evidence for a C nonconservation in the electromagnetic decay of the eta meson. To date, there is no experimental evidence for the existence of the C-nonconserving decay $\eta\rightarrow\pi^0e^+e^-$ (Table I). Furthermore, although measurements of the asymmetry in the Dalitz plot $\eta\rightarrow\pi^+\pi^-\pi^0$ show disagreement (Table II), the most precise of these[9] gives a null result. Similarly, the decay $\eta\rightarrow\pi^+\pi^-\gamma$ shows no charge asymmetry.[12] We would like to consider whether these experimental results are compatible with an electromagnetic C nonconservation of strength sufficient to account for the observed CP nonconservation in K_2^0 decay.[14] In doing so, we take special note of the following considerations:

(1) The width for the decay $\eta\rightarrow\gamma\gamma$ as measured experimentally[15] is an order of magnitude larger than earlier theoretical estimates.[16]

(2) The decay $\eta\rightarrow\pi^0e^+e^-$ and the asymmetry in

Table I. Results of experiments to detect $\eta \to e^+e^-\pi^0$.

Group	Reference	Method	Production	Normalization	Effective η sample	Quoted results[d]
Foster, Good, and Meer	2	H_2BC^a	$\pi^+p \to \pi^+p\eta$	$\eta \to \pi^+\pi^-\pi^0$	611	$\frac{e^+e^-\pi^0}{\pi^+\pi^-\pi^0} < 0.5\times10^{-2}$
Price and Crawford	3	H_2BC^a	$\pi^\pm p \to \pi^\pm p\eta$	$\eta \to \pi^+\pi^-\pi^0$	406	$\frac{e^+e^-\pi^0}{\pi^+\pi^-\pi^0} = (0.7\pm0.7)\times10^{-2}$
Rittenberg and Kalbfleisch	4	H_2BC^a	$K^-p \to \Lambda X^0$, $X^0 \to \eta\pi^+\pi^-$	$\eta \to \pi^+\pi^-\pi^0$	330	$\frac{e^+e^-\pi^0}{\text{all }\eta} < 0.7\times10^{-2}$
Berley et al.	5	H_2BC^a	$K^-p \to \Lambda\eta$	$\eta \to \pi^+\pi^-\pi^0$	226	$\frac{e^+e^-\pi^0}{\gamma\gamma} < 4.7\times10^{-2}$
Baglin et al.	6	$HLBC^b$	$\pi^-p \to n\eta$	$\eta \to \pi^+\pi^-\pi^0$	2460	$\frac{e^+e^-\pi^0}{\text{all }\eta} < 0.09\times10^{-2}$
Billing et al.	7	$HLBC^b$	$\pi^+n \to p\eta$	$\eta \to \pi^+\pi^-\pi^0$	6300	$\frac{e^+e^-\pi^0}{\text{all }\eta} < 0.037\times10^{-2}$
Bazin et al.	13	D_2BC^c	$\pi^+d \to pp\eta$	$\pi^0 \to e^+e^-\gamma$	3910	$\frac{e^+e^-\pi^0}{\text{all }\eta} < 0.084\times10^{-2}$ [e]

[a] H_2BC: hydrogen bubble chamber.
[b] HLBC: heavy-liquid bubble chamber.
[c] D_2BC: deuterium bubble chamber.
[d] All limits are for 90% confidence level.
[e] This is the final result of this experiment including three- and four-prong events from the production reaction $\pi^+ \to pp\eta$ ($\eta \to e^+e^-\gamma$ or $\gamma \to e^+e^-\pi^0$) in the bubble chamber.

Table II. Comparison of asymmetry measurements for two different eta decays.

Group	Reference	Method	Production	Measured asymmetry
		$\eta \to \pi^+\pi^-\pi^0$		
Baltay et al.	8	D_2BC	$\pi^+d \to ppn$	0.072 ± 0.028
Cnops et al.	9	Spark chamber	$\pi^-p \to n\eta$	0.003 ± 0.01
Larribe et al.	10	D_2BC	$\pi^+d \to pp\eta$	-0.061 ± 0.040
		$\eta \to \pi^+\pi^-\gamma$		
Crawford and Price	11	H_2BC	$\pi^+p \to \eta^+p^+\pi^+$	0.02 ± 0.17
Bowen et al.	12	Spark chamber	$\pi^-p \to n\eta$	0.015 ± 0.025

the $\eta \to \pi^+\pi^-\pi^0$ decay may be suppressed by requirements of SU(3) symmetry on the hypothesized C-even electromagnetic current.[17,18]

(3) The decay $\eta \to \pi^0e^+e^-$ may be forbidden (to lowest order in α) if the C-even electromagnetic current is an isoscalar.[19]

$\eta \to e^+e^-\pi^0$.—Consider first the experimental limits on the ratio $R \equiv \Gamma(\eta \to \pi^0e^+e^-)/\Gamma(\eta \to \text{all})$. Combining the experimental results of Refs. 2, 5-7, and 13[20] listed in Table I yields the upper limit $R < 2\times10^{-4}$ (90% confidence). Using the measured width[15] $\Gamma(\eta \to \gamma\gamma) = 1.21\pm0.26$ keV, we obtain the upper limit

$$\Gamma(\eta \to \pi^0e^+e^-) < 0.63 \text{ eV}. \quad (1)$$

To interpret this result, we first use a crude rate calculation[1,21] based on the matrix element

$$\langle\eta|K_\mu^v|\pi^0\rangle = f(q^2)\{(\eta_\mu+\pi_\mu)-[(m_\eta^2-m_\pi^2)/q^2](\eta_\mu-\pi_\mu)\} \quad (2)$$

where K_μ^v is an isovector C-nonconserving electromagnetic current, η_μ and π_μ are the four-momenta of the η and π^0, and f is a form factor depending on $q^2 \equiv (\eta_\mu-\pi_\mu)^2$. The diagram for this decay is given in Fig. 1. For the moment, no restriction due to symmetry is taken into account. As an approximation, one sets $f(q^2) = \frac{1}{6}eq^2\langle r^2\rangle$, where $\langle r^2\rangle$ is a mean-square charge radius characterizing the inter-

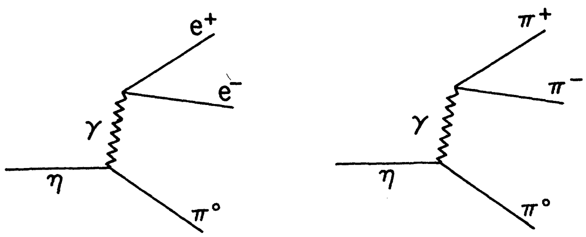

FIG. 1. Feynman diagrams for the C-nonconserving decays $\eta \to e^+e^-\pi^0$ and $\eta \to \pi^+\pi^-\pi^0$. The lower vertices are C nonconserving, while the upper ones are ordinary C-allowed electromagnetic conversions.

action. Defining $\lambda \equiv \langle r^2 \rangle / \langle r_p^2 \rangle$, where $\langle r_p^2 \rangle = 0.64$ F^2 is the mean-square charge radius of the proton, (2) leads to the rate

$$\Gamma(\eta \to \pi^0 e^+ e^-) \approx 80\lambda^2 \text{ eV}. \tag{3}$$

This together with the experimental limit (1) gives

$$\lambda < 0.1 \tag{4}$$

For a large C nonconservation, λ is expected to be of order 1; indeed, $\lambda\alpha$ can be considered to be a crude estimate of the ratio of the CP-nonconserving amplitude associated with $K_\mu J_\mu$ to the CP-conserving amplitude. If we recall that this ratio must be of order 1/500 for the C-nonconserving interaction to explain the observed $K_2^0 \to 2\pi$ amplitude, we see that the present limit on λ is at best a marginal test of the electromagnetic C-nonconservation hypothesis.

The above discussion ignores the possibility that SU(3) selection rules may act to suppress the decay. If the C-even part K_μ of the electromagnetic current transforms as a member of an SU(3) octet, then its matrix element between the π^0 and the isoscalar member η_8 of the octet vanishes in the limit of exact SU(3) symmetry. In this case the decay $\eta \to e^+e^-\pi^0$ can occur only through η-X^0 mixing.

The mixing is defined by an angle θ such that

$$X^0 = X_1 \cos\theta + \eta_8 \sin\theta,$$

$$\eta = -X_1 \sin\theta + \eta_8 \cos\theta,$$

where X_1 is the unitary singlet member of the nonet. One obtains therefore

$$\langle \eta | K_\mu^{\,v} | \pi^0 \rangle = -\sin\theta \langle X_1 | K_\mu^{\,v} | \pi^0 \rangle.$$

If θ is given by $\tan\theta = 0.19$, obtained from the mass formula,[16] then the rate (3) must be multiplied by $\sin^2(\theta) = 0.03$, giving

$$\Gamma(\eta \to \pi^0 e^+ e^-) = 2.4\lambda^2 \text{ eV}, \tag{5}$$

from which one obtains

$$\lambda < 0.5 \tag{6}$$

Thus, if the hypothesis of SU(3) suppression is essentially correct, the experimental results concerning $\eta \to \pi^0 e^+ e^-$ do not severely limit the strength of an isovector C-nonconserving electromagnetic interaction.

Recently Lee[19] has shown that the charge Q_K associated with the C-even current K_μ must be an isoscalar. This implies that the isovector part of K_μ, if it exists at all, must vanish at zero four-momentum transfer. The matrix element (2) is consistent with this requirement, so that $\eta \to \pi^0 e^+ e^-$ is not necessarily forbidden by the requirement that Q_K be an isoscalar. However, unless one is forced by direct experimental evidence into an unconventional relation between a current and its associated charge, it is natural to assume that K_μ has the same isospin properties as Q_K. Thus K_μ would be an isoscalar $K_\mu^{\,s}$, and the isovector part $K_\mu^{\,v}$ on which our previous limits of λ were based simply vanishes. It is then clear that the transition $\eta \to \pi^0 e^+ e^-$ is forbidden in first order in the C-even current since this is a $\Delta I = 1$ decay.

$\underline{\eta \to \pi^+\pi^-\pi^0}$.—Interpretation of the $\eta \to \pi^0\pi^+\pi^-$ asymmetry data is more difficult, both because of the disparity among the experimental results and because it is not clear what sort of model should be used in estimating the asymmetry. The isovector model[22] represented by the second diagram of Fig. 1 gives, rather conveniently,

$$\Delta^2 = \frac{\sin^2\varphi}{70} \frac{\Gamma(\eta \to \pi^0 e^+ e^-)}{\Gamma(\eta \to \pi^0\pi^+\pi^-)}, \tag{7}$$

where Δ is the right-left asymmetry in the Dalitz plot for $\eta \to \pi^0\pi^+\pi^-$ and φ is a phase angle determined by final-state pion interactions. Taking as an experimental limit[9] $|\Delta| < 0.013$ and assuming no SU(3) suppression of the η-π^0-γ vertex, we obtain

$$\lambda |\sin\varphi| < 0.3;$$

with SU(3) suppression taken into account one obtains the looser limit

$$\lambda |\sin\varphi| < 2.0.$$

Thus the limit set by the asymmetry measurements is even less stringent than that from the $\eta \to \pi^0 e^+ e^-$ branching ratio if we assume that C

nonconservation is caused by an isovector electromagnetic current.[23] If one however supposes that the C-nonconserving interaction proceeds via the isoscalar current $K_\mu{}^S$, as is most natural in view of Lee's results, one then has[19]

$$|\Delta| = |\sin\varphi| \times 10^{-3},$$

whereas the experimental limit above gives

$$|\Delta| < 13 \times 10^{-3}.$$

In the present case of an isoscalar interaction, the isospin of the final three-pion system must be $I=0$; therefore, the C-even amplitude will be changing sign across the boundary of each sextant of the Dalitz plot and the interference term in the decay rate will also change sign. Thus, numbering adjacent sextants (1, 2, 3, 4, 5, 6) one can form the quantity $\delta = [(N_1-N_6)+(N_5-N_2)+(N_3-N_4)]/[N_1+N_2+N_3+N_4+N_5+N_6]$. This asymmetry parameter is approximately three times as sensitive as Δ because it takes into account the particular symmetry of the $I=0$ final state. From the experimental numbers[9] we obtain $\delta = (0.28 \pm 1.0)\%$ which can still be compared with the approximate theoretical estimate $|\sin\varphi| \times 10^{-3}$.

$\eta \to \pi^+\pi^-\gamma$.—The only possible C-nonconserving transition for an asymmetry to be detected in this decay is through the isoscalar $K_\mu{}^S$ current. The theoretical prediction is[1,19]

$$|\Delta| = \sin|\delta_P - \delta_D| \times 3 \times 10^{-2},$$

where δ_P and δ_D are the P-wave and D-wave $\pi\pi$ phase shifts in the final state. The present experimental limit on this asymmetry is[12]

$$\Delta = (1.5 \pm 2.5) \times 10^{-2}.$$

Conclusion.—The experimental study of eta decays as a test of the existence of a C-even electromagnetic current does not presently set any stringent limit on the strength of the resulting interaction. This conclusion is valid for both the isovector current $K_\mu{}^V$ and the isoscalar current $K_\mu{}^S$. The isoscalar current is the most natural candidate from the theoretical point of view since the charge Q_K associated with K_μ is an isoscalar. Assuming favorable phase factors, one would still have to attain a precision of better than 0.1% in the experimental study of the asymmetry of $\eta \to \pi^+\pi^-\pi^0$ or better than 1% for $\eta \to \pi^+\pi^-\gamma$ to test with some degree of confidence the hypothesis of C-nonconservation in the electromagnetic decay of the eta meson.

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[23]Using Eq. (7), the asymmetry Δ is predicted to be less than 0.17% on the basis of the current upper limit for the rate $\eta \to e^+e^-\pi^0$.