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[13]This is not a real restriction since one could not diagonalize the particle parameters if the representation were not fully reducible. We pretend that α is discrete in the text; it can of course be continuous.

[14]This is the spirit of the game; we want the "Born approximation" to the form factors, $\bar{u}(\vec{p}'\lambda'n's')\Gamma_0 u(\vec{p}\lambda ns)$, to be more realistic than in the simple quark model using ordinary fields; for example, it should have nonkinematical singularities in $t=[p(n)-p'(n')]^2$.

[15]If Γ_μ is assumed to be p dependent, we have nothing to say, except that then there is no known technique for finding sets of u's satisfying condition (IV).

[16]More generally, we must have (symbolically)

$$\sum_n u(\vec{p}n)\bar{u}(\vec{p}n)\Gamma_0/\mathcal{E}_n + X(\vec{p}) = 1,$$

where as $\vec{p}\to\infty$, $\bar{u}\Gamma_0 X u \to 0$. It is easy to show that $X(\vec{p})$ cannot be identically zero, and the choice in the text is clearly the most obvious one.

[17]I. M. Gel'fand, R. A. Minlos, and Z. Ya. Shapiro, Representations of the Rotation and Lorentz Groups and their Applications (Pergamon Press, New York, 1963).

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[21]In particular, an equation of the form $(\gamma\cdot p-\mathfrak{M})\psi=0$, where γ_μ is the Dirac four-vector and ψ transforms under the direct product of the Dirac and any unitary representation ($\gamma_0\mathfrak{M}$ Hermitian, of course), must have spacelike solutions unless $\mathfrak{M}=m_0$, a number. This result was conjectured by Abers, Grodsky, and Norton, Ref. 12, and apparently contradicts a conclusion of Ref. 1. (We see no reason why the positive-energy spacelike solutions should decouple from all the positive-energy timelike solutions as $p_z\to\infty$.)

DIRECT PRODUCTION OF MUONS IN COSMIC RAYS?

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In a recent paper Bergeson et al.[1] have described measurements of the intensity of cosmic-ray muons as a function of zenith angle and depth underground. The results are of considerable interest in that they show almost no dependence of intensity on zenith angle, in strong contradiction to the $\sec\theta$ enhancement expected if the muons derive from pions and kaons. Bergeson et al. conclude that the majority of cosmic-ray muons of energy above 1000 GeV are produced either directly or as decay products of very short-lived secondaries ($\tau \ll 10^{-8}$ sec), and their published results indicate that direct production is dominant as low as 500 GeV.

The purpose of the present work is to examine other experimental data relevant to this conclusion, in particular, the direct measurements of near vertical and inclined muon spectra at ground level by the authors' research groups. Some of the data to be presented have been published previously and some are given here for the first time.

The early work of Barrett et al.[2] and Jakeman[3] showed that the nature of the secondaries of high-energy interactions, whose decay gives rise to the ground-level muons, could be examined by studying the relative intensities of near-vertical and inclined muons. Under the assumption that the parents of muons are a mixture of pions and kaons, the ratio of kaons to pions produced in high-energy interactions has been determined by the Durham and Nottingham groups from measurements of the momentum spectrum of muons as a function of zenith angle. The ratio has been estimated to be in the region of 20% for interactions giving muons at sea level having energies up to about 1000 GeV (see, for example, Judge and Nash,[4] Ashton et al.,[5] and MacKeown et al.[6]). The conclusion that this ratio is small arises because the muon spectrum at large zenith angles is close to what would be expected assuming that the parents of the muons are only pions. Implicit in the calculation of the expected spectrum is the assumption that the spectrum of near-vertical muons is known to fair accuracy; in point of fact, direct measurements are only available to about 400 GeV, above which the intensities have been inferred from measurements of the variation of muon intensity with depth underground. This point will be referred to again later.

Clearly, if muons are produced directly instead of by way of pions (or a mixture of kaons and pions), the $\sec\theta$ term which comes from the enhanced π-μ (or K-μ) decay probability at large zenith angles will be absent, and the expected intensity at large zenith angles will be considerably reduced, this reduction being greater as the zenith angle increases. Calculations have been made by the present authors for a mean zenith angle of 87°, and the results are shown in Fig. 1. (It is seen that the value of $R=0.1$ which corresponds to no $\sec\theta$ enhancement is only reached asymptotically at very high energies.) Comparison is made with the summarized results from the Durham horizontal spectrograph,[7] covering the range 85°-90°, and from the Nottingham

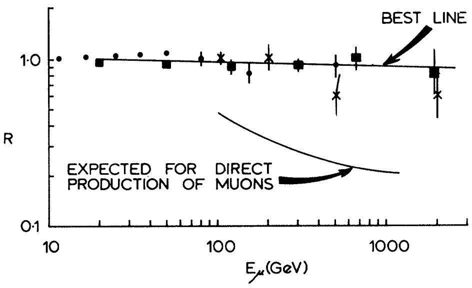

FIG. 1. Ratio of intensity at $\bar{\theta}=87°$ to that expected for pions alone as parents. Circles, Nottingham data (W. F. Nash and N. Palmer, to be published); squares, Durham data (Said, Ref. 7); crosses, Durham data (MacKeown et al., Ref. 6, included in Said, Ref. 7).

spectrograph for the range 75°-80°.[8]

The Durham results include the earlier data reported by MacKeown et al.[6] shown dotted in Fig. 1, and it will be seen that the suggestion of a reduction in the ratio above 500 GeV was not confirmed by the later measurements.

It is apparent that if the assumptions made in our calculations are correct, then the results do not support the suggestion of direct production of muons, at least for secondary muon energies below several thousand GeV. This is at variance with the conclusions of Bergeson et al., since they find virtually complete direct muon production as low as 500 GeV (for zenith angles in the range 40°-50°). The most important uncertainty in our analysis is probably the adopted vertical muon spectrum. If this were underestimated seriously in the region above 400 GeV, then the points on the figure would drop accordingly. However, a factor of 5 increase in the vertical intensity could not be achieved without destroying the near agreement between the vertical sea-level intensity inferred from the depth-intensity measurements and those derived by Krasilnikov,[9] Higashi et al.,[10] Barog et al.,[11] and more recently by Ozaki et al.[12] (It should be remarked that there is the well-known, anomalously flat, near-vertical muon spectrum, derived by Vernov et al.[13] from burst measurements. This spectrum taken with our inclined measurements would be consistent with virtually complete, direct muon production at 1000 GeV and, indeed, these authors suggested direct production as a possible explanation of their results. However, the results of Vernov et al. are clearly inconsistent with all other burst experiments.)

The only remaining source of error concerns the inclined measurements themselves. At high momenta the spectrograph measurements become increasingly uncertain due largely to Coulomb scattering in the solid iron magnets causing a spill-over of particles into neighboring momentum groups. This means that there is conceivably an additional error as 1000 GeV is approached. Furthermore, above 1000 GeV there is only the one inclined intensity measurement from the Durham results, and this cannot be regarded as having been checked in any way. Thus, we can say very little from our experiments about the behavior of muons above 1000 GeV.

In the range 500-1000 GeV, however, where Bergeson et al. find a marked inconsistency with expectation on the basis of pions as parents, the spectrum measurements are at variance. Furthermore, from the underground results one might expect a transition region below 500 GeV where direct muon production is partly responsible, but here the inclined muon data (and the vertical measurements) are thought to be reasonably firm and there is no evidence for any significant direct muon production.

We conclude that the inclined muon-spectrum measurements do not support the suggestion of an appreciable fraction of the muons of energy below 1000 GeV having been produced directly.

Further direct measurements of muon spectra as a function of angle and to higher energies are extremely desirable to resolve this problem. In addition, measurements of the charge ratio would be particularly valuable, since it is very unlikely that the measured ratio at energies to several hundred GeV (1.25) would persist if direct muon production is important.

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