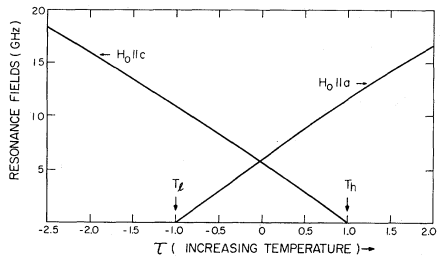

FIG. 3. Fields for resonance parallel to the a and c axes for frequencies small compared with ν_m.

The direction of maximum rf susceptibility for this resonance is the c axis.

The absorption peaks[5] in $TmFeO_3$ displayed all of the characteristics of the resonances described above. The data are schematically reproduced in Fig. 3 by showing Eqs. (7) and (11) as functions of the parameter τ.

The relative magnitudes of the two types of canting fields can be determined from the curves in Fig. 3:

$$H_C \approx \tfrac{1}{5}[(r-1)/(r+1)]H_D, \qquad (12)$$

where $-r$ is the ratio of the slopes of the two curves, and it has been assumed that $H_0 \ll H_D \pm 5H_C$. From the data of Ref. 5 we obtain $H_C \approx 0.09H_D$ for $TmFeO_3$. This should be compared with the result of Ref. 4, $H_C \approx 0.3H_D$, obtained by measuring the change in canting angle from T_l to T_h. Reasons for this disagreement will be discussed elsewhere.[11] However, if H_C were as large as $0.3H_D$, Eq. (11) shows that applied fields greater than $0.5H_D$ would be required to pull the weak moment parallel to the a axis for $T > T_l$. This is inconsistent with the experimental results of Ref. 5.

Finally, from Eqs. (7) and (12) and the value of $H_{cr}{}^c$ at $T = T_l$, the magnitude of the four-fold anisotropy field can be determined if H_E and H_D are known. Using $H_C \approx 0.09H_D$, $H_E \approx 10^7$ G, and $H_D \approx 2\times10^5$ G, we obtain from the data[5] $H_4 \approx 80$ G for $TmFeO_3$.

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[7]For the internal fields assumed, the higher frequency is 791 GHz at $\tau = 0$ and changes by less than 2% for the values of τ and H_0 used here.

[8]The magnitudes of H_E and H_D are reasonably constant for the orthoferrites (Ref. 3). Although H_1 and H_3 may vary considerably from one compound to another, only the difference $H_1 - H_3$ is important for the lower resonance frequency.

[9]The anisotropy constant K_b of Ref. 4 is related to the parameters of the model presented here by $K_b = (M/16H_E)[3H_E H_4 - 4H_C{}^2]$.

[10]The usual phase relationship between the real and imaginary parts of the susceptibility for resonance absorptions was not observed (Ref. 5). This effect may be caused by the superposition of the unresolved lines.

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INFLUENCE OF THE KONDO SCATTERING ON THE AMPLITUDE OF THE de HAAS–van ALPHEN OSCILLATIONS*

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The anomalous amplitude of the de Haas–van Alphen oscillations observed in an alloy exhibiting the Kondo effect[1] has been explained on the basis of a simple square-well energy-dependent conduction-electron relaxation time. Recently the s-d scattering theory of Kondo[2] has given an explicit expression for the energy dependence of the relaxation time which is consistent with the observed resistance minimum. In the present Letter we wish to examine the influence of Kondo scattering on the de Haas–van Alphen effect and to reinterpret the earlier data on the Zn-Mn system.

The relaxation time τ for an electron in a

metal with magnetic impurities can be written as

$$\frac{1}{\tau}=\frac{1}{\tau_0}+\frac{1}{\tau_1}+\frac{1}{\tau_2}, \tag{1}$$

where τ_0 is the normal impurity-scattering relaxation time, τ_1 is a spin-flip scattering time which is energy independent, and τ_2 is the Kondo energy-dependent relaxation time resulting from the s-d mixing. Kondo[2] has shown within the limits of a second-order Born approximation that

$$1/\tau_1 = 3\pi z J^2 s(s+1)c/2E_0\hbar, \tag{2}$$

$$1/\tau_2 = 4Jg(E_k)/\tau_1, \tag{3}$$

where z is the electron-to-atom ratio, s is the spin on the impurity, J is the s-d exchange coupling constant, c is the concentration of paramagnetic ions, and E_0 is the Fermi energy. The function $g(E_k)$ results from the intermediate-spin scattering states due to the s-d mixing and is defined as

$$g(E_k)=(1/N)\sum_q f_q/(E_q-E_k), \tag{4}$$

where N is the total number of atoms in the crystal and f_q is the Fermi distribution function. This function may be evaluated exactly[3]; however, for our purposes it is sufficient to use the low-temperature limit,

$$g(E)\simeq n(0)\ln(|E-E_0|/E_0), \tag{5}$$

where $n(0)$ is the density of electron states at the Fermi energy. For a system of electrons having an ellipsoidal Fermi surface, Dingle[4] gives the following expression for the periodic part of the electron free energy in a magnetic field:

$$F_{\text{per}} = -2\pi(2m^*)^{\frac{3}{2}}kTV\left(\frac{\beta^*H}{2}\right)^{1/2} \times \sum_{p=-\infty}^{\infty} p^{-\frac{1}{2}}(-1)^p e^{-i\pi/4}Q, \tag{6}$$

where

$$Q=\int_0^\infty \ln[1+e^{E-E_0/kT}] \times\exp[-2\pi(\pi k|p|x-ipE)/\beta^*H]dE \tag{7}$$

and $\beta^*=e\hbar/m^*c$ is an effective double Bohr magneton. The quantity x is called the Dingle or collision temperature and is given by

$$x=\hbar/\pi k\tau. \tag{8}$$

When the relaxation time in the Dingle temperature, Eq. (8), includes the Kondo scattering terms, Eq. (1), Q becomes

$$Q=\exp(-2\pi^2 k|p|x_D/\beta^*H)\int_0^\infty \ln[1+e^{E-E_0/kT}] \times\exp[-2\pi(\pi k|p|x_2-ipE)/\beta^*H]dE, \tag{9}$$

where

$$x_D=(1/\tau_0+1/\tau_1)\hbar/\pi k \tag{10}$$

and

$$x_2=\hbar/\pi k\tau_2. \tag{11}$$

The term x_D is independent of energy, but depends in magnitude on the s-d exchange integral J. The term x_2 contains the energy-dependent Kondo relaxation time τ_2 and is essentially zero except in a region kT near the Fermi energy. Thus, with negligible error, integral Q may be split into two parts given by

$$Q=Q_1+Q_2=\exp(-2\pi^2 kx_D/\beta^*H)\int_0^\infty \ln[1+e^{E-E_0/kT}]e^{2\pi ipE/\beta^*H}dE + \exp[-2\pi(\pi k|p|x_D-ipE_0)/\beta^*H]\int_0^\infty \ln[1+e^{E-E_0/kT}]e^{-2\pi^2 kx_2/\beta^*H}dE, \tag{12}$$

where the second integral is significant only in the region of E_0. The integral Q_1 when combined with Eq. (6) leads to Dingle's expression for the free energy of electrons having an energy-independent relaxation time,

$$F_{\text{per}} = 2\pi(2m^*)^{3/2}\frac{kTV(\beta^*H)^{\frac{3}{2}}}{\sqrt{2}h^3}\sum_{p=1}^{\infty}\left[\frac{(-1)^p\cos(2\pi pE_0/\beta^*H-\pi/4)}{p^{3/2}\sinh(2\pi^2 pkT/\beta^*H)} \times\cos[p\pi(m^*/m)]\exp(-2\pi^2 kx_D/\beta^*H)\right]. \tag{13}$$

The term Q_2 represents the shift in the free energy due to the addition of magnetic impurity spin scattering. Using Eqs. (3) and (5) and putting $y=(E-E_0)/kT$, Q_2 becomes

$$Q=2kT\exp[-2\pi(\pi kx_D-ipE_0)/\beta^*H]\left(\frac{kT}{E_0}\right)^{\gamma}\int_0^{\infty}y^{\gamma}\ln[1+e^{\gamma}]dy, \tag{14}$$

where[5]

$$\gamma=12\pi^2n(0)|J|^3s(s+1)c/E_0\beta^*H. \tag{15}$$

After integration by parts, the resulting integral may be expressed in terms of the gamma and Riemann ζ functions. The shift in the free energy f can be calculated by substituting the value of Q_2 into Eq. (6), giving

$$f=\frac{4\pi V(2m^*)^{3/2}}{h^3\sqrt{2}}\left(\frac{\beta^*H}{2}\right)^{3/2}\sum_{p=1}^{\infty}p^{\frac{1}{2}}\cos(p\pi m^*/m)\cos(2\pi pE_0/\beta^*H-\pi/4)e^{-2\pi^2kx_0/\beta^*H}\left(\frac{kT}{\beta^*H}\right)G(\gamma), \tag{16}$$

where[6]

$$G(\gamma)=(kT/E_0)^{\gamma}(1-2^{-(1+\gamma)})\Gamma(1+\gamma)\zeta(2+\gamma). \tag{17}$$

Adding this term to Dingle's result, Eq. (13), the free energy of a system of electrons exhibiting the Kondo effect becomes

$$F_{\text{per}}'=\frac{4\pi(2m^*)kTV(\beta^*H)^{\frac{3}{2}}}{h^3\sqrt{2}}\sum\frac{(-1)^p\cos(2\pi pE_0/\beta^*H-\pi/4)}{p^{3/2}\sinh(2\pi^2pkT/\beta^*H)}$$

$$\times\cos(p\pi m^*/m)\exp(-2\pi^2kx_D/\beta^*H)\left\{1+\frac{2pkT\sinh(2\pi^2pkT/\beta^*H)G(\gamma)}{\beta^*H}\right\}. \tag{18}$$

In the above expression the effect of the Kondo scattering is accounted for by the term $(2pkT/\beta^*H)\sinh(2\pi^2pkT/\beta^*H)G(\gamma)$. The remaining energy-independent scattering terms are included in the effective Dingle temperature x_D defined by Eq. (10) and should be temperature independent. Equation (18) has been used to reinterpret the previously published data on a Zn-0.008-wt% Mn alloy which exhibits a Kondo effect. This has been done by fitting Eq. (18) to the data using Eq. (15) with γ as an arbitrary parameter.[7] The results of this calculation, shown in Fig. 1, indicate, as expected, that the effective Dingle temperature is independent of temperature. Since γ is related to the spin S of the manganese ion and the s-d exchange coupling constant J by Eq. (15), a value of $J=-0.31$ eV was estimated from the values of γ found in the fitting procedure assuming $S=\frac{3}{2}$.[8]

The effective Dingle temperature is also related to the exchange coupling constant by Eqs. (10) and (2), yielding

$$x_0=x_0+3z|J|^2s(s+1)c/2kE_0, \tag{19}$$

where x_0 is the Dingle temperature associated with the nonmagnetic-scattering processes which has been taken as 1.3°K from measure-

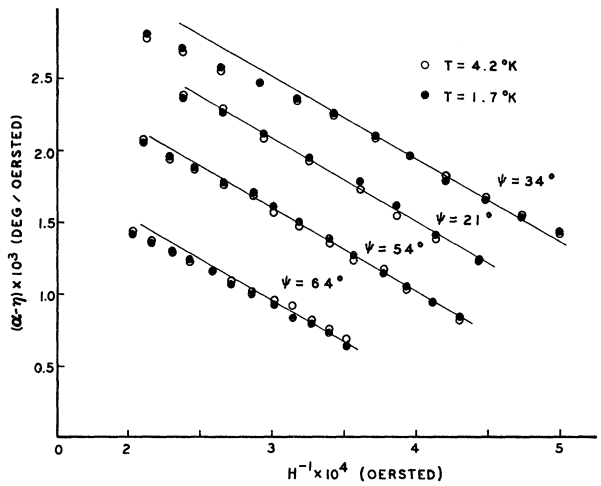

FIG. 1. The field dependence of the amplitude of the de Haas–van Alphen oscillations of the third-zone needle Fermi surface in a zinc–0.008-wt% Mn alloy. The quantities α and η are given by $\alpha=(\beta^*/2\pi^2k)\ln\{(|C|/H^{1/2}T)\sinh(2\pi^2kT/\beta^*H)\}$ and $\eta=(\beta^*/2\pi^2k)\ln\{1+G(\gamma)2(kT/\beta^*H)\sinh(2\pi^2kT/\beta^*H)\}$, where $|C|$ is the amplitude of the torque oscillations and can be derived from Eq. (18) by the method outlined in Ref. 1. The slope of this plot gives $-x_D$. The deviations at high field are due to the onset of magnetic breakdown. ψ is the angle between the field H and the c axis of the zinc crystal.

ments on pure zinc.[1] Using the value of 5.4°K found from Fig. 1 and assuming $S = \frac{3}{2}$, a J value of -0.10 eV is calculated. This smaller value of J is possibly the result of neglecting the progressive inhibition of spin-flip scattering with increasing magnetic field. Béal-Monod and Weiner[9] have shown that this effect is felt largely in the second-order magnetic-scattering term and has relatively little effect on the third-order Kondo term which appear as x_D and γ, respectively, in the present calculation.

A value of J was also estimated from the resistivity data obtained on the same material used in the de Haas–van Alphen experiments. If the resistivity arising from the impurity potential scattering is assumed to be the same size as that coming from the temperature-independent magnetic scattering then J is -0.28 eV.[10] This is in good agreement with the de Haas–van Alphen result of -0.31 eV obtained from the fitted value of γ and substantiates the proposition that the magnetic field most strongly influences the temperature-independent, second-order, spin-flip scattering.

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[5]Since the magnetic interaction in alloys exhibiting the Kondo effect is known to be antiferromagnetic, the J value is taken as $-|J|$.

[6]The major variation in $G(\gamma)$ as a function of γ comes from the term $(kT/E_0)^{\gamma}$. The remainder of the expression can be taken as 0.78 within ±2% for $0 < \gamma < 0.5$.

[7]γ is arbitrary in that it was fitted for one experimental point at each orientation ψ. The γ values for the other points were calculated using the experimental values of H, (m^*/m), and Eq. (15).

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LOCAL TEMPERATURE-GRADIENT CONTRIBUTION TO FLUX-FLOW VISCOSITY IN SUPERCONDUCTORS*

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A normal-core model of type-II superconductors is used to calculate a contribution to the flux-flow viscosity arising from local temperature gradients in the core vicinity. The flux-flow resistivity versus temperature, according to this model, exhibits a minimum for impure, but not for pure, superconductors.

We consider here a consequence of the possibility that the motion of quantized vortex lines or of magnetic flux-containing normal domains through a superconductor produces local temperature gradients in the vicinity of the normal-like regions. The following model calculation indicates that under certain conditions the dissipation associated with these local temperature gradients may be a significant fraction of the total dissipation. The temperature dependence of the resulting viscosity coefficient suggests that the thermal-gradient contribution is responsible for the minimum in the flux-flow resistivity versus temperature observed[1] for impure type-II superconductors.

We wish first to calculate the temperature distribution in the vicinity of an isolated, moving vortex line or normal domain. A convenient model to choose is that of a cylindrical normal core of radius a, oriented parallel to the z axis and moving in the y direction at constant speed v through an infinite superconductor.