A. Brandt and I. Kelson, "Single Particle Theory of Fission" (to be published).

[6]We must restrict our consideration to intrinsic variations, which are orthogonal to the total symmetries of the Hamiltonian—translations, rotations and reflections.

[7]D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953). Also Ref. 5.

NUCLEAR STRUCTURE OF Ca^{40} AND ELASTIC SCATTERING OF 750-MeV ELECTRONS

F. C. Khanna
Chalk River Nuclear Laboratories, Chalk River, Canada
(Received 12 March 1968)

It is shown that inclusion of the short-range nucleon-nucleon correlations in the usual shell-model description can provide adequate agreement with the known results of the electron elastic-scattering cross section and can make some interesting predictions.

Recently Bellicard et al.[1] published some excellent experimental data on the elastic scattering of 750-MeV electrons from calcium isotopes. It was found that a charge distribution $\rho_0(r)$ obtained by analyzing the scattering data at 250 MeV was quite inadequate at 750 MeV to explain the experimental results beyond scattering angle θ greater than 35°, i.e., in the region of large momentum transfer. An oscillating function, $\Delta\rho_0(r)$, had to be added to the charge distribution $\rho_0(r)$ to obtain a good fit at 750 MeV. We suggest in this note that such a modulating factor arises from the presence of short-range nucleon-nucleon correlations in the Ca^{40} ground-state wave function.

The usual shell-model wave function of a closed-shell nucleus is a Slater determinant of single-particle functions determined in a central potential well. In the case of Ca^{40}, the shell-model wave function is taken as the closed $1s$, $1p$, $2s$, and $1d$ shells. Such a wave function has few high-momentum components but enough low-momentum components to explain experimental results that involve momenta $p \lesssim p_F$, the Fermi momentum. For example, this wave function can provide an adequate explanation for the elastic electron scattering at 250 MeV since at this energy only low-momentum components are being studied. However, at 750-MeV electron energy and $\theta > 35°$, high-momentum components are important and the usual shell model is expected to break down. High-momentum components can arise from the strong short-range repulsion and the attractive part just outside the repulsive core in the nucleon-nucleon potential.[2] A radical approach to modify the wave function would be to do a Brueckner-type calculation for this finite nuclear system. Since such an approach is extremely difficult and has many uncertainties, we choose the phenomenological method suggested by Jastrow.[3]

The modified wave function, ψ, for the ground state of Ca^{40} is chosen to have the form

$$\psi(r_1 \cdots r_{40}) = N \prod_k \varphi_k(r_k) \prod_{i>j} f(r_{ij}), \tag{1}$$

where $\varphi_k(r_k)$ are the harmonic-oscillator-type single-particle wave functions and $f(r_{ij})$ is the Jastrow-type factor with the properties that

$$\lim_{r_{ij}\to\infty} f(r_{ij}) = 1 \text{ and } \lim_{r_{ij}\to 0} f(r_{ij}) = 0.$$

Explicitly $f(r_{ij})$ is chosen to be

$$f(r_{ij}) = 1 - \exp(-\beta^2 r_{ij}^2), \tag{2}$$

where β is a parameter to be determined. The elastic-scattering cross section for electrons of energy E on a nucleus with charge Z in the Born approximation is well known to have the form[4]

$$\frac{d\sigma}{d\Omega} = \frac{Ze^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{|F(q^2)|^2}{1 + (2E/M)\sin^2(\theta/2)}, \tag{3}$$

where θ is the scattering angle, M is the mass of the target nucleus, and $\hbar = c = 1$. $F(q^2)$ is the charge form factor of the nucleus and is given as

$$F(q^2) = \frac{1}{Z} \int \psi^* \sum_j^{\text{protons}} \exp(i\vec{q}\cdot\vec{r}_j)\psi d\tau. \tag{4}$$

Because of the complexity of ψ, this expression for $F(q^2)$ can be evaluated exactly only for simple systems like He^4. But for Ca^{40}, we use the cluster expansion as given by Iwamoto and Yamada.[5] Here only one- and two-particle cluster terms are retained and $F(q^2)$ reduces to

$$F(q^2)=\frac{1}{Z}\Big[\sum_i\langle i|e^{i\vec{q}\cdot\vec{r}_1}|i\rangle-\sum_{ij}\{\langle ij|e^{i\vec{q}\cdot\vec{r}_1}[1-f^2(r_{12})]|ij-ji\rangle-\langle i|e^{i\vec{q}\cdot\vec{r}_1}|i\rangle\langle ij|[1-f^2(r_{12})]|ij-ji\rangle\}+\cdots\Big], \quad (5)$$

where i and j represent the harmonic oscillator single-particle states in any of the shells.

The one-particle term is the usual shell-model expression for the form factor. The modification due to the correlation among nucleons is contained in the two- and more-particle cluster terms. It may be noted that the one- and two-particle terms add coherently in the expression for the form factor and they can yield interference effects in the elastic-scattering cross section.

The elastic-scattering cross section in the shell-model approximation and in the case when short-range correlations ($\beta=275$ MeV) are included is shown in Fig. 1. It may be pointed out that the ground-state function has correlations due both to the Pauli exclusion principle and to the short-range nucleon-nucleon correlations. These two types of correlations will yield distinct effects in the elastic-scattering cross section which should be distinguished. In the shell-model approximation, we get two diffraction minima at $\theta\sim17°$ and $\theta\sim30°$; these arise from the Pauli exclusion principle. Beyong $\theta=35°$ the cross section falls off exponentially. In the second case when the correlations are included, the diffraction minima at $\theta\sim17°$ and $\theta\sim30°$ are still reproduced but three additional diffraction minima due to the coherence of the one-particle and the two-particle cluster terms appear at $\theta\sim45°$, 55°, and 67.5°. The first of these minima at $\theta\sim45°$ is observed experimentally but the remaining two have yet to find an experimental verification.

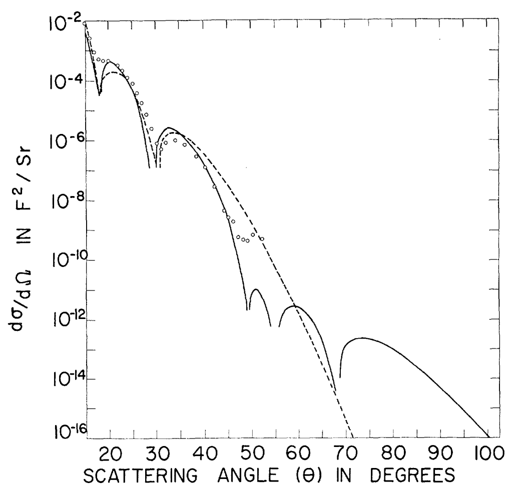

FIG. 1. Elastic-scattering cross section for 750-MeV electrons on Ca^{40} in Born approximation. The dashed lines are obtained in the usual shell model with oscillator parameter $b=1.968$ F and the full lines are obtained in the modified shell model that includes two-particle correlations with $\beta=275$ MeV.

The shell-model and the modified charge distributions are shown in Fig. 2. The effect of the short-range correlations is to decrease the charge density near zero and to smooth out the effects of the oscillations in the radial functions of the shell model. The tendency of the modified charge distribution is to appear constant in the center and to cut off more sharply than in the case of the shell model. This agrees quite well with the phenomenological adjustments made by Bellicard et al. to fit the data at 750 MeV.

We would like to make a number of comments with regard to the approximations made and some of the conclusions reached.

(i) The Born approximation has the shortcomings[6] that (a) there appear zeros instead of shallow diffraction minima; (b) the magnitude of the cross section near the diffraction minima is unreliable; and (c) the positions of the diffraction minima are shifted to slightly larger angles. But all the essential features in the structure of the cross section do appear in this approximation. Inclusion of higher Born terms only fills up the zeros of the first Born approximation so as to make the diffraction minima shallower.

(ii) The cluster expansion[5] is convergent so long as the range c of the two-particle correlation function $f(r_{ij})$ is much smalller than the interparticle separation r_0 in the nucleus, the expansion parameter being $(c/r_0)^3$. We have studied in detail the cluster expansion[7] for He^4 for the same value of the parameter β chosen for Ca^{40}. Here the form factor can be evaluated exactly and we find that the cross section, assuming only the two-particle cluster terms, differs from the exact calculation by 25 %. The position of the diffraction minimum in He^4 is unaltered by cutting off the cluster expansion after two-particle cluster terms.

(iii) The appearance of the diffraction minima as a result of the nucleon-nucleon short-range correlation function $f(r_{ij})$ appears to be fairly

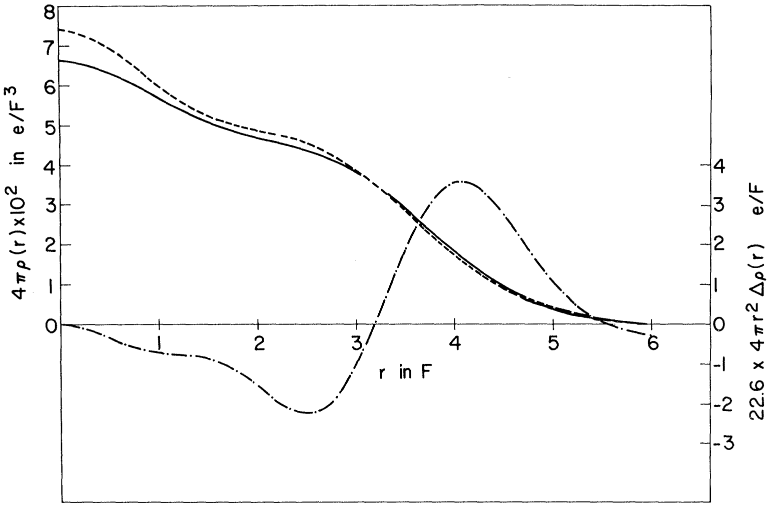

FIG. 2. Charge distribution arising from the shell model (dashed lines) and from the modified shell model with two-particle correlations (full lines). The dash-dotted curve is the modification introduced by the nucleon-nucleon correlation terms in the charge distribution of Ca^{40}. The scale for this curve is shown on the right-hand side.

general. We have calculated such minima for H^2, H^3, He^3, He^4, O^{16}, and Ca^{48}.[7] Minima have been found experimentally for the cases of H^2, He^4, and Ca^{48}.[8] The parameter β chosen for Ca^{40} is only slightly smaller than the one that gives the best fit to the diffraction minimum in He^4.

(iv) It is generally believed that inelastic electron scattering will provide information on the nucleon-nucleon correlation and this has been shown to be true in some cases.[9] But the present analysis seems to indicate that the elastic electron scattering also can be a useful tool for this study.

To summarize, the modification of the shell-model wave function by the inclusion of the nucleon-nucleon correlation function seems to provide an adequate description for the known data on the elastic scattering of electrons by Ca^{40} and makes predictions for the behavior of the cross section at larger momentum transfer. Further experimental results are needed to test the veracity of our conjecture.

We would like to express our thanks to M. Harvey and T. Sebe for discussions. We wish to thank Professor K. Gottfried for a very useful conversation.

[1]J. B. Bellicard, P. Bounin, R. F. Frosch, R. Hofstadter, J. S. McCarthy, F. J. Uhrhane, M. R. Yearian, B. S. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 19, 527 (1967).

[2]K. A. Brueckner, C. A. Levinson, and H. M. Mahmoud, Phys. Rev. 95, 217 (1954). The velocity-dependent potential can also introduce high-momentum components in the wave function; S. A. Moszkowski, Phys. Rev. 129, 1901 (1963).

[3]R. Jastrow, Phys. Rev. 98, 1479 (1955).

[4]R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957).

[5]F. Iwamoto and M. Yamada, Progr. Theoret. Phys. (Kyoto) 17, 543 (1957).

[6]D. R. Yennie, D. G. Ravenhall, and R. N. Wilson, Phys. Rev. 92, 1325 (1953).

[7]F. C. Khanna, to be published.

[8]D. J. Drickey and L. N. Hand, Phys. Rev. Letters 9, 521 (1962); R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearian, Phys. Rev. 160, 874 (1967); Bellicard et al., Ref. 1.

[9]K. Gottfried, Ann. Phys. (N.Y.) 21, 29 (1963); K. Gottfried and W. Czyż, Ann. Phys. (N.Y.) 21, 47 (1963); F. C. Khanna, Nucl. Phys. A97, 417 (1967).