potential used in this paper.

The single-particle energies are quite large for the G.S. 1 potential. To compare with proton separation energies as measured in $(p, 2p)$ and $(e, e'p)$ experiments which give[12] $E(1s) \sim 43$ MeV, $E(1p_{3/2}) \sim 19$ MeV, and $E(1p_{1/2}) \sim 12$ MeV, one would have to make "rearrangement" corrections. Allowing for a Coulomb energy of ~5 MeV for each single-particle state[9] we would thus have $V_{1s}{}^R \sim 13$ MeV, $V_{1p_{1/2}}{}^R \sim 9$ MeV, and $V_{1p_{1/2}}{}^R \sim 8$ MeV. These are in rough agreement with previous estimates[13,14] of "rearragement" corrections. We have calculated third-order "rearrangement" corrections for the G.S. 1 potential and get $V_{1s_{1/2}}{}^R = 5.1$ MeV, $V_{1p_{3/2}}{}^R = 3.4$ MeV, and $V_{1p_{1/2}}{}^R = 2.9$ MeV. Thus there would have to be further rearrangement corrections of 5-8 MeV in order to get agreement between theory and experiment.

Calculations for infinite nuclear matter with the G.S. 1 potential used here are now in progress. Further calculations for both finite and infinite systems with more refined versions of the G.S. 1 potential which are able to reproduce ${}^1P_1$ scattering data are, of course, of interest.

We wish to thank Professor A. E. S. Green for communicating data on the G.S. 1 potential and Mr. H. G. Hughes, III, for assistance with the calculations.

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ENERGY DEPENDENCE OF THE DEUTERON OPTICAL-MODEL POTENTIAL*

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It has been found[1-3] that a feature common to a number of optical-model analyses of experimental cross sections for elastic deuteron scattering is a marked increase of the radius of the imaginary part of the optical deuteron potential as the incident deuteron energy decreases. For deuteron-induced reactions there are two channels which are excited with relatively large probability—the stripping and the deuteron break-up channels. It is likely that the energy dependence of the deuteron optical-model potential is due to the large coupling which exists between the deuteron channel and the ones mentioned above. The break-up reaction is very difficult to treat in a realistic way because it leads to a three-body problem, but the stripping channels have recently been incorporated into a set of approximate coupled equations.[4] In the simple form of these equations as they are presently employed, the only stripping transition explicitly included is one with $\Delta l = 1$. The calculations have so far only been carried out for the nucleus of calcium. Despite the approximate nature of these equations, the calculations give rise to roughly the correct behavior (as a function of angle and energy) of both the elastic and the $\Delta l = 1$ stripping cross sections for the d-^{40}Ca interaction at deuteron energies between 7 and 22 MeV.[4]

It is the purpose of this Letter to show that for the case of ^{40}Ca a large portion of the energy dependence of the deuteron optical-model parameters is due to the presence of the stripping channels.

The calculation is as follows: The elastic deuteron cross section is calculated by means of the coupled-channel procedure at incident deuteron energies of 4, 7, and 11 MeV. The result is then fitted by means of a conventional optical-model search calculation. The potentials in the coupled equations are assumed to be energy independent and the parameters have the same value as those described previously.[4] The present comparison with optical-model results differs from a previous one[4] in that previously the optical-model potentials were taken from the literature so as to fit experiment, while in the present case they are chosen so as to fit the coupled-channel results. The present procedure resembles one used previously by Stamp and Rook[5] in the investigation of (p,p) and (p,p') calculations. The deuteron optical potential has a real part of a Woods-Saxon form with depth, radius, and diffuseness given, respectively, by V_0, r_0, and a. The imaginary part is given by the radial derivative of a Woods-Saxon potential, with parameters $4a'W$, r_0', and a'. The fitting procedure is done with the search code ABACUS.[6] The starting values of the optical parameters are given by the energy-independent potential $Z2$, obtained by Bassel et al.[3] First, a succession of three-parameter searches in (V, r_0, W) and (W, r_0, r_0') is carried out. The resulting value of r_0 is found to be nearly energy independent and the average value of 1.005 F is adopted. Next, a succession of one-parameter searches is undertaken in order to explore the correlation between parameters. The correlation between parameters X and Y is obtained by varying X in finite steps and searching Y so as to minimize[7] χ^2. An increase of χ^2 by a factor of 2 from its minimum value leads to changes in X and Y which are taken as a measure of their correlation. The parameter pairs tested successively are (V_0, a), (W, a'), and (W, r_0'). The final values of the parameters thus obtained are indicated by means of the stars in Fig. 1, and the correlations are indicated by means of the error flags. For the parameter a the flags are too small to be vis-

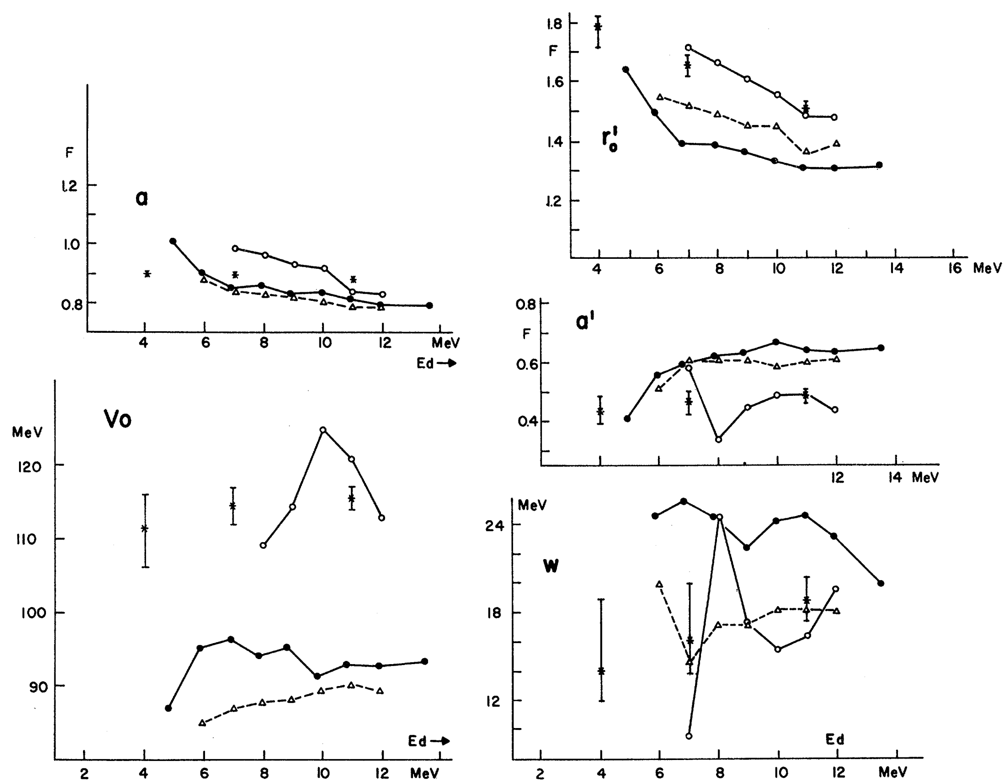

FIG. 1. Deuteron optical-potential parameters. The open circles and triangles refer to target nuclei of ^{40}Ca and ^{48}Ti, respectively (Refs. 2 and 3). The full circles represent ^{60}Ni data (Ref. 8). The stars are the results of the present analysis. The error flags represent correlations as described in the text.

ible in Fig. 1. For comparison, Fig. 1 also illustrates the parameters obtained by various authors in their fits to experimental elastic cross sections. The open circles, filled circles, and triangles represent, respectively, fits with ^{40}Ca data,[3] ^{60}Ni data,[8] and ^{48}Ti data.[2]

Figure 2 illustrates the fits of the optical-model cross section (continuous lines) to the coupled-channel results (circles). At 4 MeV the elastic cross section is relatively featureless, with the result that the χ^2 valleys are more shallow than at the higher energies.

The most striking feature shown in Fig. 1 is the pronounced energy dependence found for the radius of the imaginary potential, in confirmation of the trend seen experimentally. This behavior reflects the energy-dependent feedback of the stripping channel into the deuteron channel and lends credibility to this coupled-channel method of calculation. It would also be desirable to compare stripping cross sections with the distorted-wave approximation (DWA) calculations based on the equivalent optical potentials. So far this comparison has been carried out only for an angular momentum transfer $\Delta l = 1$ (p states in ^{41}Ca). When the stripping cross sections for $\Delta l = 3$ are available, these results as well as comparison of the deuteron wave functions will be presented in detail in a future publication. For the $\Delta l = 1$ case it is found that the peaks of the stripping cross sections agree to within 10% at all three energies. At 2 MeV the stripping cross section increases with angle, and beyond 50° the DWA result is about a factor of 1.5 times larger than the coupled-channel result.

A brief description of the comparison of the radial deuteron wave functions as obtained by the two methods of calculation will now be given. In the asymptotic region (beyond 8 F, in this case) they agree remarkably well. At the surface of the interaction region the coupled-channel wave function is about 10% larger than the optical-model wave function, but at distances smaller than about 5 F the latter is larger than the former by factors of approximately 2 and at still smaller distances the factor is between 3 and 4. This damping of the wave function is a result of the nonlocality of the optical potential which is obtained via the coupled equation. Although the coupled-channel deuteron radial waves are not strictly proportional to the equivalent local optical-potential wave functions, it is nevertheless interesting

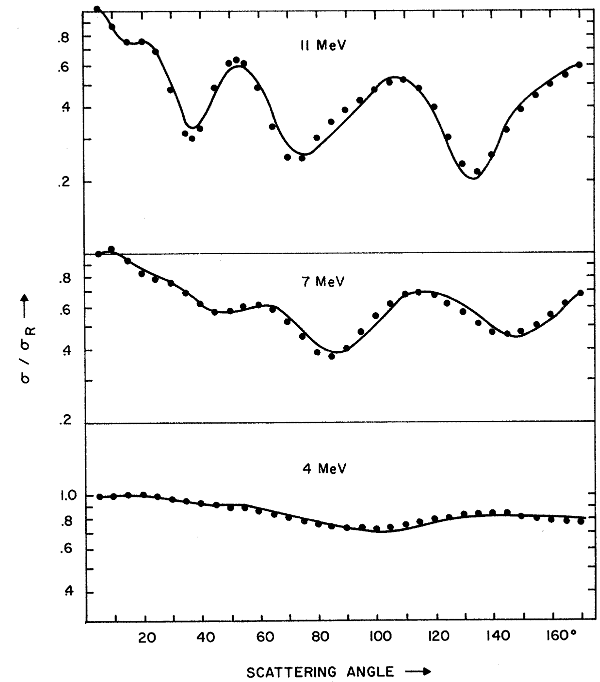

FIG. 2. Optical model fits to the elastic d-Ca scattering cross sections calculated by means of the coupled equations described in the text. Plotted is the ratio of the elastic cross section to the Rutherford value at the energies indicated. The dots represent the coupled-channel results; the curves illustrate the optical-model results. The parameters for the optical-model potentials are represented by the stars in Fig. 1.

to express the approximate magnitude of the damping in terms of the "Perey damping factor" used in the local energy approximation (LEA) of a nonlocal potential. Yntema and Ohnuma[9] found recently that at 23 MeV the j dependence for the $\Delta l = 1$ (d,p) stripping reaction in ^{54}Fe is qualitatively reproduced by the DWA calculation provided that considerably greater damping of the contributions from the interior of the nucleus are assumed than was previously thought necessary. They express this increased damping in terms of LEA nonlocality parameters $\beta = 2$ and 1 F for the proton and deuteron, respectively. Using these values of β, the combined damping factor due to the deuteron and proton nonlocalities turns out to be equal to 3.75 for the nucleus of calcium here discussed, at distances near the center of the nucleus. The damping factor is in nice agreement with the damping described above.

In summary, the present results indicate that the coupled-channel approach suggested for the d-Ca interaction produces a nonlocality in the optical potential which gives the right type of energy dependence for the equivalent, local optical potential, and it also gives rise to a damping of the deuteron wave function in the interior of the nucleus which is consistent with that suggested recently[9] for the case of d-^{54}Fe. It is a pleasure to thank Dr. R. Siemssen for many stimulating discussions and for making available unpublished results. Conversations with Dr. L. J. B. Goldfarb and Dr. A. F. Jeans at a Gordon Research Conference in the summer of 1967 are also much appreciated.

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[7]The chi squared is defined as

$$\frac{1}{N}\sum_{1}^{N} W_{\mathcal{J}}(\sigma_{\mathrm{th}}-\sigma_{\mathrm{exp}})^2,$$

where $W_{\mathcal{J}}=\sigma_{\mathrm{exp}}^{-2}$ and where σ_{th} and σ_{exp} denote the optical-model and the coupled-channel cross sections, respectively. The smallest values for chi squared obtained at deuteron energies of 4, 7, and 11 MeV are, respectively, 0.1×10^{-3}, 0.44×10^{-3}, and 1.48×10^{-3}.

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INELASTIC ELECTRON SCATTERING FROM EVEN TIN ISOTOPES AND MICROSCOPIC THEORIES OF VIBRATIONAL STATES

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Two- and four-quasiparticle microscopic theories are applied to fit the most recent data on the reactions $\mathrm{Sn}^{116}(e,e')\mathrm{Sn}^{116}(2_1^+,3_1^-)$. The effective nuclear force is the realistic nucleon-nucleon potential of Tabakin renormalized by the core-polarization corrections. The observed angular distributions and the absolute values of the form factors can be reproduced for quite reasonable values of the single-particle parameters.

In a recent Letter Barreau and Bellicard[1] published the first experimental data on the inelastic electron scattering from the even tin isotopes 116, 120, and 124 with the excitation of the 2_1^+ and the 3_1^- collective states. The bombarding electron energy was 150 MeV and the scattering angle varied between 45° and 80°. The corresponding electric quadrupole and octupole form factors, $|F_{\mathrm{in}}(Q)|^2$, have been extracted from the differential cross sections as $|F_{\mathrm{in}}|^2=\sigma(E_0,\theta)/Z^2\sigma_{\mathrm{Mott}}(Z=1)$. Both the absolute values of $|F_{\mathrm{in}}|^2$ and their angular distributions should, particularly when combined with the static electromagnetic moments and the corresponding values of $B(E\lambda)$, serve as a sensitive test of any microscopic or other nuclear wave functions of the excited states in question.

The low-lying excited states of even tin isotopes, particularly those of collective charac-