The K^- yields are shown as a function of lab momentum in Fig. 1(b). The Tsai-Whitis[16] computer program was used to calculate the K^- flux coming from φ decay under the assumptions that

$$d\sigma(\gamma p\rightarrow\varphi p)/dt\propto e^{-At},$$

with $A=3$ or 7 GeV^{-2}, a total φ cross section of 1.0 μb, φ helicity $=\pm1$, and $(\varphi\rightarrow K^+K^-)/(\varphi\rightarrow\mathrm{all})=0.48$; the results of the program are also shown in Fig. 1(b).

The φ cannot contribute to the K yield at 10 BeV/c and the yield at this momentum is a measure of the background from other processes such as the Drell mechanism,[17] Y^* decay, etc. The yield from these background processes might be expected to increase as the momentum is lowered. Indeed, a straight line can be drawn through the points and the data do not show a need for a peak in the distribution corresponding to φ production. If, however, we assume that the K^- yield from other processes at 8 GeV/c is just that given by the point at 10 GeV/c, a total $\gamma p\rightarrow\varphi p$ cross section of 0.38 ± 0.18 μb is obtained if the φ's are produced with an e^{-7t} distribution; this becomes 0.52 ± 0.25 μb for e^{-3t}. If the K^- yield from the other processes falls off rapidly going from 10 to 8 GeV/c, then an upper limit for $\gamma p\rightarrow\varphi p$ can be obtained by assuming that all the K^- yield at 8 GeV/c comes from φ decay; this drastic assumption results in upper limits (95% confidence) of 1.0 and 1.4 μb for e^{-7t} and e^{-3t} distributions, respectively.

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REALITY OF THE SCHWARZSCHILD SINGULARITY*

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A spherically symmetric solution of the Einstein equations is presented that coincides with the exterior ($r>2m$) Schwarzschild solution, but where the Schwarzschild "sphere" becomes a point singularity. The possible relevance of this solution to the question of gravitational collapse is discussed.

Recently, Israel[1] presented a proof of an interesting theorem: Among all static, asymptotically flat, vacuum solutions of the Einstein equations with closed simply connected equipotential surfaces, $g_{00}=\mathrm{const}$, Schwarzschild's solution is the only one that has a nonsingular event horizon $g_{00}=0$. It has been hypothesized by both Israel[1] and Penrose[2] that this result has an important bearing on the question of asymmetric gravitational collapse—if a nonsingular event horizon is to develop during the collapse of a body with mass and some asymmetry, the body must radiate away all its higher multipole moments. Once this has happened, the Penrose theorem becomes operable and thus continued collapse is inevitable. It is necessary that the body lose its asymmetry for this to happen, as otherwise the Israel theorem prohibits the development of trapped surfaces due to the formation of singularities at the event horizon. These considerations, as well as our conclusions, depend, of course, on the assumption that time-dependent solutions do not introduce qualitatively different features.

We wish to present here an extension of Israel's idea of singular event horizons to include the Schwarzschild solution as well as the asymmetric metrics. If the Schwarzschild metric can be considered to have a singular event horizon, then collapse through it becomes impossible, thus obviating the Penrose theorem.

We shall present a metric from which, by a limiting procedure, the exterior Schwarzschild metric is obtained but where the event horizon (the usual $r=2m$) is not only singular but is also a point rather than a sphere.

The metric we first construct is the spherically symmetric solution of the coupled gravitational and zero-rest-mass scalar fields with field equations

$$G_{\mu\nu}=-\kappa T_{\mu\nu},\quad T_{\mu\nu}=\varphi_{,\mu}\varphi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\varphi_{,\alpha}\varphi_{,\beta}g^{\alpha\beta},\quad \Box\varphi=0.$$

The spherically symmetric solution can be explicitly written as a function of two parameters A and $r_0=2m$, with R as a radial coordinate, as

$$ds^2=\left[\frac{2R+r_0(\mu+1)}{2R-r_0(\mu-1)}\right]^{1/\mu}dR^2+r^2(d\theta^2+\sin^2\theta d\varphi^2)-\left[\frac{2R-r_0(\mu-1)}{2R+r_0(\mu+1)}\right]^{1/\mu}dt^2, \tag{1}$$

$$\varphi=\frac{A}{\mu}\ln\left|\frac{2R-r_0(\mu-1)}{2R+r_0(\mu+1)}\right|, \tag{2}$$

with

$$r^2=\tfrac{1}{4}[2R+r_0(\mu+1)]^{1+1/\mu}\times[2R-r_0(\mu-1)]^{1-1/\mu}, \tag{3}$$

and

$$\mu\equiv(1+4\kappa A^2/r_0^2)^{1/2}\geq 1.$$

The analysis rests on the meaning of r^2 (the surface area of a sphere of constant R is given by $4\pi r^2$ and hence $r^2=0$ is a point) and its dependence on R. Note that as R goes from its minimum value of $\frac{1}{2}r_0(\mu-1)$ to ∞, r^2 goes from 0 to ∞. At the minimum value of R, it is easily calculated that the scalar field φ [Eq. (2)], the Ricci tensor, and the curvature scalar all become singular. Thus $R=\frac{1}{2}r_0(\mu-1)$ (or alternatively $r=0$) is a singular point in the space no matter how small the coupling constant κ becomes. The problem now is to find the relation between r and R as $\kappa\to 0$, or as $\mu\to 1$. We shall see that an anomaly develops here and that the limit is not unique.

The difficulty lies in the second bracket in Eq. (3), namely $J\equiv[2R-r_0(\mu-1)]^{1-1/\mu}$. For $R>\frac{1}{2}r_0\times(\mu-1)$, the limit as $\mu\to 1$ of this term is clearly 1; however for $R=\frac{1}{2}r_0(\mu-1)$, the limit is now 0. The function $J(R,\mu)$ is indeterminate at $R=\mu-1=0$; its value depends on how the limit is approached. The function $J(R,1)$ is 1 for $R>0$ but undefined for $R=0$. If it is taken to be one for all values of R, the resulting metric is the Schwarzschild solution, with $R=0$ corresponding to $r=2m$. [This value arises from the first bracket in Eq. (3).] However, since $J(R,\mu)$ for all $\mu>1$ has a zero at $R=\frac{1}{2}r_0(\mu-1)$ (the zero getting closer to $R=0$ as $\mu\to 1$), it appears that a more reasonable limit (at least from the perturbation point of view) is to treat $J(R,1)$ as 1 for $R>0$, and as 0 for $R=0$. (See Fig. 1.) We see that from this latter viewpoint we have $r=0$, instead of $r=2m$, at $R=0$. In other words, the space suddenly collapses from a radius slightly greater than $r=2m$ to zero.

We thus have a space that coincides with the exterior Schwarzschild space but where the

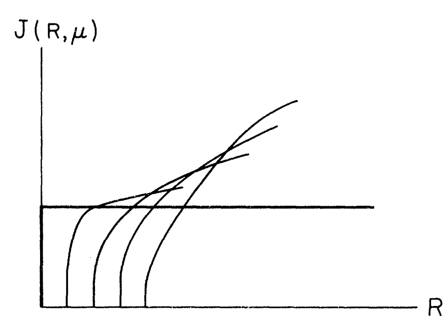

FIG. 1. Curves with smaller μ have their intercepts with the R axis closer to the J axis.

Schwarzschild "sphere" $r = 2m$ now becomes a singular point.

We had hoped to show that all the Weyl–Levi-Civita axially symmetric solutions corresponding to a point singularity with mass and higher multipole moments would have the same limit as above when the higher moments vanished. Unfortunately, to our knowledge, all the known axially symmetry solutions do not have point singularities. (They all appear to have unusual geometrical properties near their singular event horizons. The two-dimensional surface area surrounding the singularity appears to tend to infinity.) Hence the limiting case tends neither to our version of the Schwarzschild metric (called the truncated Schwarzschild metric) nor to the standard version. We nevertheless strongly feel that our conjecture is true, namely that all asymptotically flat, static solutions with a point singularity, and which possess mass and some asymmetry, do approach the truncated Schwarzschild solution as the asymmetry vanishes.

Once this solution is admitted as a spherically symmetric solution of the Einstein equations, then mathematically there is no a priori reason to choose between it and the standard Schwarzschild metric to represent physical models. However, there appears to be a good physical reason to choose the truncated solution; namely, it is apparently (if our conjecture is correct) much closer to the perturbed solutions than is the usual form. More specifically, static perturbation expansions of asymmetric solutions off the usual Schwarzschild metric do not approximate (close to the event horizon) the exact solutions they are meant to represent. However, it appears as if the perturbations off the truncated solution would lead to good approximations to exact point-singularity solutions if they could be found. (We would like to point out parenthetically that this is a good illustration of the dangers of indiscriminate use of perturbation calculations: Here the perturbations do not converge to any exact solution. It also emphasizes the importance of looking for solutions corresponding to point multipoles.)

It is clear that if our truncated Schwarzschild metric is to be considered as the physical solution corresponding to a spherically symmetric point mass, then the entire question of gravitational collapse beyond the Schwarzschild radius becomes meaningless. This point of view also obviates all discussion of the topological questions of the Schwarzschild interior, which for many people has always been disturbing.

We wish also to point out that Bel[3] has come to a similar conclusion concerning the Schwarzschild radius by totally different methods.

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FIELD THEORY, PARTIAL CONSERVATION OF AXIAL-VECTOR CURRENT, AND KAWARABAYASHI-SUZUKI-RIAZUDDIN-FAYYAZUDDIN SUM RULE*

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The identification of the divergence of the axial-vector field (or current) as the pion field has proven to be a useful and successful concept in particle physics. In this note, we formulate the notion of partially conserved axial-vector currents in a canonical Lagrangian formalism. It is shown that the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF)[1] sum rule is an immediate and simple consequence of Weinberg's[2] first sum rule and of the natural assumption that the Lagrangian is a functional of the axial-vector field.

The free fields (interaction picture fields).– We consider an axial-vector field φ^μ that describes both spin-1 particles of mass m and spin-0 particles of mass μ. In the absence of interactions we have explicitly

$$(\Box-\mu^2)\partial^\alpha\varphi_\alpha=0 \tag{1}$$

and

$$(\Box-m^2)(g^{\mu\nu}-\partial^\mu\partial^\nu/\mu^2)\varphi_\nu=0. \tag{2}$$

Thus the free axial-vector and pseudoscalar me-