DIRECT-CHANNEL RESONANCES FROM REGGE-POLE EXCHANGE*

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(Received 1 February 1968)

The ρ Regge amplitude for πN charge-exchange scattering with helicity flip is partial-wave analyzed in the direct channel. This produces resonance circles on the Argand diagram corresponding to the prominent experimental N^* resonances.

We consider the ρ Regge amplitude for πN charge-exchange scattering with helicity flip B_{cex}. The usual parametrization and fit to the high-energy data, $p_L = 5.8\text{-}18.2$ BeV/c, is[1]

$$B_{\text{cex}} = \beta\left(\frac{E}{E_0}\right)^{\alpha-1} \frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha\,\Gamma(\alpha)}, \tag{1}$$

where $\alpha(t) = 0.57 + 1.08t$, and E is the laboratory energy of the π. Choosing $E_0 = 0.7$ BeV and $\beta(t) = \text{const} = 60.3$ mb, one correctly fits the relative height of the near forward peak and the secondary peak. We now extrapolate this expression down to $p_L = 1.7$ BeV/c and check that it roughly fits $d\sigma/dt$.[2] Note in particular that the dip at $t \approx -0.6$ BeV^2 persists down to this energy and that the magnitudes of the near-forward peak and of the secondary peak are correctly given by the extrapolation of the high-energy fit.

Our partial-wave analysis of $B(E, z)$ in the direct channel gives

$$B_l(E) \equiv \tfrac{1}{2}\int_{-1}^{+1} dz\, P_l(z) B(E, z). \tag{2}$$

We integrate out to 180°, but we ignore the small backward peak coming from N exchange. Furthermore, if we want to use $\beta(t)$ in (1) only for those t values which have been measured,[3] $|t| \leq 3.0$ BeV^2, then we should stay below $p_L \approx 2.0$ BeV/c. On the other hand, we should stay above $p_L \approx 1.7$ BeV/c in order to have (1) give a rough fit to the $d\sigma/dt$ data. Actually, we let p_L vary beyond these limits and consider $1.0 \leq p_L \leq 2.4$ BeV/c.

Because $\beta(t)$ is constant it is irrelevant for most of the following considerations, and we put $\beta = 1$. The complex amplitude B_l, calculated from (1) and (2), is shown in Figs. 1(a), 1(b), and 1(c) for fixed l vs p_L. Surprisingly, the B_l describe circles on the Argand diagram. Such circles are usually associated with resonances. We read off the momentum p_L at which B_l reaches the top of the circle, and in Fig. 2 we plot these "resonance" momenta versus l and compare them with the experimental resonances (1920, 2190, 2420). In Fig. 1(d) we show B_l for fixed p_L as a function of l. Such a circle corresponds to an N^* Regge pole and indicates that the N^* resonances plotted in Fig. 2 actually belong to an N^* Regge trajectory.

Let us understand qualitatively why we obtain circles. (1) The zeros of ImB at $\alpha = 0, -1, -2$, etc. and the double zeros of ReB at $\alpha = 0, -2, -4$, etc. are crucial. Consider, for exam-

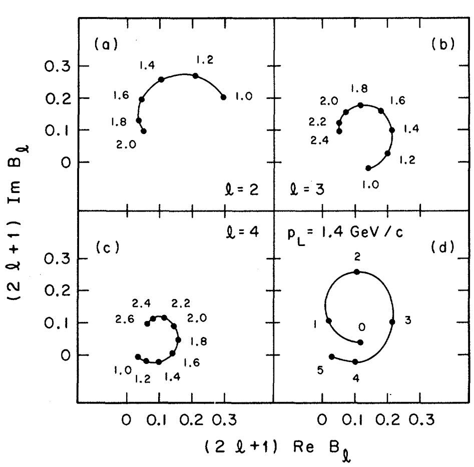

FIG. 1. (a)-(c) The complex amplitude $B_l(p_L)$ for fixed l versus the laboratory momentum p_L. The numbers along the circles give the values of p_L. (d) $B_l(p_L)$ for fixed p_L vs l. The numbers along the circle give the values of l. For normalization we have put $\beta = 1$.

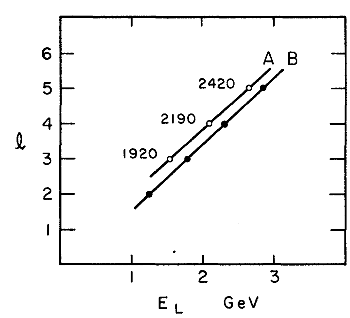

FIG. 2. Chew-Frautschi plot. Curve A is the established πN resonances and curve B is the N^* resonances from our partial-wave analysis of the ρ Regge exchange. E_L = lab energy of $\pi \approx p_L$. E_L is linear in $(M_{\text{res}})^2$.

ple, Fig. 1(d). For $p_L = 1.4$ we have $\alpha = -1.5$ at 180^0. Therefore ImB contains two zeros in the physical region, $-1 \leq z_s \leq +1$, and the $l = 2$ wave becomes particularly strong. (2) The signature factor $(1-e^{-i\pi\alpha})$ induces the correct counterclockwise motion on the Argand diagram because the phase of $B(E, t)$ moves counterclockwise as $(-t)$ increases, and the partial-wave amplitude $B_l(E)$ includes larger $(-t)$ values at higher energies. (3) The arguments given in (1) and (2) are mainly relevant at intermediate energies, $p_L \leq 3.0$ BeV/c, since at high energies the secondary peak is so much suppressed that the presence of the nodes becomes less relevant.

Are the circles in Fig. 1 really resonances? The problem is that (1) and (2) are regular at $s = m_{\mathrm{res}}^2 - i(m\Gamma)_{\mathrm{res}}$; they do not contain poles corresponding to resonances. Let us give a mathematical analog. Take $\psi(z)$, the logarithmic derivative of the Γ function. It has poles (resonances) at $z = 0, -1, -2, \cdots$. The asymptotic representation (Regge representation) is the Stirling expansion. ψ_{asy} is a good approximation to ψ for $|z| \to \infty$ as long as we exclude a wedge $|\arg z| < \pi - \epsilon$. But ψ_{asy} does not contain the poles at $z = 0, -1, -2, \cdots$; the approximation breaks down if we penetrate the wedge. Similarly the Regge asymptotic form B_{asy} is a good approximation to the full amplitude B for real s. Both amplitudes contain the resonance circles. But if we go below the physical axis the approximation breaks down, and B_{asy} does not contain the resonance poles. In a sense we are in a situation similar to that of the phase-shift analyst. We must stick to real energies, and if we see a circle, we never know with absolute certainty whether or not the extrapolation to Im$s < 0$ would lead to a pole.

We now make the comparison of our circles with the established N^* resonances more quantitative, and we clarify the meaning of the parameter l. The connection between B_l and the partial waves $f_{l\pm}$ is[4]

$$\frac{1}{4\pi}B_l = \sum_{n=0}^{\infty}\left[\frac{1}{E+M}\left(f_{(l+2n)+} - f_{(l+2+2n)-}\right) + \frac{1}{E-M}\left(f_{(l+1+2n)-} - f_{(l+1+2n)+}\right)\right]. \quad (3)$$

For our comparison we approximate the right-hand side of this equation by the resonances,[5] we leave out the low-energy tails of high-energy resonances, corresponding to $n \neq 0$, and we neglect the term with $(E+M)^{-1}$. This reduces (3) to

$$\frac{1}{4\pi}B_l \approx \frac{1}{E-M}\left[f_{(l+1)-} - f_{(l+1)+}\right]. \quad (4)$$

Note that the parameter l in B_l mainly corresponds to an orbital angular momentum of $(l+1)$. Using the established resonances, we obtain from (4)

$$\frac{1}{4\pi}B_3^{\mathrm{cex}} \approx \frac{\sqrt{2}}{3}\frac{1}{E-M}f_{2190}, \quad (5)$$

$$\frac{1}{4\pi}B_4^{\mathrm{cex}} \approx \frac{\sqrt{2}}{3}\frac{1}{E-M}f_{2420}, \quad (6)$$

$B_2^{\mathrm{cex}} \approx \cdots (f_{1920} + f_{1688})$. From Fig. 1 we read off the widths $M\Gamma = (0.7, 0.8\ \mathrm{GeV}^2)$ for $l = (3, 4)$. The corresponding experimental N^* widths are $M\Gamma = (0.44, 0.67\ \mathrm{GeV}^2)$. The elasticities are determined by the height of the circles in Fig. 1, and using $\beta = 60.4$ mb and (5) and (6) we obtain $M\Gamma x = (0.12, 0.09\ \mathrm{GeV}^2)$. This agrees well with Rosenfeld's values $M\Gamma x = (0.13, 0.07\ \mathrm{GeV}^2)$.

Let us compare our procedure with the N/D scheme.[6] Both times one uses as an input the particles in the crossed channel (forces), in the N/D scheme elementary ρ exchange, in our case Regge ρ exchange. One computes the contribution of this exchange to a definite partial wave in the direct channel $a_l(s)$. In our scheme this already produces the direct-channel resonances. In the N/D scheme this only gives a real Born amplitude, and the additional step of unitarization generates the resonance. In contrast, the Regge ρ exchange amplitude is automatically unitary, since it roughly represents the full amplitude. For the same reason it automatically includes absorption.

We have shown (Horn et al.[7]) how finite-energy sum rules (FESR) imply that direct-channel resonances are, on the average over the low- and intermediate-energy region, already contained in the Regge amplitude of the crossed channel. The present Letter shows that the equivalence between t-channel Regge poles and s-channel resonances does not only hold on the average, but even locally at each intermediate energy. (At low energies the equivalence must break down; the resonances no longer overlap.) The equivalence $B_{\rho\mathrm{Regge}} \approx B_{N^*\mathrm{res}}$ suggested by Fig. 1 shows that the interference model,[8] which puts $B \approx B_{\mathrm{Regge}} + B_{\mathrm{res}}$ in the intermediate energy region, involves severe double counting.

We do not know yet whether this equivalence is a very general feature of strong interactions. But we might mention one additional example, $K^{\pm}p$. Assume for the moment that the Pomeranchon is a special case with $\alpha_P(t)\equiv 1$, and that for purposes of describing $K^{\pm}p$ we can lump together all odd-signature meson trajectories (ρ, ω, φ) into one trajectory X, and all even signature trajectories (A_2, f, f') into Y. We further assume that X and Y are exchange degenerate.[9] If we now partial-wave analyze the Regge amplitudes, we find that the Pomeranchon does not generate resonance circles because it is purely imaginary. The meson signature factor in K^+p factor is real, $(+1-e^{-i\pi\alpha})_X -(-1-e^{-i\pi\alpha})_Y=2$, and we cannot obtain resonances. On the other hand, for K^-p the phase of the Regge amplitude rotates, $(+1-e^{-i\pi\alpha})_X +(-1-e^{-i\pi\alpha})_Y=-2e^{-i\pi\alpha}$, the imaginary part has zeros at $\alpha=0, -1, -2, \cdots$, and the real part has zeros at $\alpha=\cdots, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \cdots$. Therefore we expect that resonances will be generated in K^-p.

The argument in the opposite direction is well known.[9] No resonances in K^+p (and pp, KK) means that there are only direct forces, but no exchange forces, in $K\bar{K}\to p\bar{p}$ (and $p\bar{p}\to p\bar{p}$, $K\bar{K}\to K\bar{K}$). Therefore (ρ, ω, φ) are "exchange degenerate" with (A_2, f, f'), and the total cross sections in K^+p and K^+n must be constant and equal.

*This work was supported in part by the U. S. Atomic Energy Commission.

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TEST OF TIME-REVERSAL INVARIANCE USING THE MÖSSBAUER TRANSMISSION OF THE 73-keV TRANSITION IN Ir^{193} †

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(Received 6 November 1967)

The real and imaginary parts of the $E2$-$M1$ mixing ratio of the 73-keV transition in Ir^{193} have been determined in an arrangement that requires no correction for Faraday rotation. The values $\mathrm{Re}\delta=+0.556\pm0.010$ and $\mathrm{Im}\delta=(+0.6\pm2.1)\times10^{-3}$ result in a phase angle between $E2$ and $M1$ of $\eta=(+1.1\pm3.8)\times10^{-3}$, thus showing no evidence for T nonconservation.

The CP nonconservation observed in the K_2^0 decay[1] led Bernstein, Feinberg, and Lee[2] to conjecture that T invariance is violated in the electromagnetic interaction of hadrons. As a result of such a violation, nuclear matrix elements would contain an admixture of a T-odd amplitude. Under favorable conditions, the fractional admixture could be as large as 10^{-3} to 10^{-2} and lead to effects observable in interference terms, $\langle f|L|i\rangle\langle f|L'|i\rangle^*+\mathrm{c.c.}$, of mixed gamma transitions. If T invariance holds, the ratio of reduced matrix elements is real[3]; a complex mixing ratio indicates failure of T invariance.[4] Experiments so far have been done on $E2$-$M1$ mixed transitions where we write for the ratio of reduced matrix elements[5]

$$\delta=\langle f\|E2\|i\rangle/\langle f\|M1\|i\rangle=|\delta|e^{i\eta}.$$

A deviation of the phase angle η from 0 or π indicates violation of T invariance. We have performed a measurement of δ for the 73-keV $\frac{1}{2}^+\to\frac{3}{2}^+$ transition in Ir^{193} using the Mössbauer effect in a geometry that requires no correction for Faraday rotation. Our result, $\mathrm{Re}\delta=+0.556\pm0.010$, $\mathrm{Im}\delta=(+0.6\pm2.1)\times10^{-3}$, $\eta=(+1.1$