able in our experimental conditions.

Amplitude oscillations are expected to be observed when $k_{osc}/k_i \gtrsim (2\pi)^{1/2}$,[3,4] where k_i is the imaginary part of the wave number of the small-amplitude waves. When the damping of the small-amplitude waves is small, so that $k_{osc}/k_i \gg (2\pi)^{1/2}$ is easily satisfied, the effect of the trapped particles becomes remarkable and strong amplitude oscillations appear. In our case, the waves are observed to be damped slowly accompanied by amplitude oscillations, in contrast with the experiment of Malmberg and Wharton,[3] in which $k_{osc}/k_i \gg (2\pi)^{1/2}$. The difference is mainly attributed to the fact that the experimental values of k_{osc}/k_i, 1.8-3.2, are comparable with $(2\pi)^{1/2}$. The amplitude oscillations disappear gradually with the increase of the plasma density. The trapped ions which sustain the amplitude oscillations are scattered by the ion-ion collisions because the mean free path (several tens of centimeters) is only about ten times as large as the values of $2\pi/k_{osc}$. The nonlinear effects which have been described above are also found in the recent computer experiment[8] on the large-amplitude electron plasma waves.

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FERMI-LIQUID EFFECTS IN MAGNETOPLASMA MODES IN ALKALI METALS*

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Platzman and Walsh[1] have looked for Fermi-liquid effects in potassium in plasma modes propagating perpendicular to a uniform magnetic field, at frequencies on the order of the cyclotron frequency. However, they have had difficulty accounting for their experimental results.[2]

We wish to clarify the rather subtle structure of the spectrum of long-wavelength plasma modes propagating perpendicular to the field and to show how the Landau-Silin kinetic equation for an isotropic, charged Fermi liquid does predict a quadratic small-k dependence for the resonant frequencies.

We start with the linearized collisionless ($\omega_c\tau \gg 1$) Boltzmann equation with Fermi-liquid corrections[3]:

$$\omega\nu - kv_x\bar{\nu} - \omega_c \frac{1}{i}\frac{\partial}{\partial\varphi}\bar{\nu} = ie\vec{\mathcal{E}}\cdot\vec{\mathbf{v}} + \frac{3\omega_p^{*2}}{\omega}\left[\hat{v}_x\langle\hat{v}_x\bar{\nu}\rangle + \frac{\omega^2}{\omega^2 - k^2c^2}(\hat{v}_y\langle\hat{v}_y\bar{\nu}\rangle + \hat{v}_z\langle\hat{v}_z\bar{\nu}\rangle)\right]. \quad (1)$$

Here the deviation of the distribution function from equilibrium is written as

$$\delta n = -(\partial n_0/\partial\epsilon)\nu e^{i(\vec{k}\cdot\vec{r} - \omega t)}.$$

$\vec{H}$ is along the z axis, $\vec{k}$ is along the x axis, $\omega_c = eH/m^*c$, and $\omega_p^{*2} = 4\pi ne^2/m^*$, where m^* contains Fermi-liquid corrections, and spherical coordinates are used so that $v_x = v_F\hat{v}_x$, $\hat{v}_x = \sin\theta\cos\varphi$, etc.; $\vec{\mathcal{E}}$ is a fictitious driving field, and the term in square brackets is the scalar product of $ie\vec{v}$ and the induced field, the form of which follows from solving Maxwell's equations with $\vec{j} = e(m^*k_F/\pi^2)\langle\vec{v}\bar{v}\rangle$. The angular brackets represent an average over angles [e.g., $\langle\nu\rangle = \int d\Omega\nu(\theta,\varphi)/4\pi$]; and[4] $\bar{v} = (1+F)\nu$. For example, if

$$\nu = \sum\nu_{lm}Y_{lm}(\theta,\varphi),$$

then

$$\bar{v} = \sum\gamma_l\nu_{lm}Y_{lm}(\theta,\varphi), \quad \gamma_l = 1+F_l/(2l+1).$$

We wish to treat the k dependence of (1) by perturbation theory and therefore transform it to a Hermitian form by introducing $\psi = (1+F)^{1/2}\nu$ (i.e., $\psi_{lm} = \gamma_l^{1/2}\nu_{lm}$).[5] We also project out the $l=0, m=0$ subspace[6] by using the equation of continuity. The resulting equation is to be solved in the space spanned by all Y_{lm} except Y_{00} and has the form

$$(H_c + H_e + \lambda V)\psi = E\psi + ie\gamma_1^{1/2}\vec{\mathcal{E}}\cdot\vec{v}/\omega_c, \tag{2}$$

where

$$H_c\psi = (1+F)\frac{1}{i}\frac{\partial}{\partial\varphi}\psi \quad [(H_c\psi)_{lm} = m\gamma_l\psi_{lm}], \tag{3}$$

$$V\psi = (1+F)^{1/2}\hat{v}_x(1+F)^{1/2}\psi, \tag{4}$$

$$H_e = \frac{3\omega_p^2}{\omega\omega_c}\left\{\hat{v}_x\langle\hat{v}_x\psi\rangle\left[1+\tfrac{1}{3}(kv_F/\omega_p)^2\gamma_0\right] + \frac{\omega^2}{\omega^2-k^2c^2}(\hat{v}_y\langle\hat{v}_y\psi\rangle + \hat{v}_z\langle\hat{v}_z\psi\rangle)\right\},$$

$$\lambda = kv_F/\omega_c, \quad E = \omega/\omega_c, \quad \omega_p^2 = 4\pi ne^2/m. \tag{5}$$

Note that (2) does not mix the subspaces $l+m$ even and $l+m$ odd. They contain modes polarized perpendicular and parallel to $\vec{H}$, respectively.[7]

Let $H_0 = H_c + H_e$, and regard λV as a perturbation. H_e acts only in the $l=1$ subspace. Outside of this space $H_0 = H_c$ and its eigenvalues are[8] $E_{lm} = m\omega_c\gamma_l$, with eigenfunction Y_{lm}. Since Fermi-liquid effects ($\gamma_l \neq 1$) split the (infinite) degeneracy of the $m\omega_c$ free-electron level, we can do ordinary perturbation theory in λ.

We are interested in levels near $m\omega_c$ for rather small m, and to leading order may therefore replace the ω occurring explicitly in H_e by this value. The eigenvalues of H_e in the $l=1$ subspace are then of order ω_p^2/ω_c^2 or $\omega_p^2/(\omega^2-k^2c^2)$ and are enormous, provided only $(\omega_p/\omega_c)^2$ and $(\omega_p/kc)^2 \gg 1$. (This is, of course, because any mode containing a substantial amount of Y_{1m} will be dominated by electromagnetic restoring forces and go at the longitudinal or transverse plasma frequencies.) In the $l=1$ space, H_c is therefore a small perturbation on H_e and the eigenvalues of H_0 are substantially the same as those of H_e. Consequently, in corrections to levels with $l>1$, intermediate states with $l=1$ have enormous energy denominators and can be ignored.

We find that all levels are corrected in second order:

$$\omega_{lm}/\omega_c = E_{lm} = m\gamma_l + \lambda^2\gamma_l\sum_{\substack{l'\neq 0,1\\ m'}}\gamma_{l'}\frac{|\langle lm|\sin\theta\cos\varphi|l'm'\rangle|^2}{m\gamma_l - m'\gamma_{l'}} \tag{6}$$

or, for l greater than 2,

$$\omega_{lm} = m\omega_c\gamma_l + \frac{\lambda^2\omega_c\gamma_l}{4(2l+1)}\left\{\frac{\gamma_{l-1}}{2l-1}\left[\frac{(l-1+m)(l+m)}{m\gamma_l-(m-1)\gamma_{l-1}} + \frac{(l-1-m)(l-m)}{m\gamma_l-(m+1)\gamma_{l-1}}\right]\right.$$

$$\left. + \frac{\gamma_{l+1}}{2l+3}\left[\frac{(l+1-m)(l+2-m)}{m\gamma_l-(m-1)\gamma_{l+1}} + \frac{(l+1+m)(l+2+m)}{m\gamma_l-(m+1)\gamma_{l=1}}\right]\right\}. \tag{7}$$

Since V has the selection rule $\Delta l = \pm 1$, the exclusion of the $l=1$ level only affects the second-order

correction to E_{2m}:

$$\omega_{2m}=m\omega_c\gamma_2+\frac{\lambda^2\omega_c\gamma_2\gamma_3}{140}\left[\frac{(3-m)(4-m)}{m\gamma_2-(m-1)\gamma_3}+\frac{(3+m)(4+m)}{m\gamma_2-(m+1)\gamma_3}\right]. \tag{8}$$

Equation (8) has small corrections of order $(\omega_c/\omega_p)^2$ or $(kc/\omega_p)^2$, while (7) is valid even when these are not small. Equations (6) and (7) are not valid when $l=1$, these modes being slightly modified plasmons.[9]

Note the following: (a) When all $\gamma_l=1$, the second-order corrections to all but the $l=2$ modes vanish identically.

(b) A measure of the power absorbed in the lm mode is the residue of $\vec{\mathcal{E}}\cdot\vec{j}$ at the corresponding pole, and thus the residue at E_{lm} of E_{lm} of

$$\left\langle\hat{v}\cdot\vec{\mathcal{E}}\,\frac{1}{H_0-E}\sum_{n=0}^{\infty}\left(\lambda V\frac{1}{H_0-E}\right)^n\hat{v}\cdot\vec{\mathcal{E}}\right\rangle. \tag{9}$$

Since V changes l by ±1, the pole at E_{lm} will not contribute until Y_{lm} appears as an intermediate state. Since $\hat{v}\cdot\vec{\mathcal{E}}$ is in the $l=1$ subspace, this requires that $n=2(l-1)$. Thus the power absorbed in the lm mode goes as $\lambda^{2(l-1)}$, $l>1$.

(c) Therefore the $l=2$ modes are the easiest to excite. For polarization parallel to $\vec{H}$ ($l=2$, $m=1$), this is the mode discussed earlier (without Fermi-liquid corrections) by Walsh and Platzman.[10] They found a k^2 dependence, but unfortunately, when $l=2$ the coefficient of the k^2 term does not vanish as $F\to0$. Thus it is not easy to distinguish free-electron effects from Fermi-liquid corrections in these modes. They do, however, provide additional parameters for fitting the data of Ref. 10 besides the effective mass and sample thickness. Similarly, for polarization perpendicular to $\vec{H}$, although the $l=2$ mode [at $2\omega_c\gamma_2+o(\lambda^2)$] is most strongly coupled, the coefficient of the k^2 correction is quite likely to be dominated by the free-electron contribution.

(d) Therefore Fermi-liquid effects are most easily detected in the $l=3$ modes, since these are the most strongly coupled ones for which the k^2 corrections vanish when $F\to0$. There are three such modes, two polarized perpendicular to $\vec{H}$ (31 and 33) and one polarized parallel (32). Equation (7) gives the coefficient of the k^2 term in these cases, and should replace Eq. (8) of Ref. 1. Since the free-electron theory first gives corrections to the $l=3$ modes in order k^4, Platzman and Walsh's finding that the dispersion curve is quadratic rather than quartic for small k is confirmed; however, F_2, F_3, and F_4 are involved rather than F_0, F_1, and F_2. One can in principle determine F_2, F_3, and F_4 separately if one does the experiment with polarization parallel to $\vec{H}$ for the 32 mode, and distinguishes the 31 and 33 modes in the perpendicular case.

(e) The two-branched dispersion relation displayed by Platzman and Walsh (Ref. 1, Fig. 2) will be characteristic of correct calculations using truncated F functions, but is rather misleading. However it is the upper (quartic) rather than the lower (quadratic) branch that is suspect. If all F were kept we should find many $m=1$ branches, all going as k^2 for small k [with the 31 branch dominating because of (b)]. If, however, all F_l with $l>l_0$ are set equal to zero, then the $l1$ modes with $l>l_0+1$ will have no second-order Fermi-liquid corrections. The $(l_0+2)1$ mode will have a fourth-order Fermi-liquid correction due to F_{l_0}, and this will give the quartic branch. However it is an artifact of the truncation of F. All branches should be quadratic (although the quadratic character may not become manifest until very small λ for the higher l modes if the higher γ_l are very close to unity).

(f) The significance of an exact solution to (1) keeping only a single F_l when λ is not small is dubious, even as a curve-fitting procedure, since increasingly larger numbers of F_l enter into the coefficients of higher powers of λ. Perhaps the least illegitimate procedure is to hope that the F_l decline rapidly with increasingly l. If this were true, one could extract the λ dependence of the l modes by an exact solution retaining all the F_l from F_2 up to F_{l-1}. Thus a numerical fit to the mode of interest in Ref. 1 should keep not F_0 but F_2. (However one should not then take seriously the quartic branch that would emerge from the calculation, since the inclusion of subsequent F_l would both improve the fit to the $l=3$ branch, and convert successively higher l branches from quartic to quadratic.)

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[6]Regard ψ as a vector in the space spanned by the Y_{lm}.

[7]Mixing occurs when $\vec{k}$ is not perpendicular to $\vec{H}$. We shall describe elsewhere the rather different structure emerging in that case.

[8]This and the quadratic form of the small-k corrections were first pointed out by Silin, Ref. 3. It should be emphasized that for each m there are infinitely many distinct modes, but see also remark (b).

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NMR STUDY OF THE SPIN-REORIENTATION BEHAVIOR OF $Mn_{2-x}Cr_xSb$†

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Spin reorientation due to change in sign of the temperature-dependent anisotropy energy has been studied in $Mn_{2-x}Cr_xSb$ ($x = 0$ and 0.03) by nuclear spin echo measurements. The behavior of the dipole and quadrupole hyperfine interactions and T_2 is reported in the region 180 to 280°K. The data suggest the possibility that a phase transition(s) is occurring.

We have studied ferrimagnetic $Mn_{2-x}Cr_xSb$ ($x = 0$ and 0.03) in the spin-reorientation temperature region by nuclear spin-echo observations. A large change (by a factor of 2) in the quadrupole interaction upon the reorientation of the magnetization from perpendicular to parallel to the c axis is observed as a change in the modulation of the spin-echo spectrum excited at the magnetic hyperfine frequency. The results indicate a broad region of instability as the spin-reorientation region is approached and should provide a strong test of any dynamical model which might be developed.

Neutron-diffraction studies have shown Mn_2Sb to have two molecules in each tetragonal unit cell with magnetic moments of $(+2.3 \pm 0.20)\mu_B$ and $(-3.87 \pm 0.40)\mu_B$ on the two types of manganese sites, I and II, respectively.[1] Measurements by Darnell, Cloud, and Jarrett[2] show the anisotropy energy increases nearly linearly between 200 and 300°K passing through zero ($K_1 = -K_2$) at $T_f = 247$°K. For $Mn_{2-x}Cr_xSb$ ($0 \leq x \leq 0.05$), T_f monotonically decreases with increasing x down to 200°K, where a first-order transition to antiferromagnetism occurs. The magnetization is parallel to the c axis for $T > T_f$ and perpendicular for $T < T_f$.

The spin-echo measurements are made using a wide-band hybrid junction and wide-band mixer. The sample, bulk pieces or powder dispersed in paraffin, is placed in a copper block at the shorted end of a 50-Ω stainless-steel coaxial line 25 cm long. The line is capacitor tuned and critically coupled to 50 Ω. A heater and thermocouple in the copper block allow measurements to below 77°K. The oscillator is able to provide rf fields H_1 up to 3 Oe and pulses 0.5 μsec wide. The powdered sample is Mn_2Sb ground to 325 mesh (44 μ) size under a liquid. The bulk sample is $Mn_{1.97}Cr_{0.03}Sb$, cleaved into thin flakes with surfaces perpendicular to the c axis.[3] These flakes were carefully stacked into piles to maintain uniform c-axis orientation and were molded in paraffin.

Mn_2Sb at 77°K has NMR signals due to Mn at 143.7 and 126.26 MHz.[4] Applying fields up to 10 kOe increases the higher and decreases the lower resonant frequencies, and at fields greater than the demagnetization field, the slope $d\nu/dH$ agrees within measurement error with the gryomagnetic ratio of ^{55}Mn. Assuming a negative effective hyperfine magnetic field, the electronic moment associated with the lower frequency is aligned parallel to the field and is therefore the larger-moment site, II. Thus the frequencies 143.7 and 126.26 MHz