not detectable under index-matching conditions. At present, theory remains to be developed to explain these various effects. The present experimental capability also needs to be refined to observe deviations of α/β from the constant value, thus providing a new tool for studying the optical properties of surfaces.

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[1]N. Bloembergen, R. K. Chang, and C. H. Lee, Phys. Rev. Letters 16, 986 (1966); C. H. Lee, R. K. Chang, and N. Bloembergen, Phys. Rev. Letters 18, 167 (1967).

[2]N. Bloembergen and R. K. Chang, in Physics of Quantum Electronics, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Company, Inc., New York, 1966), p. 80.

[3]F. Brown, R. E. Parks, and A. M. Sleeper, Phys. Rev. Letters 14, 1029 (1965).

[4]F. Brown and R. E. Parks, Phys. Rev. Letters 16, 507 (1966).

[5]R. W. Terhune, Solid State Design 4, 38 (1963).

[6]P. S. Pershan, Phys. Rev. 130, 919 (1963).

[7]E. Adler, Phys. Rev. 134, A728 (1964).

[8]H. Cheng and P. B. Miller, Phys. Rev. 134, A683 (1964).

[9]S. S. Jha, Phys. Rev. Letters 15, 412 (1965), and Phys. Rev. 140, A2020 (1965); S. S. Jha and C. W. Warke, Phys. Rev. 153, 751 (1967), and 162, 854(E) (1967).

[10]N. Bloembergen and Y. R. Shen, Phys. Rev. 141, 298 (1966).

[11]N. Bloembergen and P. S. Pershan, Phys. Rev. 128, 606 (1962).

[12]P. D. Maker, in Physics of Quantum Electronics, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Company, Inc., New York, 1966), p. 60.

[13]H. Holloway, private communication.

[14]American Institute of Physics Handbook, edited by D. E. Gray (McGraw-Hill Book Company, Inc., New York, 1963), Sect. 6; H. R. Philipp and E. A. Taft, Phys. Rev. 113, 1002 (1959), and 120, 37 (1960); L. G. Schulz, J. Opt. Soc. Am. 44, 357 (1954); L. G. Schulz and F. R. Tangherlini, J. Opt. Soc. Am. 44, 362 (1954); International Critical Tables (McGraw-Hill Book Company, Inc., New York, 1929), Vol. 5, p. 249.

BINDING ENERGY OF O^{16} WITH A VELOCITY-DEPENDENT NUCLEON POTENTIAL*

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The binding energy and single-particle energies have been calculated for O^{16} using reaction matrix theory and a meson-theoretical nucleon potential due to Green and Sawada. The binding obtained is 7.2 MeV/nucleon after Coulomb and center-of-mass corrections, in reasonable agreement with the experimental 7.98 MeV/nucleon.

Recent developments in the reaction-matrix (K-matrix) theory of the nuclear many-body problem indicate that potential energies in the particle spectrum should be neglected when defining the K matrix.[1] The binding energy is then calculated with good accuracy to first order in the K matrix thus defined.

Hard-core potentials like the Brueckner-Gammel-Thaler (BGT) or Hamada-Johnston (HJ) potentials then give much too little binding for nuclear matter[2] or for O^{16}.[3-6] Softening of the core, while preserving the free-scattering properties, increases the binding somewhat but, as it seems, not sufficiently. The static hard-core and soft-core potentials currently in use reproduce two-body scattering data quite well but are mainly phenomenological. In this Letter we report on some calculations using one of the meson-theoretical potentials derived by Green and coworkers.[7,8] The "one-parameter, almost relativistic, regularized potential" of Green and Sawada[8] (to be referred to as G.S. 1) was chosen for simplicity.

The method of calculating the K-matrix elements has been presented in previous papers.[4-6] In Fig. 1, curve A, the results are shown for the energy per particle of O^{16} for the potential used. There is no sign of saturation for reasonable values of the oscillator parameter β ($\hbar\omega = \beta\hbar^2/M$). However, the G.S. 1 potential that we used reproduces all but the $^1P_1$ phase shifts quite well. The total contribution of $^1P_1$ K-matrix elements to the energy of O^{16} is shown in Fig. 2 in the curve labeled "G.S. 1." This contribution is positive and decreases with increasing β. For comparison is shown the corresponding $^1P_1$ contribution to the energy for the HJ potential. The latter is seen to increase quite

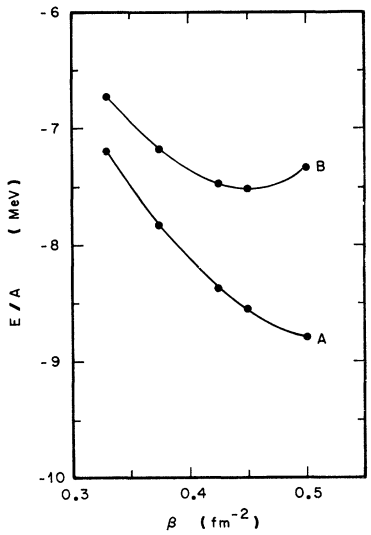

FIG. 1. Energy/nucleon of O^{16} for the G.S. 1 potential (curve A) and corrected for the $^1P_1$ contribution (curve B) versus the oscillator parameter $\beta(\beta=\omega M/\hbar)$.

sharply with β and is, in fact, important to obtain saturation for the HJ potential. Correcting the binding energy curve for the G.S. 1 potential by replacing the $^1P_1$ energy with that for the HJ potential yields curve B in Fig. 1. This curve saturates at $\beta\sim 0.45$ fm^{-2} yielding a binding energy per particle (E_B/A) around 7.52 MeV/nucleon.

The radius thus obtained ($\beta = 0.45$ fm^{-2} corresponds to $R_{\rm rms} = 2.23$ fm) for O^{16} may seem small compared with the experimental value $R_{\rm rms} = 2.57$ fm. There are, however, several corrections to this radius. The use of self-consistent orbitals would increase the radius by ~0.1 fm.[9] The Coulomb field and some correlation effects[10] would also increase the radius. In general, the calculated radius or saturation density is of course quite sensitive to small corrections, since the energy minimum is defined by varying this parameter.

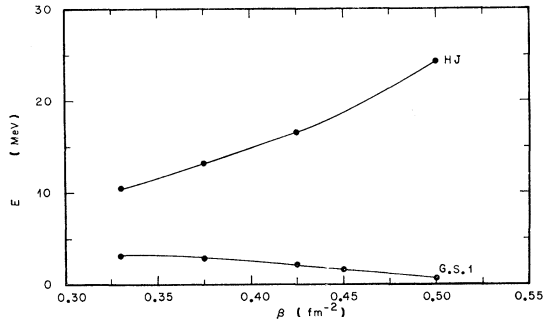

FIG. 2. Contribution of the $^1P_1$ K-matrix elements to the total energy of O^{16} for the HJ and G.S. 1 potentials.

It should be emphasized that the saturation depends critically on the $^1P_1$ contribution to the energy. The HJ K-matrix elements used probably underestimate the correction term since they are calculated using single-particle potential energies appropriate to HJ self-consistency.[5] Furthermore, had we used the BGT $^1P_1$ contribution the correction terms would be substantially larger and saturation would occur for a smaller value of β. Finally, we note that the corrected saturation curve is not calculated completely self-consistently since the single-particle potentials should also be corrected for the $^1P_1$ contributions. This correction is fairly small for small values of β but becomes important for values greater than $\beta = 0.5$ fm^{-2}.

More significant is the ability of the G.S. 1 potential (corrected for the $^1P_1$ contribution) to reproduce reasonably well the observed binding energy of O^{16}, which is 7.98 MeV/nucleon. The curve B shows a minimum at ~7.52 MeV/nucleon. Correcting for Coulomb energy and center-of-mass motion we get ~7.2 MeV/nucleon. Ring diagrams,[11] higher order insertions,[3,6] and single-particle excitations may increase this value by 1-2 MeV/nucleon.

Table I provides a summary of binding energies, single-particle energies, and rms radii of O^{16} for three hard-core potentials, the Bressel soft-core potential, and the G.S. 1

Table I. Binding energy per nucleon, single-particle energies, and radius of O^{16} for a number of nucleon potentials. Binding energy is corrected for Coulomb energy and center-of-mass motion. All units in MeV unless otherwise noted.

Nucleon potential	$R_{\rm rms}$ (fm)	$E(s_{1/2})$	$E(p_{3/2})$	$E(p_{1/2})$	E_B/A
SHC[a]	2.62	−39.2	−22.1	−22.1	3.4
BGT[b]	2.62	−39.7	−22.6	−15.9	4.0
HJ[b,c]	3.00	−28.7	−13.7	−12.1	2.3
Bressel[d]	2.62	−40.7	−20.2	−16.8	4.2
G.S. 1	2.23	−61.0	−33.2	−25.1	7.2

[a]Ref. 4.
[b]Ref. 5.
[c]Ref. 6.
[d]Preliminary results of calculations now in progress.

potential used in this paper.

The single-particle energies are quite large for the G.S. 1 potential. To compare with proton separation energies as measured in $(p,2p)$ and $(e,e'p)$ experiments which give[12] $E(1s)\sim 43$ MeV, $E(1p_{3/2})\sim 19$ MeV, and $E(1p_{1/2})\sim 12$ MeV, one would have to make "rearrangement" corrections. Allowing for a Coulomb energy of ~5 MeV for each single-particle state[9] we would thus have $V_{1s}{}^{R}\sim 13$ MeV, $V_{1p_{1/2}}{}^{R}\sim 9$ MeV, and $V_{1p_{1/2}}{}^{R}\sim 8$ MeV. These are in rough agreement with previous estimates[13,14] of "rearragement" corrections. We have calculated third-order "rearrangement" corrections for the G.S. 1 potential and get $V_{1s_{1/2}}{}^{R}=5.1$ MeV, $V_{1p_{3/2}}{}^{R}=3.4$ MeV, and $V_{1p_{1/2}}{}^{R}=2.9$ MeV. Thus there would have to be further rearrangement corrections of 5-8 MeV in order to get agreement between theory and experiment.

Calculations for infinite nuclear matter with the G.S. 1 potential used here are now in progress. Further calculations for both finite and infinite systems with more refined versions of the G.S. 1 potential which are able to reproduce $^1P_1$ scattering data are, of course, of interest.

We wish to thank Professor A. E. S. Green for communicating data on the G.S. 1 potential and Mr. H. G. Hughes, III, for assistance with the calculations.

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[1]One should then also explicitly calculate three-body diagrams.

[2]P. C. Bhargava and D. W. L. Sprung, Ann. Phys. (N.Y.) 42, 222 (1967), and private communications.

[3]C. W. Wong, to be published.

[4]H. S. Köhler and R. J. McCarthy, Nucl. Phys. 86, 611 (1966).

[5]R. J. McCarthy and H. S. Köhler, Nucl. Phys. A99, 65 (1967).

[6]H. S. Köhler and R. J. McCarthy, Nucl. Phys. A106, 313 (1968).

[7]A. E. S. Green, Phys. Rev. 75, 1926 (1949), and 76, 870 (1949); A. E. S. Green and R. D. Sharma, Phys. Rev. Letters 14, 380 (1965).

[8]A. E. S. Green and T. Sawada, Rev. Mod. Phys. 39, 606 (1967).

[9]H. S. Köhler, Phys. Rev. 138, B831 (1965).

[10]G. E. Brown and G. Jacob, Nucl. Phys. 42, 177 (1963).

[11]G. E. Brown and C. W. Wong, Nucl. Phys. A100, 241 (1967).

[12]G. Jacob and Th. A. J. Maris, Rev. Mod Phys. 38, 121 (1966).

[13]H. S. Köhler, Nucl. Phys. 88, 529 (1966).

[14]L. R. B. Elton, to be published.

ENERGY DEPENDENCE OF THE DEUTERON OPTICAL-MODEL POTENTIAL*

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It has been found[1-3] that a feature common to a number of optical-model analyses of experimental cross sections for elastic deuteron scattering is a marked increase of the radius of the imaginary part of the optical deuteron potential as the incident deuteron energy decreases. For deuteron-induced reactions there are two channels which are excited with relatively large probability—the stripping and the deuteron break-up channels. It is likely that the energy dependence of the deuteron optical-model potential is due to the large coupling which exists between the deuteron channel and the ones mentioned above. The break-up reaction is very difficult to treat in a realistic way because it leads to a three-body problem, but the stripping channels have recently been incorporated into a set of approximate coupled equations.[4] In the simple form of these equations as they are presently employed, the only stripping transition explicitly included is one with $\Delta l=1$. The calculations have so far only been carried out for the nucleus of calcium. Despite the approximate nature of these equations, the calculations give rise to roughly the correct behavior (as a function of angle and energy) of both the elastic and the $\Delta l=1$ stripping cross sections for the d-^{40}Ca interaction at deuteron energies between 7 and 22 MeV.[4]

It is the purpose of this Letter to show that for the case of ^{40}Ca a large portion of the energy dependence of the deuteron optical-model parameters is due to the presence of the