$K_{\pi 3}$ BRANCHING RATIOS: MASS-DIFFERENCE CORRECTIONS AND AMBIGUITIES IN INTERPRETATION*

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One of the methods of testing isospin selection rules is the comparison of partial rates for the decay of K mesons into three pions. The processes we consider here are

$$K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}, \tag{1}$$

$$K^{\pm} \to \pi^0\pi^0\pi^{\pm}, \tag{2}$$

$$K_2^0 \to \pi^+\pi^-\pi^0, \tag{3}$$

$$K_2^0 \to \pi^0\pi^0\pi^0. \tag{4}$$

Because of the low kinetic energy in the final state (75-90 MeV in the center-of-mass system), the mass difference between charged and neutral mesons exerts a substantial influence on the phase-space volume available for each process.[1] This may be expressed as

$$R_i = \int_{\text{phase space}} |M_i(t_1, t_2)|^2 dt_1 dt_2, \tag{5}$$

where R_i is the partial rate of M_i, the matrix element for the ith process, and it includes trivial factors: vector addition coefficients, etc. The kinetic energies t_1, t_2, and t_3 for the pions in any given process are in the order listed in (1)-(4) above. Since the sum of kinetic energies is a constant, Q, any pair is sufficient to define a point in phase space.

The various models of the decay interaction (e.g., the $|\Delta T| = \frac{1}{2}$ isospin selection rule[2]) make predictions about the ratios formed among the various matrix elements M_i. The technique of applying the correction for the volume of phase space has been dealt with by a number of authors.[1,3,4] The assumptions made are the following: (1) The matrix element can be expanded in a power series in the Dalitz energy variables of the final-state pions; (2) second- and higher-order terms in the squared matrix element are negligible; and (3) the integral over phase space of the first-order term is zero.

One of the purposes of this paper is to point out that, in the cases of the $\pi^0\pi^0\pi^{\pm}$ and $\pi^+\pi^-\pi^0$ final states, the third assumption is incorrect. Because of the unequal masses of charged and neutral pions, the exact kinematic symmetry of the Dalitz plot is destroyed and the integral of the first-order term is not zero. This affects some of the branching ratios at the 10% level. Since present experimental accuracy is about 3%, the effect is far from negligible.

A consideration of this aspect of the phase-space correction raises nontrivial questions about the validity of the branching-ratio tests at the present level of accuracy. All the tests assume invariance of the initial and final states under rotations in isospin space. The mass difference between charged and neutral mesons is manifestly a violation of this invariance. The usual assumption is that the violation affects only the kinematics of the final state, i.e., the volume of phase space available, and that the matrix element itself obeys the invariance principle. We shall have some remarks concerning this assumption below. For the present we accept it.

We expand the matrix element to second order in a manner similar to the recipe of Weinberg,[5]

$$M_i(X, Y) = M_i[1 + a_i Y + b_i(Y^2 + X^2) + c_i(Y^2 - X^2)], \tag{6}$$

where X and Y are the Dalitz energy variables and $M_i = M_i(0, 0)$. We retain the symbols X and Y in our formulas, but whenever a numerical calculation is made we use the Lorentz-invariant quantities $S_j = -(P_K - P_j)^2$, where P_K and P_j are the four-momenta of the K meson and the jth pion, respectively. The relationships are

$$Y = 3(S_0 - S_3)/2MQ, \quad X = \sqrt{3}(S_1 - S_2)/2MQ, \tag{7}$$

where $S_0 = \frac{1}{3}(S_1 + S_2 + S_3)$. The relationships between the slopes a_i used here and those of Trilling,[3] λ_i, are given by

$$|M_i(S_3)|^2 \propto 1 + 2\lambda_i(S_0 - S_3)/m^2,$$

$$\text{Re}(a_i) = \lambda_i(2MQ)/(3m^2). \tag{8}$$

In these formulas, M is the mass of the K meson and m is the mass of one of the two like pions. There is a question of which pion mass to use at this point, but the effect is absorbed in the definition of the coefficients.

We square the matrix element and retain only terms to second order in X and Y:

$$|M_i(X,Y)|^2 = |M_i|^2[1+2(\mathrm{Re}a_i)Y+|a_i|^2Y^2 + 2(\mathrm{Re}b_i)(Y^2+X^2) + 2(\mathrm{Re}c_i)(Y^2-X^2)]. \quad (9)$$

The coefficients of the energy-dependent terms have been measured to first order[3] and limits have been placed on the second-order terms.[6] Thus, to second order, we can perform the phase-space integral and compare the matrix elements at $X=Y=0$.

For convenience we define a number of quantities which are shown in Table I. The integrals of the various terms in (9) are

$$I_0=\int dt_1dt_2,\quad I_1=\int Ydt_1dt_2,\quad I_2=\int Y^2dt_1dt_2,$$
$$I_+=\int(Y^2+X^2)dt_1dt_2,\quad I_-=\int(Y^2-X^2)dt_1dt_2. \quad (10)$$

We use these in the integration of the matrix element to obtain the expressions

$$R_i=|M_i|^2(I_0)_i[1+2(\mathrm{Re}b_i)\gamma_i+2(\mathrm{Re}a_i)\alpha_i + |a_i|^2\beta_i]\propto|M_i|^2\varphi_i, \quad (11)$$

where we have factored out the zero-order phase-space integral, and

$$\alpha=I_1/I_0,\quad \beta=I_2/I_0,\quad \gamma=I_+/I_0,\quad \delta=I_-/I_0. \quad (12)$$

We have dropped the term involving δ because it is small for all states. Also, the coefficient b is the same for all final states provided that the $|\Delta T|=\frac{1}{2}$ rule is approximately true. Since b is not large[6] the effects of this term cancel in the formation of all branching ratios.

Thus we find that we need know only the linear part of the odd-pion energy spectrum in order to correct the branching ratios for all first- and second-order effects. This must be qualified to account for $\mathrm{Im}a_i$ which affects only the second-order terms. However, pres-

Table I. Various quantities involved in mass difference corrections to branching ratios.

Quantity	Units	$\pm\pm\mp$	$00\pm$	$+-0$	000
M	BeV/c^2	0.4939	0.4939	0.4977	0.4977
$m=m_1=m_2$	BeV/c^2	0.1396	0.1350	0.1396	0.1350
m_3	BeV/c^2	0.1396	0.1396	0.1350	0.1350
Q	BeV	0.0751	0.0843	0.0835	0.0927
$I_0(\times 10^3)$	BeV^2	1.572	1.960	1.926	2.349
$I_1(\times 10^3)$	BeV^2	0	0.135	-0.135	0
$I_2(\times 10^3)$	BeV^2	0.363	0.449	0.459	0.533
$I_+(\times 10^3)$	BeV^2	0.727	0.907	0.893	1.067
$I_-(\times 10^3)$	BeV^2	0	-0.009	0.027	0
λ	---	0.093±0.011[a]	-0.25±0.02[a]	-0.24±0.02[a]	0
a(Eq. 8)	---	0.118	-0.356	-0.341	0
$2\mathrm{Re}(a)\alpha$ (Eq. 12)	---	0	-0.049	0.048	0
$\|a\|^2\ \beta$ (")	---	0.009	0.029	0.028	0
$1+2\mathrm{Re}(a)\alpha+\|a\|^2\beta$	---	1.009	0.980	1.076	1.000
Φ (Eq. 11)	---	1.00[c]	1.21[b]	1.30[b]	1.48[b]
Φ (")	---	1.000[c]	1.214±0.001[d]	1.312±0.004[d]	1.484±0.001[d]

[a]Ref. 3.
[b]Results based on slopes on Ref. a.
[c]Factors normalized to Φ++−.
[d]Results calculated by the author using all data on first- and second-order terms in the energy spectrum.

ent data (of limited accuracy) are consistent with $\mathrm{Im}a_i = 0$, and the uncertainty does not affect our conclusions.

It should be emphasized that the magnitude of the first-order terms, neglected in prior work, is of order $(m_3-m_1)/Q$.

All that remains is to apply the measured values of the terms in the energy spectrum to Eq. (11) in order to evaluate the corrected phase-space factor φ_i. The results of two separate calculations are shown in Table I. The first ignores the second-order terms in the matrix element and uses the values of the slopes quoted by Trilling.[3] In order to make clear the contribution of each term, its fractional correction to the phase space is shown explicitly. The most striking feature is the comparison between the 00± and the +−0 states. Although the spectra for these two states are identical within experimental accuracy (and exactly so if the $|\Delta T| = \frac{1}{2}$ rule is true), the charged-neutral mass difference enters with opposite sign, and the effects on the phase space are in opposite directions. The contribution of the second-order term tends to cancel that of the first-order term in the case of 00±, and enhance it in the case of +−0.

The last row of Table I presents a somewhat more elaborate calculation by the author. Both first- and second-order terms in the spectrum for all processes were fitted to existing data.[3,6] Data on the coefficients of the second-order terms are consistent with zero; they were used primarily to allow a calculation of the uncertainties quoted for the phase-space factors. The fits to the data were made under the assumption that the coefficients for the various final states can be related by the $\Delta T = \frac{1}{2}$ rule. While this is probably not strictly true,[7] the small value of the violation (~5 %) would have no appreciable effect on the results presented here.

The procedure which has been carried out above is directed toward a comparison of the various matrix elements $M_i(X, Y)$ at the points where $X = Y = 0$ for each of the processes, i.e., where $S_1 = S_2 = S_3 = S_0$. A more recent calculation of the branching ratios reported in Ref. 7 yields

$$\tfrac{1}{2}|M_{+-0}|^2/|M_{00+}|^2 = 0.816 \pm 0.034. \qquad (13)$$

The choice of this point for the branching-ratio test seems a natural one, but, because of the violations of isospin invariance inherent in the mass differences, it is no longer obvious that it is the correct one.

As an example, we might have chosen to compare the matrix elements at the point where, for each final state, the kinetic energies of all three pions are equal, i.e., where $t_1 = t_2 = t_3 = \frac{1}{3}Q$. This corresponds in the unequal-mass cases to a displacement in Y by an amount

$$\Delta = 2(M-m)(m_3-m)/MQ, \qquad (14)$$

and the displacement is toward higher Y in the case of 00± and lower Y in the case of +−0. The effect on the branching ratio is given by the factor

$$(1-2a_{+-0}\Delta)/(1+2a_{00+}\Delta) = 1.12, \qquad (15)$$

where the measured slopes have been used. Combining (13) and (15) we have

$$\tfrac{1}{2}|M_{+-0}(0, -\Delta)|^2/|M_{00+}(0, \Delta)|^2 = 0.91 \pm 0.038. \qquad (16)$$

We do not propose this alternative seriously. We mention it merely to illustrate the magnitude of the ambiguities encountered as a result of electromagnetic corrections. Further, we do not interpret the ratio in Eq. (16) as weakening, in any way, the conclusions drawn in Ref. 7. The branching ratio 000/++−, independent of the ambiguity discussed in the present paper, is evidence of the violation of the predictions of the $|\Delta T| = \frac{1}{2}$ rule.

We do wish, however, to emphasize strongly that there is no reason to believe that the matrix elements themselves are immune to corrections of the order discussed here, i.e., we can no longer believe the predictions of the $|\Delta T| = \frac{1}{2}$ rule to better than a few percent (and perhaps as much as 10 % in an extreme interpretation). Thus increased experimental accuracy in these branching ratios cannot strengthen present conclusions until a much more comprehensive prescription for electromagnetic corrections is available.

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OBSERVATION OF A $pp\bar{p}$(3755) ENHANCEMENT IN THE REACTION $\pi^+p \to \pi^+pp\bar{p}$ AT 8.4 BeV/c*

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The existence of nucleon isobars having masses of 3030, 3230, 3245, and 3695 MeV has been reported in different experiments.[1-3] We report the observation of a narrow enhancement seen in the $pp\bar{p}$ invariant-mass spectrum in a study of the reaction $\pi^+p \to \pi^+pp\bar{p}$. If this enhancement is interpreted as the $pp\bar{p}$ decay of a nucleon isobar, then the resonance parameters are $M = 3755 \pm 8$ MeV and $\Gamma = 40 \pm 20$ MeV.

The sample of events for this study is obtained from a systematic investigation of π^+p interactions at 8.45 BeV/c in a 65 000-picture exposure[4] of the Brookhaven National Laboratory 80-in. hydrogen bubble chamber. The events are part of a sample of approximately 15 000 events having the topology of four prongs and no kinks. In addition, all events in the sample have at least one outgoing track clearly identifiable as a proton on the scanning table. This restriction corresponds to a proton momentum below about 1 BeV/c, and it was made to enrich the sample of events for the study of pionic resonances.[5]

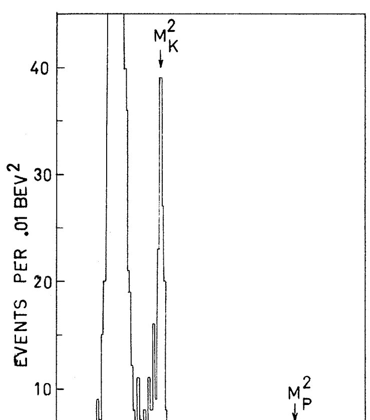

FIG. 1. Distribution in the square of the $X^\pm$ rest mass for 2964 events of the type $\pi^+p \to \pi^+pX^+X^-$. X^+ and X^- here represent any long-lived particle-antiparticle pair. Each event is plotted twice due to the X^+-π^+ ambiguity. The peak centered at 0.02 BeV^2 has a height of 2356 and a half-width of 0.007 BeV^2.

Events corresponding to the reaction $\pi^+p \to \pi^+pp\bar{p}$ were identified in the following way: All four-prong events not consistent with momentum conservation, assuming no neutrals to be present, were first rejected. The remaining events were assumed to correspond to reactions of the type $\pi^+p \to \pi^+pX^+X^-$, where X^+ and X^- represent any long-lived particle-antiparticle pair. After adjusting only the incoming momentum, so as to conserve momentum exactly, energy conservation was then used to calculate the rest mass of the X particle.[6] The resulting distribution in the X mass squared, in Fig. 1, shows three clearly separated peaks, corresponding to the following reactions:

$$\pi^+p \to \pi^+p\pi^+\pi^-, \text{ 2779 events;} \quad (1)$$

$$\pi^+p \to \pi^+pK^+K^-, \text{ 162 events;} \quad (2)$$

$$\pi^+p \to \pi^+pp\bar{p}, \text{ 21 events.} \quad (3)$$

Each event is plotted twice, as the X^+ and the π^+ tracks are generally indistinguishable. Calculating the X mass using the wrong assignment of tracks will introduce some background only for events corresponding to Reactions (2) and (3).

All events outside the pion peak in Fig. 1 were kinematically fitted in GRIND, resulting