[3]H. H. Heckman and G. H. Nakano, to be published.

[4]J. Valerio, J. Geophys. Res. 69, 4949 (1964).

[5]D. C. Jensen and J. C. Cain, J. Geophys. Res. 67, 3568 (1962).

[6]S. D. Freden, J. B. Blake, and G. A. Paulikas, private communication, 1967.

[7]J. D. Gabbe and W. L. Brown, in Radiation Trapped in the Earth's Magnetic Field, edited by B. M. McCormac (D. Reidel Publishing Company, Dordrecht, Holland, 1966), p. 165.

[8]R. Filz and E. Holeman, J. Geophys. Res. 70, 5807 (1965).

[9]R. C. Blanchard and W. N. Hess, J. Geophys. Res. 69, 3927 (1964).

[10]I. Harris and W. Priester, J. Geophys. Res. 67, 4585 (1962). Also National Aeronautics and Space Administration Technical Note No. D-1444, 1962 (unpublished).

[11]I. Harris and W. Priester, J. Geophys. Res. 68, 5891 (1963).

POSSIBILITY OF STRONG INTERACTIONS FOR THE INTERMEDIATE BOSON*

C. G. Callan, Jr.†

Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138

(Received 26 February 1968)

It has been pointed out in the past that the hypothetical vector mesons of weak-interaction theory could well enjoy strong interactions of the form $fW_\mu^\dagger W^\mu H$, where H is some function of hadron fields, without directly contradicting experimental information on low-energy processes.[1] This possibility now seems much more attractive since it provides a "simple" explanation for the recently discovered failure of the high-energy cosmic-ray muon flux to obey the "$\sec\theta$" law.[2] On the other hand, it is not clear that it is consistent with the rather detailed picture of low-energy weak processes which has been built up over recent years, so the question must be re-examined with some care. It is the purpose of this paper to present a theory of strongly interacting W's which maintains all the important features of the conventional theory for low-energy weak processes, while permitting significant deviations from them at high energies. The appearance of new physics at high energies seems an inescapable consequence of giving the W's strong interactions, and should be regarded as a challenge to experiment rather than as a defect of the theory.

We wish to describe a system comprising leptons, hadrons, and a single, charged, vector meson. The Lagrangian should differ from the conventional one only by the addition of a strong-interaction term quadratic in the vector-meson field. For reasons which will become apparent as we go along, we choose

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}^{\dagger}F^{\mu\nu}(1+S)-\tfrac{1}{2}m_W^{2}W_\mu^{\dagger}W^{\mu}+g(W_\mu^{\dagger}J_\mu+W_\mu J_\mu^{\dagger})+\mathcal{L}_h+\mathcal{L}_l, \qquad (1)$$

where W is the vector-meson field; $F_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu$; S is a scalar object constructed out of hadron fields, involving no derivatives, and invariant under SU(3)⊗SU(3) transformations on the hadron fields; $\mathcal{L}_h$ is the usual hadronic Lagrangian; $\mathcal{L}_l$ is the free Lagrangian for leptons; and J_μ is the usual weak current,

$$J_\mu=J_\mu^{h}+J_\mu^{l},$$

$$J_\mu^{l}=\bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu+\bar{e}\gamma_\mu(1-\gamma_5)\nu_e,$$

$$J_\mu^{h}=\cos\theta(V_\mu^{\pi\dagger}-A_\mu^{\pi\dagger})+\sin\theta(V_\mu^{K\dagger}-A_\mu^{K\dagger}).$$

This differs from the usual theory of boson-mediated weak interactions only through the addition of $\mathcal{L}_I^W=-\frac{1}{4}F_{\mu\nu}^\dagger F^{\mu\nu}S$. Such a theory is, of course, not renormalizable because of the derivative coupling of the vector meson. In spite of this, we shall be able to prove interesting theorems about the theory, and show that, if it exists, it does not lead to a contradiction with what we know about weak interactions. The same does not seem to be true of the simple renormalizable theories which one can write down. In any event, recent progress in quantum field theory[3] leads one to hope that nonrenormalizable fields may eventually be put on a sound mathematical basis, so that there is no real reason to abandon the richness provided by nonrenormalizable theories.

Let us first study the strong-interaction properties of this theory and verify that the usual conservation laws hold. The hadron fields are presumed to provide a representation of SU(3)⊗SU(3). If we set $g=0$ and make an infinitesimal transformation on the fields such that $\delta W_\mu = 0$ and $\delta\psi_h$ is given by the appropriate representation of $SU(3)\otimes SU(3)$, then $\delta\mathcal{L} = \delta\mathcal{L}_h$. This is because S is taken to be an $SU(3)\otimes SU(3)$ invariant. Since S contains no derivatives of hadron fields, Noether's theorem guarantees the existence of currents whose commutation and divergence properties are the same as if the total Lagrangian were just $\mathcal{L}_h$. In particular, if $\mathcal{L}_h$ conserves isospin and strangeness, so does $\mathcal{L}(g=0)$. Partial conservation laws retain their usual form: If canonical partial conservation of axial-vector currents holds for $\mathcal{L}_h$, it holds for the full strong-interaction part of $\mathcal{L}$ as well. There is, of course, one new conservation law—conservation of W charge—since vector mesons interact strongly in oppositely charged pairs. We make the usual identification of the weak hadronic currents with the generators of $SU(3)\otimes SU(3)$ defined by Noether's theorem.

There is one new feature as far as electromagnetic interactions are concerned. The electromagnetic current of strongly interacting particles is $Q_\mu = Q_\mu{}^h + Q_\mu{}^W$, where $Q_\mu{}^h$ is the charge current of hadrons and $Q_\mu{}^W$ is the charge current of W's. The SU(3) properties of $Q_\mu{}^h$ are as usual, while $Q_\mu{}^W$ commutes with the generators of SU(3). The electromagnetic current therefore has an SU(3)-singlet piece in this theory. This does not affect any predictions based on U-spin invariance. It does affect leptonic decays of the φ and ω mesons, as well as some hyperon magnetic moments. At present there is no conclusive evidence against the existence of a singlet term.[4]

We now turn to the question of semileptonic weak interactions. Since the coupling constant g is small, we may safely write the matrix element for the process $A \to B + l^- + \bar{\nu}_l$ as

$$g\bar{u}_l\gamma^\mu(1-\gamma_5)v_{\nu_l}\langle B|W_\mu|A\rangle.$$

On the other hand, the equation of motion for W implied by the Lagrangian of Eq. (1) is

$$W_\mu = (g/m_W^2)J_\mu + (g/m_W^2)\partial^\nu G_{\nu\mu},$$

$$gG_{\mu\nu} = F_{\mu\nu}(1+S). \tag{2}$$

Therefore, the matrix element may be rewritten as

$$(g^2/m_W^2)\bar{u}_l\gamma^\mu(1-\gamma_5)v_{\nu_l} \times\{\langle B|J_\mu{}^h|A\rangle + iq^\nu\langle B|G_{\nu\mu}|A\rangle\},$$

where q_μ is the four-momentum transfer between B and A. If we set $g^2/m_W^2 = G_F/\sqrt{2}$ and let $q_\mu = 0$, this is identical to the usual expression for semileptonic-decay amplitudes. Therefore, for zero momentum transfer, we recover the predictions of Cabibbo theory and conserved vector currents (CVC). When $q \neq 0$, the picture is more complicated. We evidently do not have strict equality of the electromagnetic and strangeness-conserving vector weak current. Indeed, for a transition between two spin-one-half states, $q^\nu\langle B|G_{\nu\mu}|A\rangle$ will look just like a vector magnetic moment and would lead to deviations from the weak magnetism expected from standard CVC theory. On the other hand, it is perfectly easy to choose the strong coupling of the W so that, at least in the lowest order of perturbation theory, $\langle B|G_{\mu\nu}|A\rangle = O(m_W^{-1})$. There seems no reason to doubt that the full theory could not have this property, in which case the anomaly would be suppressed in low-energy experiments but would be visible in sufficiently high-momentum-transfer neutrino experiments.

Similar remarks hold concerning soft-pion theorems for semileptonic decays. In all interesting reactions, q/m_W is small enough that the contribution of $G_{\mu\nu}$ to the matrix element may be neglected. To a good approximation, the amplitude for $A \to B + l^- + \bar{\nu}_l$ becomes

$$2^{-\frac{1}{2}}G_F\bar{u}_l\gamma^\mu(1-\gamma_5)v_{\nu_l}\langle B|J_\mu{}^h|A\rangle, \tag{3}$$

which is all one needs for the soft-pion theorems. Notice also that, according to Eq. (2), $\partial^\mu W_\mu = \partial^\mu J_\mu{}^h$. Therefore, any theorem depending only on the divergence properties of the weak current (such as Adler's result on the connection between forward lepton production by neutrinos on protons and pion-proton scattering) remains true.

The theory of nonleptonic weak interactions differs in no way from the usual picture. The soft-pion theorems on s-wave hyperon decays depend only on the fact that generators of the form $V+A$ commute with the weak Hamiltonian. This remains true because the currents to which W couples are again of the form $V-A$, and the SU(3)⊗SU(3) generators commute with W_μ.

We can also verify that the effective coupling constant for muon decay is the same as that for semileptonic decays in spite of the strong W interactions. The matrix element for $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ is, to lowest order in g,

$$g^2 \bar{u}_{\nu_\mu} \gamma^\mu (1-\gamma_5) u_\mu \bar{u} \gamma^\nu (1-\gamma_5) v_{\nu_l} P_{\mu\nu}(q),$$

where $P_{\mu\nu}$ is the covariant W propagator and q is the momentum transfer between μ and ν_μ. In terms of the spectral function $\sigma(\mu^2)$ defined by

$$\langle 0 | W_\mu^\dagger(x) W_\nu(0) | 0 \rangle = (2\pi)^{-3} \int d^4p\, \theta(p^0) e^{ip\cdot x} \sigma(p^2)(g_{\mu\nu} - P_\mu P_\nu / p^2),$$

we have

$$P_{\mu\nu}(q) = \int d\mu^2 \sigma(\mu^2) \frac{(g_{\mu\nu} - q_\mu q_\nu/\mu^2)}{q^2 - \mu^2}.$$

On the other hand, we conclude from Eq. (2) and the fact that $F_{0i}(1+S)$ is the canonical momentum conjugate to $W_i^\dagger$ that

$$\delta(x_0)[W_0^\dagger(x), W_i(0)] = (i/m_W^2)\partial_i \delta(x).$$

Taking the vacuum expectation value of this commutator, we get the familiar-looking sum rule

$$\int d\mu^2 \sigma(\mu^2)/\mu^2 = 1/m_W^2,$$

which implies that

$$P_{\mu\nu}(0) = g_{\mu\nu}/m_W^2.$$

Since σ is positive and equal to zero for μ^2 less than the physical mass μ_W of the W (remember that in calculating $P_{\mu\nu}$ we include only the strong interactions), we easily see that $P_{\mu\nu}(q)$ differs from $P_{\mu\nu}(0)$ by at most $O(q^2/\mu^2{}_W)$. For muon decays this difference is certainly negligible, and the effective matrix element is, to a good approximation,

$$g^2 m_W^{-2} \bar{u}_{\nu_\mu} \gamma^\mu (1-\gamma_5) u_\mu \bar{u}_e \gamma_\mu (1-\gamma_5) v_{\nu_e}$$

On comparing with the semileptonic-decay amplitude for small momentum transfer [Eq. (3)], we see that the coupling constants are equal in the usual sense (remember that $G_F/\sqrt{2} = g^2/m_W^2$). Therefore lepton-hadron universality survives.

Finally, we must consider the new weak processes which are induced by a strong W coupling. These have been considered in some detail by Ericson and Glashow,[1] so we shall restrict our attention to what seems the most important of them: neutrino-proton scattering. In a theory of strongly interacting W's this process is superficially of the same order in the weak coupling as $\bar{\nu} + p \to l^+ + n$ and could well be seen in high-energy neutrino experiments. We can show, however, that the amplitude for $\nu + p \to \nu + p$ must vanish at zero energy and will be suppressed relative to the usual process at moderate energies.

Consider the amplitudes for forward νp and $\bar{\nu} p$ scattering, in the approximation of neglecting the lepton masses:

$$M(\nu p) = A(W) \bar{u}_\nu \not{P}(1-\gamma_5) u_\gamma,$$

$$M(\bar{\nu} p) = \bar{A}(W) \bar{v}_\nu \not{P}(1-\gamma_5) v_{\bar{\nu}},$$

where p and q are the proton and neutrino momenta, respectively, and $W = q \cdot p$. Crossing symmetry tells us that $A(W) = -\bar{A}(-W)$. However, the total strong interaction Lagrangian of our system is invariant under $W_\mu \leftrightarrow W_\mu^\dagger$. This means that, to lowest order in the weak interactions (order g^2), $A(W) = \bar{A}(W)$. Therefore, we have the crossing property $A(W) = -A(-W)$, which means that $A(W) \propto W$, for $W \simeq 0$. Within the regime of strong interactions for the W it is easy to arrange that the scale of variation for A be determined by the mass of the W (at least to lowest order in perturbation theory, as in the discussion of CVC). Then one would expect $\bar{\nu} p$ scattering to be suppressed relative to $\bar{\nu} + p \to l^+ + n$ by $O(W/m_W^2)$. If the mass of the W is large enough, this is consistent with the limits set by high-energy neutrino experiments.

We can summarize these remarks as follows: It is possible to give the intermediate boson strong interactions and to maintain all the usual theorems about weak interactions which apply to zero momentum transfer. Theorems about the semileptonic interactions which hold at nonzero momentum transfer to leptons are no longer exact, but receive corrections which can be suppressed by the W mass, and may well be small in the energy range so far explored. At sufficiently high energies, however, one should find significant deviations from the usual picture of weak interactions, e.g., the appearance of first-order-weak neutrino-proton scattering. It does not seem possible to avoid these unpleasant features if one is forced to introduce strong interactions for the W. It is at least a comfort that many of the features of standard weak-interaction theory can be maintained while opening up the possibility of many new types of weak processes to explore at high energies.

It is a pleasure to thank Professor S. Glashow for many conversations essential to the genesis of this paper.

*Research supported in part by the U. S. Office of Naval Research, Contract No. Nonr-1866(55) (Mod. 1).

†Alfred P. Sloan Foundation Fellow.

[1]T. Ericson and S. L. Glashow, Phys. Rev. 133, B130 (1964).

[2]H. E. Bergeson et al., Phys. Rev. Letters 19, 1487 (1967).

[3]A. M. Jaffe, Phys. Rev. 158, 1454 (1967).

[4]H. Harari, "Electromagnetic Interaction and SU(3)," in Proceedings of the Symposium on the Present Status of SU(3) Symmetry, Argonne National Laboratory, July, 1967 (to be published).

REMARKS ON ISOSPIN SELECTION RULES AND CP-NONCONSERVING PHASES IN GLASHOW'S THEORY*

Jogesh C. Pati
Center for Theoretical Physics, Department of Physics and Astronomy,
University of Maryland, College Park, Maryland
(Received 27 December 1967)

Using algebra of currents and partial conservation of axial-vector currents (PCAC), Suzuki[1] has shown that in a CP-conserving theory with Cabibbo-type vector–axial-vector charged current-current interaction, one obtains a strict $\Delta I=\frac{1}{2}$ rule for K-meson nonleptonic decays in the limit of soft pions, even if the interaction was a mixture.

The present note is to remark that in a theory in which CP nonconservation is attributed to phase angles between normal octets of vector and axial-vector currents, as proposed by Glashow,[2] Suzuki's argument regarding the validity of the $\Delta I=\frac{1}{2}$ rule will hold for the CP-conserving part only if $\xi=\pm\varphi$, and it will hold for the CP-nonconserving part only if $\xi=-\varphi$; φ and ξ are the phase angles for the strangeness-preserving and strangeness-changing axial currents, respectively. This will suggest, as φ is known to be small[3] from β-decay experiments, that ξ should also be small, contrary to Glashow's conjecture that it could be very large. The choice of $\xi=\varphi$ will have the interesting consequence that it will allow finite $\Delta I=\frac{3}{2}$ transition in the soft-pion limit only for the CP-nonconserving amplitude but not for the CP-conserving one; both facts are supported by present experiments.[4] Furthermore, the success of current-algebra applications to the decays $K\rightarrow 3\pi$[5] and $Y\rightarrow N+\pi$[6] will be preserved if $\xi=\varphi$. Further consequences of the choice $\xi=\varphi$ (with both of them small) are discussed.

To prove the statements mentioned above, we define our current in the form proposed by Glashow[2]:

$$J_\mu=[(V_\mu{}^1+iV_\mu{}^2)+(A_\mu{}^1+iA_\mu{}^2)e^{i\varphi}]\cos\theta+[(V_\mu{}^4+iV_\mu{}^5)+(A_\mu{}^4+iA_\mu{}^5)e^{i\xi}]\sin\theta. \tag{1}$$

The weak interaction is assumed to have the current-current form[7]

$$H_W=G(J_\mu J_\mu{}^\dagger+J_\mu{}^\dagger J_\mu). \tag{2}$$

To establish Suzuki's result on isospin selection rules in $K\rightarrow 3\pi$ and $K\rightarrow 2\pi$ decays, it is sufficient to consider only the $K\rightarrow 2\pi$ decay amplitude, for which one needs to consider only the $|\Delta S|=1$ pari-