UNIVERSAL VECTOR AND AXIAL-VECTOR INTERACTION THEORY FOR STRONG INTERACTIONS*

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A primary universal vector and axial-vector interaction theory is proposed for strong-interaction phenomena. It is shown that such a hypothesis leads to a definite relation between vector and pseudoscalar coupling constants, which predict the S- and P-wave pion-nucleon scattering lengths, in good agreement with experiments. The ratio of π-meson to ρ-meson masses is also predicted, which agrees well with the observed value.

Though a quantitative understanding of the salient features of p-wave pion-nucleon interaction emerged from the work of Chew and Low,[1] a systematic computation of s- and p-wave scattering lengths has only recently been undertaken within the framework of current algebra.[2] It has been noted that many of the results deduced from current algebras could be derived from simple Lagrangian models involving vector mesons,[3] particularly as far as the s-wave computation is concerned. On the other hand, for p-wave scattering it is necessary to have an effective pseudovector Yukawa coupling of the pion. It is therefore of interest to develop a theoretical scheme in which both vector and axial-vector couplings play equally important roles. This is particularly of interest since the $V-A$ universal weak-interaction structure is suggestive of a primary universal vector–axial-vector strong interaction structure. One of the authors[4] has constructed such a theory of primary interactions which seems to be able to correlate a vast body of particle phenomena in electromagnetic, weak, and strong interactions. In this Letter we show that such a hypothesis leads to a definite relation between the vector and pseudoscalar coupling constants which predict s- and p-wave scattering lengths in agreement with experiments. The ratio of the rho-meson and pion masses is also predicted in good agreement with the observed value.

Interaction structure.—The primary strong-interaction structure involves the direct coupling of the vector V^λ and the axial-vector A^λ fields to the hadrons. As far as pion-nucleon scattering is concerned, the relevant hadrons are the nucleon and the $(I=J=\frac{3}{2})$ nucleon resonance. The vector-meson field is divergence free and is therefore associated with vector particles only. But the axial-vector field has a divergence which may be taken to be proportional to the pion field. We may write

$$A^\lambda = B^\lambda + (\xi/m_\pi)\partial^\lambda \varphi_\pi,$$

$$\partial^\lambda B_\lambda = 0. \quad (1)$$

The field B_λ describes a pseudovector particle of mass m_A. If it is identified with A_1 meson we have

$$m_A \simeq \sqrt{2}\, m_\rho. \quad (2)$$

The dimensionless parameter ξ is characteristic of strong interactions and can be determined either from the p-wave pion-nucleon coupling constant or from the observed ratio of the π and ρ meson masses.

We postulate that the primary hadron weak interaction is given by a direct coupling of the vector and axial-vector meson fields with the leptons,

$$G'(m_A{}^2 A^\lambda + m_\rho{}^2 V^\lambda) \times [\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e + \bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu]. \quad (3)$$

Then, by considering the effective nuclear beta decay, we get the relation

$$g_A = f/g, \quad (4)$$

where $g_A \simeq (25/18)^{1/2}$ is the ratio of the axial-vector to vector coupling constants and f and g are the axial-vector and vector coupling constants for strong interactions. The relevant strong-interaction coupling to the baryons is given by

$$\tfrac{1}{2}g\bar{N}\{\gamma^\lambda(\vec{\tau}\cdot\vec{V}_\lambda) + (g'/g)\sigma^{\lambda\nu}(\vec{\tau}\cdot\tfrac{1}{2}\vec{V}_{\lambda\nu}) + (f/g)\gamma^\lambda\gamma_5(\vec{\tau}\cdot\vec{A}_\lambda) + (f'/g)\sigma^{\lambda\nu}(\vec{\tau}\cdot\tfrac{1}{2}\vec{A}_{\lambda\nu})\}N, \quad (5)$$

with

$$f=f'=g'/\sqrt{2}=(25/18)^{1/2}g. \quad (6)$$

This includes the coupling of the vector-isovector ρ meson to the isospin current of the nucleons. There is a pseudovector coupling of the pion to the nucleon which, according to (1) and (5), is

$$(f_1/m_\pi)\bar{N}\gamma^\lambda\gamma_5(\vec{\tau}\cdot\partial_\lambda\vec{\varphi}_\pi)N,$$
$$f_1=\tfrac{1}{2}g_A g\xi. \quad (7)$$

We must also consider a coupling of the vector mesons with pions and the pseudovector mesons. The pion-vector meson coupling is given by

$$g\vec{\rho}_\lambda\cdot(\partial^\lambda\vec{\varphi}_\pi\times\vec{\varphi}_\pi)$$
$$+(g''/m_\rho^{\ 2})\tfrac{1}{2}\vec{\rho}_{\lambda\nu}\cdot\partial^\lambda\vec{\varphi}_\pi\times\partial^\nu\vec{\varphi}_\pi, \quad (8)$$

where g is the same coupling constant as in (5) and g'' is a new dimensionless coupling constant. These couplings induce an effective (nonlocal) s-wave meson-nucleon coupling[5] of the form

$$-(f_2/m_\pi)^2\bar{N}\gamma^\lambda\vec{\tau}N\cdot\partial^\lambda\vec{\varphi}_\pi\times\vec{\varphi}_\pi, \quad (9)$$

with the dimensionless coupling constant

$$f_2^{\ 2}=\tfrac{1}{2}g_0^{\ 2}(m_\pi/m_\rho)^2;\quad g_0=g-\tfrac{1}{4}g''. \quad (10)$$

Scattering length for s waves.—We are now in a position to calculate s-wave scattering lengths. This gets contributions from vector-meson exchange and nucleon-resonance exchange. We obtain

$$a_1=2(\lambda_1-\lambda_0+\lambda_\rho)(1+m_\pi/m)^{-1},$$
$$a_3=-(\lambda_1+2\lambda_0+\lambda_\rho)(1+m_\pi/m)^{-1}, \quad (11)$$

with

$$\lambda_0=(f_1^{\ 2}/4\pi)\{2-\tfrac{4}{3}(f^*/f)^2\}m_\pi^{\ -1},$$
$$\lambda_1=(f_1^{\ 2}/4\pi)(m/m_\pi)$$
$$\times\{2-\tfrac{4}{3}(m^*/m)(f^*/f)^2\}m_\pi^{\ -1},$$
$$\lambda_\rho=(g^2/4\pi)(m_\pi/m_\rho)^2m_\pi^{\ -1}, \quad (12)$$

where m_π, m_ρ, m, and m^* are the masses of the π, ρ, N, and N^*. To get the numerical value of the scattering lengths we must specify the ratio (f^*/f) of the coupling constants of $N^*N\pi$ and $NN\pi$. This can be determined from the width of the N^* and is in good agreement with the "unitarity prediction"[6]

$$(f^*/f)^2=\tfrac{3}{2}. \quad (13)$$

With this choice λ_0 vanishes and the remaining terms lead to the prediction

$$a_3/a_1=-\tfrac{1}{2}, \quad (14)$$

which has been known to be fairly well satisfied experimentally. With the choice

$$g^2/4\pi=6.5,\quad f_1=0.85, \quad (15)$$

we predict

$$a_1=+0.20m_\pi^{\ -1},$$
$$a_3=-0.10m_\pi^{\ -1}. \quad (16)$$

These are to be compared with the experimental values

$$a_1=+0.183m_\pi^{\ -1},$$
$$a_3=-0.109m_\pi^{\ -1}. \quad (17)$$

The value (17) for the vector coupling is somewhat larger than the value estimated from the width of the ρ meson with g'' in (8) put equal to zero. But with $g''=g$, the value (15) yields a ρ-meson decay width of ~120 MeV in agreement with the experimental values. It is to be pointed out that our computation of the s-wave scattering length is not affected by such a term.

Scattering lengths for p waves.—The p-wave scattering length gets contributions from the nucleon and nucleon-resonance exchange. It is simplest to calculate this in the static limit. It has been noted that in this limit, independent of the absolute value of the coupling constant, it is possible to show that

$$a_{13}=a_{31}=\tfrac{1}{4}a_{11}, \quad (18)$$

which is in reasonable agreement with experiment.[7] The nucleon and nucleon resonance exchange together give

$$a_{11}=4a_{13}=4a_{31}=\left(\frac{f_1^2}{4\pi}\right)\left\{-8/3+16/9\left(\frac{f^*}{f}\right)^2\frac{m_\pi}{m^*-m+m_\pi}\right\}\left(1+\frac{m_\pi}{m}\right)^{-1}m_\pi^{-1},$$

$$a_{33}=\left(\frac{f_1^2}{4\pi}\right)\left\{\frac{4}{3}+\left(\frac{f^*}{f}\right)^2\frac{m_\pi}{m^*-m+m_\pi}+\frac{1}{9}\left(\frac{f^*}{f}\right)^2\frac{m_\pi}{m^*-m+m_\pi}\right\}\left(1+\frac{m_\pi}{m}\right)^{-1}m_\pi^{-1}. \quad (19)$$

With the values given by (13) and (15), these expressions give

$$a_{11}=-0.091m_\pi^{-1},$$

$$a_{13}=a_{31}=-0.022m_\pi^{-1},$$

$$a_{33}=+0.133m_\pi^{-1}. \quad (20)$$

These numbers are to be compared with the observed values

$$a_{11}=-0.101m_\pi^{-1},$$

$$a_{13}=-0.029m_\pi^{-1},$$

$$a_{31}=-0.039m_\pi^{-1},$$

$$a_{33}=0.215m_\pi^{-1}. \quad (21)$$

With the exception of the scattering length a_{33} for the resonant channel, the predicted scattering lengths are in good agreement. It is to be expected that a theoretical calculation[8] of the scattering length for a resonant channel may not be satisfactory.

Given the values of the universal strong (vector) coupling constant g and the p-wave coupling constant f_1 we can calculate the pion strength parameter ξ according to the formula

$$\xi=2f_1/f=0.16. \quad (22)$$

This value of ξ could be related to other strong-interaction data, particularly meson mass ratios.

Nucleon-nucleon interaction.—Consider the nucleon-nucleon interaction obtained from the exchange of the pseudoscalar, vector, and pseudovector mesons. The resultant nuclear force consists of components with three distinct ranges. The longest range contributions come from pion exchange which provides both tensor and central potentials. The vector-meson terms yield the intermediate-range potentials; and this includes the leading contribution to the spin-orbit potential. It has been realized that the spin-orbit potential has a shorter range than the central or tensor potentials. The shortest range contributions come from the pseudovector-meson exchange.

Within the framework of the universal primary interaction theory all the strong-interaction coupling ratios are uniquely determined. Hence, given the value of the vector coupling constant the absolute nuclear force can be computed. A detailed study of the two-nucleon interaction and its comparison with experiment will be discussed elsewhere.

Ratio of π and ρ masses.—It is well known that the tensor force from pseudoscalar exchange is singular as r^{-3} at the origin and that such a singularity is not acceptible in a Schrödinger potential. We should therefore seek a cancellation of these singular terms[9] (by a proper choice of the meson masses) by contributions from the vector and pseudovector meson exchanges. With the interaction structure (5) and (6) the pseudovector contribution vanishes; and we obtain the restriction

$$(f\xi/m_\pi)^2-(g/m_\rho)^2=0. \quad (23)$$

Using the value (22) for ξ, we could predict the the ratio of the π and ρ masses to get

$$(m_\pi/m_\rho)=0.188. \quad (24)$$

This is in remarkable agreement with the actual mass ratio

$$(m_\pi/m_\rho)=0.182. \quad (25)$$

Discussion.—We have shown above how the pion-nucleon scattering lengths can be obtained using a single coupling constant. The fundamental idea is that the primary interactions of the hadrons are the couplings of the vector and axial-vector fields with hadrons or with leptons. The strong-interaction scheme itself should be capable of predicting the correct nucleon-nucleon force. In the present theory all coupling constants and coupling types are already specified; hence there are no unknown parameters in the theory. Elsewhere[4] it has been shown that this scheme is able to relate the three universal vector couplings in strong, electromagnetic, and weak interactions, and to predict the electromagnetic properties of the nucleons.

We would like to emphasize that the present work provides an explanation of s and p-wave pion-nucleon scattering lengths in terms of a single input parameter, the universal coupling constant of the isospin current. The decay width of N^* is not a free parameter of the theory, since it can be related to the pseudovector pion-nucleon coupling constant by the requirement of unitarity (see Ref. 6). The resulting pion-nucleon scattering lengths are seen to be in good agreement with experiment. The s-wave sum rule, though known for some time, was recently derived by several authors using current algebra. The cancellation of the $1/r^3$ term in nuclear potential gives rise to a ratio of pi-meson to rho-meson mass which agrees very well with experiments.

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$\Delta T = \frac{3}{2}$ ADMIXTURE OF NONLEPTONIC DECAYS OF K MESONS

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That the $\Delta T = \frac{1}{2}$ rule[1] cannot be a strict selection rule for nonleptonic weak decays is manifested by the presence of the pure $\Delta T = \frac{3}{2}$ decay $K^+ \rightarrow \pi^+ + \pi^0$. Recently, there have been experimental indications[2] that $\Delta T = \frac{3}{2}$ amplitudes are also present in other, predominantly $\Delta T = \frac{1}{2}$, nonleptonic decays of K mesons. In this Letter, we present a simple theoretical model, which satisfactorily correlates all these observed $\Delta T = \frac{3}{2}$ amplitudes. The implications of this model will be briefly discussed in the latter part of this Letter.

In addition to a dominant $\Delta T = \frac{1}{2}$ term $\mathcal{L}_{1/2}$, we assume[3] that there is in the phenomenological weak Lagrangian another term, which is of the current×current form:

$$\mathcal{L}_{\frac{3}{2},\frac{1}{2}} = C[V_\lambda^{(K^+)} + A_\lambda^{(K^+)}]\cdot[V_\lambda^{(\pi^-)} + A_\lambda^{(\pi^-)}] + \text{adjoint}, \qquad (1)$$

where C is a parameter to be phenomenolog-