MESON DECAYS AND THE PION MASS DIFFERENCE*

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We discuss a Lagrangian model of π-A_1 mixing in the presence of divergenceless vector fields, which permits the calculation of satisfactory rho- and A_1-meson decay widths. The pion mass difference (for physical pions) is finite and approximately 5 MeV when the A_1 has unit magnetic moment.

Based on the assumption that the vector and axial-vector currents are dominated by mesons, a number of phenomenological Lagrangian theories[1-3] have been proposed which involve the mixing of the pion with the A_1 meson. These theories, which are able to produce in simpler fashion a number of results of current algebra, involve the breaking of chiral symmetry by partially conserved axial-vector currents (PCAC), implying a mixing of the pion and A_1 fields. Since there is not a unique way of breaking the chiral symmetry, we consider instead a nonchiral Lagrangian in which, however, the divergence-free property of the ρ-meson field and the mixing of the A_1 and π fields are retained.

In our model this mixing is introduced at the outset by means of the free Lagrangian density

$$\mathcal{L} = \mathcal{L}_1(\varphi_\mu) + \alpha\mathcal{L}_2(\pi) + \mathcal{L}_3(\varphi_\mu, \pi), \quad (1)$$

where

$$\mathcal{L}_1 = -\tfrac{1}{2}(\partial_\mu \varphi_\nu{}^i \partial_\mu \varphi_\nu{}^i - \partial_\mu \varphi_\nu{}^i \partial_\nu \varphi_\mu{}^i) - \tfrac{1}{2}\kappa(\partial_\mu \varphi_\mu{}^i \partial_\nu \varphi_\nu{}^i - \partial_\mu \varphi_\nu{}^i \partial_\nu \varphi_\mu{}^i) + \tfrac{1}{2}m^2 \varphi_\mu{}^i \varphi_\mu{}^i, \quad (2)$$

$$\mathcal{L}_2 = \tfrac{1}{2}\partial_\mu \pi^i \partial_\mu \pi^i - \tfrac{1}{2}\mu_0{}^2 \pi^i \pi^i, \quad (3)$$

and

$$\mathcal{L}_3 = m_0 \varphi_\mu{}^i \partial_\mu \pi^i. \quad (4)$$

$\mathcal{L}_1$ alone corresponds to an ordinary spin-1 field whose positively charged component has magnetic moment $(1+\kappa)(e/2m)$. The index i denotes an internal symmetry which, for the present work, we regard as isospin. $\mathcal{L}_2$ is the free Lagrangian density for a scalar field of mass μ_0, while $\mathcal{L}_3$ provides the simplest possible mixing of the π and φ_μ fields.

From the Lagrangian density $\mathcal{L}$ follow the equations of motion

$$(\Box + m^2)\varphi_\mu - \partial_\mu \partial_\sigma \varphi_\sigma + m_0 \partial_\mu \pi = 0 \quad (5)$$

and

$$(\Box + \mu_0{}^2)\pi + (m_0/\alpha)\partial_\mu \varphi_\mu = 0. \quad (6)$$

The coupled Eqs. (5) and (6) are solved uniquely by

$$\varphi_\pi = \mathcal{A}_\mu - (m_0/m^2)\partial_\mu \pi, \quad (7)$$

where $\mathcal{A}_\mu$ is divergenceless and satisfies the equation

$$(\Box + m^2)\mathcal{A}_\mu = 0, \quad (8)$$

while π satisfies

$$(\Box + \mu^2)\pi = 0, \quad (9)$$

with

$$\mu^2 = \alpha\mu_0{}^2(\alpha - m_0{}^2/m^2)^{-1}. \quad (10)$$

Thus, we take the spin-1 field $\mathcal{A}_\mu$ to represent the A_1 meson.

The canonical quantization procedure can be applied consistently to $\mathcal{L}$, yielding the condition

$$\alpha = 1 + m_0{}^2/m^2, \quad (11)$$

so that the pion mass is

$$\mu = (1 + m_0{}^2/m^2)^{1/2}\mu_0. \quad (12)$$

We see that when the fields are free, the mixing is purely formal, in the sense that it can be transformed away by redefining the fields in terms of the A_1 field $\mathcal{A}_\mu$ of mass m and a pion field of mass $\mu = \alpha^{1/2}\mu_0$. When interactions are introduced, the mixing ceases to be formal and provides the minimal physical coupling between the A_1 and the pion.

The theory presented so far can be considered fundamental in that it contains no worse divergences than the standard Proca theory of vector mesons. A single ξ term appears sufficient to renormalize the theory in the ξ-

limiting sense,[4] for minimal electromagnetic interactions and also for nuclear interactions of π and A_1 mesons, provided that in the nuclear case there are no derivative interactions other than those of the pion implicitly contained in φ_μ.

We introduce the interaction with the rho field by the minimal substitution[5-7]

$$\partial_\mu \delta_{ij} \to \partial_\mu \delta_{ij} + g\rho_\mu^k \epsilon_{ijk}. \tag{13}$$

The three-particle interaction terms obtained in this way are

$$\mathcal{L}_I^{(3)} = g\epsilon_{ijk}\rho_\mu^k[-\partial_\mu \varphi_\nu^i \varphi_\nu^j + \partial_\nu \varphi_\mu^i \varphi_\nu^j - \kappa(\partial_\nu \varphi_\nu^i \varphi_\mu^j - \partial_\nu \varphi_\mu^i \varphi_\nu^j) + (1+m_0^2/m^2)\partial_\mu \pi^i \pi^j + m_0 \varphi_\mu^i \pi^j]. \tag{14}$$

This gives in momentum space the following vertices (with $p=q+k$) involving a neutral rho meson:

$$\mathcal{L}(\rho\mathfrak{A}\mathfrak{A}) = g\rho_\lambda(k)\mathfrak{A}_\gamma^\dagger(p)\mathfrak{A}_\mu(q)T_{\lambda\nu\mu},$$

$$T_{\lambda\nu\mu} = g_{\mu\nu}(p+q)_\lambda - (p_\nu g_{\mu\lambda} + q_\mu q_{\gamma\lambda}) + (1+\kappa)(k_\nu g_{\mu\lambda} - k_\mu g_{\nu\lambda}); \tag{15}$$

$$\mathcal{L}(\mathfrak{A}\rho\pi) = ig(m_0/m^2)\rho_\lambda(k)\mathfrak{A}_\nu^\dagger(p)\pi(q)T_{\lambda\nu} + \text{H.c.},$$

$$T_{\lambda\nu} = p_\lambda q_\nu - (p\cdot q - m^2)g_{\nu\lambda} + \kappa(k_\nu q_\lambda - k\cdot q g_{\nu\lambda}); \tag{16}$$

$$\mathcal{L}(\rho\pi\pi) = g\rho_\lambda(k)\pi^\dagger(p)\pi(q)T_\lambda,$$

$$T_\lambda = -(p+q)_\lambda[1-(\kappa m_0^2/2m^4)k^2]. \tag{17}$$

From (17) the decay rate of the rho meson is found to be

$$\Gamma(\rho\to\pi\pi) = \frac{g^2}{4\pi}\left(1-\frac{\kappa m_0^2}{4m^2}\right)^2 \times 56 \text{ MeV}, \tag{18}$$

where we have used the empirical relation $m^2 = 2m_\rho^2$. From (16), the A_1 decay rate is

$$\Gamma(A_1\to\rho\pi) = 0.001\,64\left(\frac{g^2}{4\pi}\right)\frac{m_0^2}{m_\rho}(52-32\kappa+5\kappa^2). \tag{19}$$

Analogously, the radiative decay rate is

$$\Gamma(A_1\to\gamma\pi) = \left(\frac{e^2}{4\pi}\right)\frac{m_0^2}{24m}(1-\kappa)^2. \tag{20}$$

We calculate the mass difference of the charged and neutral pion following the ground rules laid down by Kroll, Lee, and Zumino[6,7]; that is, the virtual photon is effectively coupled to the electromagnetic current of the π-A_1 system through an intermediary neutral rho meson. This provides an effective photon propagator

$$G_{\mu\nu} = -\left(g_{\mu\nu} - \frac{k_\mu k_\gamma}{k^2}\right)\frac{m_\rho^4}{k^2(k^2-m_\rho^2)^2}. \tag{21}$$

In the soft-pion limit the interaction terms which contribute to the pion mass difference are, from Eq. (16),

$$-iem_0 A_\mu(\pi^\dagger \mathfrak{A}_\mu - \pi\mathfrak{A}_\mu^\dagger) \tag{22}$$

and, from Eqs. (1) and (13), the four-vertex term

$$(1+m_0^2/m^2)e^2 A_\mu A_\mu \pi^\dagger\pi. \tag{23}$$

The result, therefore, is independent of the magnetic moment and it turns out to be convergent for any choice of m_0. It is interesting to note that this convergence is also independent of the ratio m/m_ρ.

By straightforward application of the Feynman rules, using (21)-(23), we obtain

$$\delta\mu^2 = \mu_{\pi^+}^2 - \mu_{\pi^0}^2 = \frac{3ie^2 m_\rho^4}{(2\pi)^4}\int\frac{d^4k}{k^2(k^2-m_\rho^2)^2} \times\left[\left(1+\frac{m_0^2}{m^2}\right)+\frac{m_0^2}{k^2-m^2}\right]. \tag{24}$$

Integration gives

$$\delta\mu^2=\left(\frac{e^2}{4\pi}\right)\frac{3m_\rho^{\,2}}{4\pi}\left[1+\frac{m_0^{\,2}}{m^2}-\frac{m_0^{\,2}}{m^2-m_\rho^{\,2}}\left(1-\frac{m_\rho^{\,2}}{m^2-m_\rho^{\,2}}\ln\frac{m^2}{m_\rho^{\,2}}\right)\right]. \tag{25}$$

If m_0 is chosen equal to m, Eq. (24) is just the result obtained previously using broken chiral symmetry,[8-10] yielding $\delta\mu\approx 5$ MeV; while for $m_0=m_\rho$ we get $\delta\mu\approx 4.3$ MeV. Both results are in fair agreement with the experimental value of 4.6 MeV.

We consider now the calculation of the pion mass difference when the pion is on its mass shell. Although there occurs a logarithmic divergence, arising from the interaction terms (16) and (17), its coefficient depends on the A_1 magnetic moment in such a way that it vanishes for $\kappa=0$. A similar logarithmic divergence arises also in the other treatments of this problem[9,10]; however, it cannot be made to vanish for any choice of the A_1 magnetic moment.

Thus, for $\kappa=0$, we obtain the finite mass squared difference to $O(\mu^2/m_\rho^{\,2})$:

$$\delta\mu^2=\frac{3}{4\pi}\left(\frac{e^2}{4\pi}\right)m_\rho^{\,2}\left\{\left[1+\left(\frac{m_0}{m}\right)^2\left(1-\frac{3\mu^2}{4m^2}\right)\right]+\left[\frac{\mu^2}{m_\rho^2}\left(\frac{1}{2}+\ln\frac{m_\rho^{\,2}}{\mu^2}\right)\right]+\frac{m_0^{\,2}}{m_\rho^{\,2}}\left[\left(1-\frac{9}{8}\frac{\mu^2}{m_\rho^{\,2}}\right)(\ln 2-1)-\frac{\mu^2}{4m_\rho^{\,2}}\right]\right\}. \tag{26}$$

The first term in square brackets arises from the four vertex, the second from the diagram with an intermediate pion, and the last from the diagram with an intermediate A_1 meson.

Numerically, for $m_0=m$, $\delta\mu\approx 5.5$ MeV; on the other hand, for $m_0=m_\rho$, $\delta\mu\approx 4.8$ MeV, in good agreement with the experimental result 4.6 MeV.

For $\kappa=0$, Eq. (18) for the ρ-meson decay width becomes the conventional one and, with $m_0=m_\rho$, we get the ratio

$$\frac{\Gamma(A_1\to\rho\pi)}{\Gamma(\rho\to\pi\pi)}=1.15. \tag{27}$$

A recent compilation of the experimental widths gives, for comparison,

$$\Gamma(A_1\to\rho\pi)=130\pm 40\text{ MeV},$$

$$\Gamma(\rho\to\pi\pi)=140\pm 20\text{ MeV}.$$

If we compare with results obtained from current algebra by Schnitzer and Weinberg[11] for the same processes, we see that our vertices $A_1A_1\rho$ and $\pi\pi\rho$ have the same form as theirs. On the other hand, the form of our $A_1\pi\rho$ vertex differs in the sign of $\delta\equiv\kappa-1$ as well as in the occurrence of the mixing mass m_0, which they take to be the A_1 mass as a consequence of Weinberg's second sum rule.[12] It is easy to see that the $A_1\pi\rho$ vertex given by our Lagrangian could be obtained by the following alternative prescription: Take the $A_1A_1\rho$ vertex, substitute $-(m_0/m^2)\partial_\mu\pi$ for one of the A_1 fields, and preserve gauge invariance by adding a term which is the minimal term, in the sense that it contains no derivatives. The $A_1\pi\rho$ vertex of Schnitzer and Weinberg could be obtained by an additional gauge-invariant but momentum-dependent term, which is responsible for the logarithmic divergence in the mass difference which was found by previous authors.

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