π^+-He^4 ELASTIC SCATTERING AT 610 MeV/c *

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In this paper we present the results of the analysis of π^+-He^4 elastic scattering at an incident pion laboratory momentum of 610 MeV/c. The data for this investigation were obtained from 17 500 frames exposed in the Argone National Laboratory–Carnegie Institute of Technology 10-in. He^4 bubble chamber[1] at the Argonne National Laboratory.

Events were measured on conventional digitized projectors and processed through geometry (NP54) and kinematic fitting (GRIND) programs. To insure good measurements, fiducial volume and beam track criteria were imposed in selecting the data. Acceptable events were required to have beam momentum in the range 550-650 MeV/c to avoid as much as possible the variations of energy-dependent parameters and still have a statistically significant sample of events. The mode of the beam distribution was 610 MeV/c. The total number of events that satisfied the elastic criteria was 869, of which 194 had a discernible recoil track and were fitted as a four-constraint (4C) class, while the rest were 1C fits. The χ^2 cuts for both topologies were based on a 1% confidence level. Because of the large scanning losses involved with small-angle scattering, we did not accept events with $\theta_{\text{lab}} < 10°$. This cut eliminated the background from $\pi^+ \to \mu^+ + \nu_\mu$ decays and at the same time made the contribution of the Coulomb amplitude vanishingly small. The events in each individual bin of the differential cross section (Fig. 1) were corrected for scanning biases by requiring isotropic distributions for the azimuthal angle of the interaction plane. Since the muon contamination of the beam was unknown, we have normalized our total cross section to 111.4 mb, the result of the counter experiment of Chavanon et al.[2] The experimental elastic cross section is then 30.7 mb.

Two models were used in fitting the experimental results: a strong-absorption model suggested by Palevsky's analysis of p-He^4 elastic scattering,[3] and the multiple-scattering model.[4]

(a) Strong-absorption model.[5]–In the strong-absorption model the elastic-scattering amplitude is given by

$$f(\theta)=\frac{i}{2k}\sum_{l=0}^{\infty}(2l+1)[1-\eta_l \exp(2i\sigma_l)]P_l(\cos\theta), \quad (1)$$

where σ_l is the Coulomb phase shift (taken equal to zero), and η_l the nuclear reflection coefficient which is parametrized to give a closed-form expression for the differential cross section. In this analysis we used for the parametrization of η_l the Woods-Saxon functional form $g(l)$[6]:

$$\eta_l=\{(1-\epsilon)g(l)+\epsilon\}+i\mu dg/dl \quad (2)$$

and

$$g(l)=\{1+\exp[(L_0-l)/\xi]\}^{-1}. \quad (3)$$

The parameters L_0 and ξ are related, respectively, to the nuclear radius R and spatial diffuseness d of the interaction region by

$$L_0+\tfrac{1}{2}=kR, \quad \xi=kd. \quad (4)$$

The parameter μ represents the effect of the real part of the nuclear phase shift while ϵ is

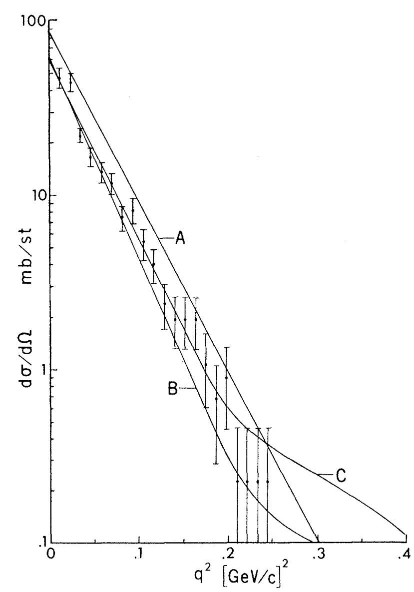

FIG. 1. Differential cross section for π^+-He^4 elastic scattering. Curve A is the impulse approximation, curve B is the multiple-scattering model, and curve C is the strong-absorption model.

the transparency coefficient. The differential cross section is then

$$\frac{d\sigma}{d\Omega}=R^2\left(\frac{\theta}{\sin\theta}\right)[F(\zeta\theta)]^2\left\{(1-\epsilon)^2\left[\frac{J_1(kR\theta)}{\theta}\right]^2+\mu[J_0(kR\theta)]^2\right\}, \tag{5}$$

where $F(\zeta\theta)$ is the Fourier transform of dg/dl and is given by

$$F(\zeta\theta)=\pi\zeta\theta/\sinh(\pi\zeta\theta). \tag{6}$$

The total cross section is

$$\sigma_{\text{total}}=2\pi R^2(1-\epsilon)[1+\tfrac{1}{3}\pi^2(d/R)^2], \tag{7}$$

while the complex potential in the high-energy approximation is given by[7]

$$V(r)=\frac{2}{\pi}\frac{k^2}{(m^2+k^2)^{1/2}}\int_{kr}^{\infty}\frac{\delta'(l)dl}{[l^2-(kr)^2]^{1/2}}. \tag{8}$$

(b) Multiple-scattering model.—In the multiple-scattering model the basic assumption is the additivity of the phase shifts of the individual interactions that a particle undergoes as it traverses the nucleus. Thus the phase shift is given by

$$\exp\{i\chi(\vec{b},\vec{s},\cdots\vec{s}_A)\}=\exp\{i[\chi_1(\vec{b}-\vec{s}_1)+\cdots+\chi_A(\vec{b}-\vec{s}_A)]\}, \tag{9}$$

where χ_j is the phase-shift function for the jth nucleon, $\vec{b}$ is the impact parameter, and $\vec{s}_j$ is the fixed position of the jth nucleon relative to the axis of collision. The scattering amplitude as a function of momentum transfer $\vec{q}$ can be written as

$$F_{fi}(\vec{q})=\frac{ik}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\int\varphi_f^*(\{\vec{r}_l\})\left\{1-\prod_{j=1}^{A}\left[1-\frac{1}{2\pi ik}\int\exp[i\vec{q}_j\cdot(\vec{b}-\vec{s}_j)]f_j(\vec{q}_j)d^2q_j\right]\right\}$$
$$\times\varphi_i(\{\vec{r}_l\})\delta(A^{-1}\sum_{n=1}^{A}\vec{r}_n)\prod_{m=1}^{A}d\vec{r}_m d^2b. \tag{10}$$

We assume the ground state of He^4 to be adequately described by a Gaussian wave function

$$\varphi(\{\vec{r}_l\})=C\exp\left(-\frac{9}{64R^2}\sum_{i<j}(\vec{r}_i-\vec{r}_j)^2\right), \tag{11}$$

where C is a normalization factor and R is the rms radius. For the individual π^+-N scattering amplitudes we have used the form

$$f_j(\vec{q})=\left(\frac{i+\rho_j}{4\pi}\right)k\sigma_j^{\text{total}}\exp\{-b_jq^2\}, \tag{12}$$

where

$$\rho_j=\frac{\mathrm{Re}f_j(\vec{q})}{\mathrm{Im}f_j(\vec{q})}. \tag{13}$$

The parameters b_j and ρ_j were determined by performing a least-squares fit of $|f_j(\vec{q})|^2$ to the known Legendre polynomial expansions of $\pi^\pm$-p elastic-scattering differential cross sections.[8] Our fits show that the "shape" parameters b_p and b_n are nearly the same, so that we may write $b_p=b_n=b$. The expansion for the differential cross section is

$$\frac{d\sigma}{d\Omega}=|\sum_{n=1}^{4}F^{(n)}(\vec{q})|^2, \tag{14}$$

where the n-fold scattering amplitude is

$$F^{(n)}(\vec{q})=\frac{1}{n}\left(\frac{1}{2k}\right)^{n-1}\left(b+\frac{2}{9}R^2\right)^{1-n}e^{q^2R^2/18}\exp[-q^2(b+\frac{2}{9}R^2)/n]\sum_{j_1<\cdots<j_n=1}^{4}A_{j_1}\cdots A_{j_n},\tag{15}$$

and $A_j=[(i+\rho_j)/4\pi]k\sigma_j^{\text{total}}$ is the coefficient in Eq. (12). The $n=1$ term in Eq. (15) is the single scattering term and represents the contribution of the impulse approximation.

Results.—The experimental differential cross section is shown in Fig. 1. The error bars include the statistical errors and the uncertainties of the corrections.

The impulse approximation (curve A) and multiple scattering (curve B) are obtained using $R=1.41$ F which is in agreement with electron-scattering results after the correction for the finite size of the proton.[9] The parameters ρ_p, ρ_n, and b are determined directly from π^+-p experiments at this energy. The values are ±1.80, ±0.25, and 2.73 $(\text{GeV}/c)^{-2}$, respectively; the sign of ρ_j is indeterminate. The multiple-scattering model yields an elastic cross section of 28.9 mb and a total cross section using the optical theorem of 95.2 mb. It is clear that both the experimental angular distribution and elastic cross section are in reasonable agreement with the theoretical results; it should be emphasized that this agreement is produced with no adjustable parameters. Figure 2 shows the sensitivity of the calculations with respect to the parameters.

In the strong-absorption model analysis the theoretical expression Eq. (5) is fitted by least squares to the experimental distribution. The fitted values of the parameters are $R=1.41$ F, $d=0.30$ F, $\epsilon=0.24$, and $\mu=\pm0.69$ compared with the p-He^4 results[3] $R=1.56$ F, $d=0.29$ F, $\epsilon=0.188$, and $\mu=\pm0.55$. The sign of μ is indeterminable, because the Coulomb scattering is insignificant in the range of interest. To obtain a repulsive real part to the potential $V(r)$, μ must be negative. The potential shown in Fig. 3 resembles very much that obtained from the p-He^4 experiment.[3] The elastic cross section is 30.6 mb and the total cross section using Eq. (7) is 113.1 mb. At this point it should be stated that this is the first time that either of these models has been applied to π^+-He^4 elastic scattering and it seems that either gives a satisfactory fit to the experimental results.

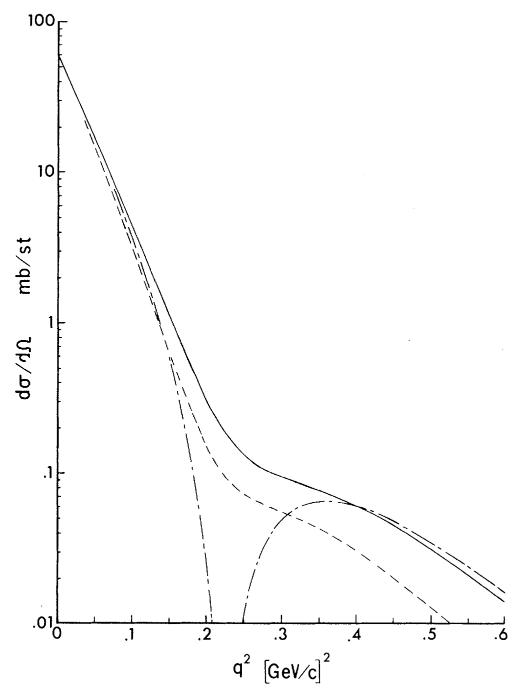

FIG. 2. Variation of the differential cross section for elastic π^+-He^4 scattering with different parameters in the multiple-scattering model. The solid curve is the result of the theory with $R=1.41$ F, $\rho_p=1.80$, $\rho_n=0.25$, and $b=2.73$ $(\text{GeV}/c)^{-2}$; the dashed curve is the result of increasing R to 1.56 F; the dot-dashed curve is the result with $R=1.41$ F and $\rho_p=\rho_n=0$.

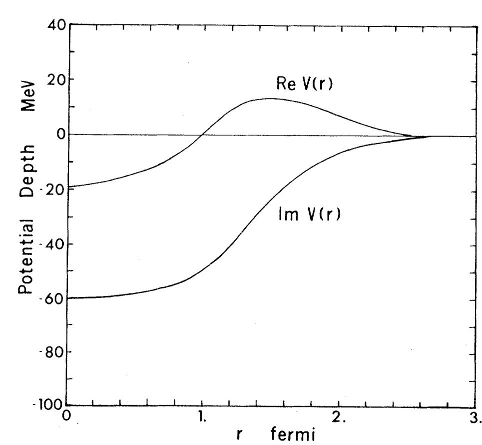

FIG. 3. The real and imaginary parts of the optical potential $V(r)$ given by Eq. (8).

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PION ELECTROMAGNETIC MASS DIFFERENCE FOR PHYSICAL PIONS. I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer [Phys. Rev. Letters 19, 1064 (1967)].

The coefficient of δ^2 in Eq. (9) should read

$$[-\tfrac{1}{4}+\tfrac{1}{4}\ln 2+\tfrac{1}{8}\ln(\Lambda^2/m_\rho^{\,2})]$$

instead of

$$[-\tfrac{3}{4}+\tfrac{1}{4}\ln 2+\tfrac{1}{8}\ln(\Lambda^2/m_\rho^{\,2})].$$

This correction makes a negligible change in our result. This was brought to our attention by Professor T. D. Lee, who informed us of the calculation of K. Ng.

EVIDENCE FOR ELECTRON–TO–PHONON INTERACTION IN InSb. D. H. Dickey and D. M. Larsen [Phys. Rev. Letters 20, 65 (1968)].

Figure 1, on p. 66, was inadvertently printed upside down.

ON INFINITIES IN ELECTROMAGNETIC MASS DIFFERENCES TO ANY ORDER IN α. P. Oelsen [Phys. Rev. Letters 20, 525 (1968)].

Reference 1 should also contain the following paper: M. B. Halpern and G. Segrè, Phys. Rev. Letters 19, 611 (1967).