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SEARCH FOR THE DIRECT DECAY $K^+ \to \pi^+ + \gamma + \gamma$ †

M. Chen, D. Cutts, P. Kijewski, R. Stiening, and C. Wiegand
Lawrence Radiation Laboratory, University of California, Berkeley, California

and

M. Deutsch
Massachusetts Institute of Technology, Cambridge, Massachusetts
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The measurement of the branching ratio for the direct decay $K^+ \to \pi^+ + \gamma + \gamma$, where the invariant mass of the two gamma rays is not equal to the π^0 mass, has considerable theoretical importance. We have experimentally established an upper limit of 1.1×10^{-4} for this branching ratio. We have used our results to set a limit on the off-the-mass-shell variation of the $K^+ \to \pi^+ + \pi^0$ amplitude.

As a part of our Bevatron spark chamber study of K^+ decays at rest, we have searched for examples of the $K^+ \to \pi^+ + \gamma + \gamma$ decay in which the kinetic energy of the π^+ was between 60 and 90 MeV. Our apparatus is shown in Fig. 1. We recognized candidates for the decay $K^+ \to \pi^+ + \gamma + \gamma$ by the relation of the conversion points of two gamma rays to the momentum of a charged particle which was assumed to be a pion. The momentum of the pion was determined from its range. We have observed 29 events which are acceptable candidates for the decay. A study of the background, which is due to $K^+ \to \mu^+ + \pi^0 + \nu$ events, indicates that there should be 30 ± 3 background events among the candidates. The excess of the observation over the background prediction is -1 ± 6 events. We have based our branching ratio limit on the assumption that there are fewer than 11 events in our sample. As we do not observe all of the phase space available to the reaction, the total branching ratio we determine is somewhat dependent on the model that we assume for the decay. We shall discuss some of the various possibilities.

One possible interpretation of our results is a limit on the off-mass-shell behavior of the $K^+ \to \pi^+ + \pi^0$ amplitude. The mechanism causing the decay $K^+ \to \pi^+ + \pi^0$ has been the subject of considerable theoretical speculation. If the two-pion final state were a pure isospin-2 state, and if the weak interaction satisfied the $|\Delta T| = \frac{1}{2}$ law, this decay would be forbidden. Indeed, the amplitude for this decay is about 20 times smaller than the amplitude for the decay $K_S^0 \to 2\pi$ which does not violate the $|\Delta T| = \frac{1}{2}$ selection rule.

It has been suggested that the $K^+ \to \pi^+ + \pi^0$ decay may occur because the 2π final state may not be a pure isospin state on account of the π^+-π^0 mass difference. In this picture the weak

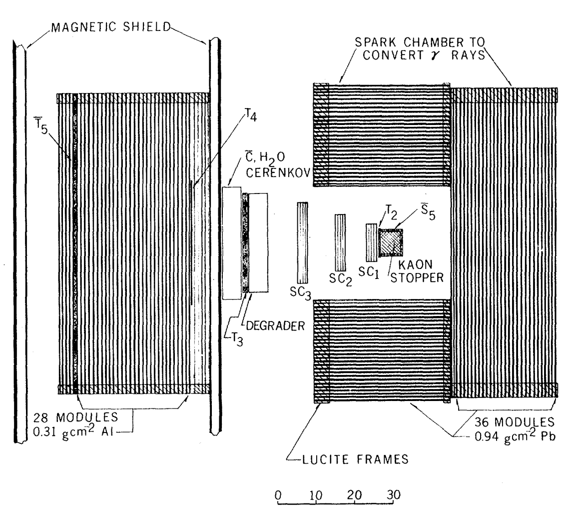

FIG. 1. The apparatus used in this experiment. K^+ mesons entered the stopper in a direction perpendicular to the plane of the diagram.

interaction is thought to obey the $|\Delta T| = \frac{1}{2}$ law exactly. We can use our results on the rate of $K^+ \to \pi^+ + \gamma + \gamma$ to test this hypothesis. If we imagine that the two gamma rays constitute a virtual π^0 intermediate state, the mass difference between the π^+ and this virtual π^0 is much greater than the real $\pi^+ - \pi^0$ mass difference. It follows that the effect suggested above as a mechanism causing the decay $K^+ \to \pi^+ + \pi^0$ would be greatly enhanced. We would therefore expect a large rate for $K^+ \to \pi^+ + \gamma + \gamma$. This model has been suggested by many authors.[1-4]

In order to describe our results, we shall assume that the decay $K^+ \to \pi^+ + \gamma + \gamma$ is due to a single process in which there is a π^0 intermediate state. We shall assume that the amplitude for $\pi^0 \to 2\gamma$ does not vary off the mass shell, and that the amplitude for $K^+ \to \pi^+ + \pi^0$ varies in the following manner:

$$M(q^2) = M(m_{\pi^0}^2)\left[1 - \xi \frac{q^2 - \bar{m}_{\pi^0}^2}{m_{\pi^0}^2}\right].$$

In this expression, $M(m_{\pi^0}^2)$ is the on-mass-shell amplitude for the decay $K^+ \to \pi^+ + \pi^0$; $\bar{m}_{\pi^0}$ is the complex mass of π^0, $\bar{m}_{\pi^0} = m_{\pi^0} + \frac{1}{2} i \Gamma(\pi^0 \to 2\gamma)$. q is the invariant mass of the two gamma rays.

Cabibbo and Gatto[1] and Fujii[3] have related the parameter ξ to the amplitude for the decay $K_S^0 \to \pi^+ + \pi^-$. Fujii finds that

$$|\xi| \cong \frac{M(K^0 \to \pi^+ + \pi^-)}{M(K^+ \to \pi^+ + \pi^0)} \cong 20.$$

In order to compare our results with this prediction, we have calculated the differential spectrum of the π^+ energy in the decay $K^+ \to \pi^+ + \gamma + \gamma$ for various values of ξ. The result is as follows:

$$\frac{1}{\Gamma(K^+ \to \pi^+ + \pi^0)} \frac{d\Gamma(K^+ \to \pi^+ + \gamma + \gamma)}{dE_{\pi^+}} = \frac{2}{\pi} \frac{M_K}{m_{\pi^0}^3} \Gamma(\pi^0 \to 2\gamma) \frac{P_{\pi^+}}{P_\theta} q^4 \left| \frac{1}{q^2 - m_{\pi^0}^2 + \frac{1}{4}\Gamma(\pi^0 \to 2\gamma)^2 - i m_{\pi^0} \Gamma(\pi^0 \to 2\gamma)} - \frac{\xi}{m_{\pi^0}^2} \right|^2.$$

In this expression P_θ is the momentum of the π^+ in the decay $K^+ \to \pi^+ + \pi^0$. For the purpose of comparing this calculation with our experimental results we have assumed that $\Gamma(\pi^0 \to 2\gamma) = (7.4 + 0.15) \times 10^{-6}$ MeV as given in the table of Rosenfeld et al.[5]

The upper limit of 11 events in our sample, combined with the number of stopping K^+ mesons and the experimental efficiencies, implies that $|\xi| < 30$. Our experiment therefore is not sensitive enough to rule out the prediction of Fujii.[3]

As another method of analysis, we have considered a model in which the two gamma rays result from the decay of a spin-zero, positive-parity intermediate state, as has been suggested by various authors.[6-8] We shall refer to this state as a σ meson.

In order to compare our results with this model we have computed the fraction of σ-meson-mediated $K^+ \to \pi^+ + \gamma + \gamma$ decays that would be detected by our apparatus. In making this calculation we have used a Breit-Wigner function:

$$\frac{d\Gamma(K^+ \to \pi^+ + \gamma + \gamma)}{dE_{\pi^+}} = \lambda \frac{P_{\pi^+} q^4}{(q^2 - m^2 + \frac{1}{4}\Gamma_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma^2}.$$

m_σ is the mass of the σ, assumed to be 400 MeV, and Γ_σ is its width, assumed to be 100 MeV. λ is a constant. Our result, which is relatively insensitive to the assumed σ-meson parameters, is that if the decay $K^+ \to \pi^+ + \gamma + \gamma$ proceeds via a σ-meson intermediate state, the total branching ratio of the K^+ meson into this channel is less than 3.3×10^{-4}. If, instead, we use the parameters $m_\sigma = 700$ MeV, $\Gamma_\sigma = 100$ MeV, our results limit the total branching ratio to be less than 1.8×10^{-4}.

Finally, if we assume that the hypothetical decay $K^+ \to \pi^+ + \gamma + \gamma$ is governed by a phase-space model, that is, if the distribution of the π^+ is

$$d\Gamma(K^+ \to \pi^+ + \gamma + \gamma)/dE_\pi = \lambda P_{\pi^+},$$

where λ is a constant, our experimental result limits the total branching ratio of the K^+ into this channel to be less than 1.1×10^{-4}.

The vector-meson-dominant model[8,9] and the η-pole model[10] both predict branching ratios for $K^+ \to \pi^+ + \gamma + \gamma$ that are much lower than the upper limits which we have been able to set in this experiment.

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POSITIVE DEFINITENESS OF GRAVITATIONAL FIELD ENERGY*

D. Brill
Sloane Physics Laboratory, Yale University, New Haven, Connecticut

and

S. Deser
Department of Physics, Brandeis University, Waltham, Massachusetts

(Received 20 September 1967)

The total gravitational field energy functional is shown to have only one extremum under variation of the metric field variables. At the extremum the energy vanishes and space is flat; second variation shows that the vacuum state is also a local minimum.

It has been increasingly recognized in recent years that the gravitational field, as described by general relativity, shares many of the basic physical properties of Lorentz-covariant field theories, particularly of massless gauge systems. Thus, asymptotically flat spaces, which describe isolated physical systems, can be assigned a well-defined total energy-momentum P^μ, satisfying the physical requirement that P^μ is covariant under Lorentz transformations and invariant under interior coordinate transformations.[1] The fact that the field is self-coupled, i.e., that gravitational field energy is itself a source of the gravitational field, is reflected in the nonlinear nature of the Einstein equations, particularly of the constraints that determine the energy. This is also the reason that the following fundamental problem had resisted solution until now[2]: Is the total energy of the gravitational field positive definite?[3] The difficulty of this question is due to the implicit nature of the Hamiltonian of the theory—it can be given explicitly only as an infinite series in the metric field variables. On the other hand, an affirmative answer is clearly essential for a satisfactory physical interpretation of the theory.

In this Letter we regard the energy as a functional of the gravitational field variables and consider its variational properties under change of geometry. We thereby establish that the functional has only one extremum, flat space. Further, the second variation about this "point"—the energy of a weak field—is shown to be positive. From these results, and to the extent that our functional behaves like a function of a finite number of variables,[4] the positiveness problem is resolved in the affirmative: The vacuum (absence of any field excitations) is the lowest energy state, and, as a corollary, vanishing energy implies flatness. We restrict ourselves here to the source-free field, but the proof holds also in the presence of normal matter sources that have positive energy and are minimally coupled to gravitation. We consider here only "nonpathological" systems,