PARITY DOUBLING, ASYMMETRIC FERMION TRAJECTORIES, AND LOW-ENERGY πN DYNAMICS

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We suggest that fermion Regge trajectories will be strongly asymmetric functions of w (mass) for $w \lesssim 1$ GeV, in contrast to a recent suggestion of Barger and Cline. This suggestion fits the observed spectrum of πN resonances and would also be predicted by conventional low-energy πN dynamics.

On the basis of the MacDowell symmetry of meson-baryon scattering amplitudes,[1] one expects a fermion Regge trajectory to be analytic in w (mass), rather than in s (mass squared) as for bosons[2]; positive values of w correspond (conventionally) to even-parity states and negative values to odd-parity states. As a first approximation, one would then expect the trajectories to be linear in w:

$$\alpha(w) \simeq a + bw. \tag{1}$$

Since only positive half-integral values of α can correspond to physical particles, a trajectory would then describe either only odd- or only even-parity states.

However, if a Chew-Frautschi plot[2] is made of the known and conjectured πN resonances,[3] three sets of even-parity resonances are seen to lie on straight lines in the variable $s = w^2$, rather than in w. This then implies for the positive w values involved approximately parabolic trajectories:

$$\alpha(w) \simeq a + cw^2. \tag{2}$$

If this form is extrapolated to negative w, this in turn implies the existence of corresponding odd-parity resonances of the same mass, and recently Barger and Cline[4] have pointed out that some of these may appear in the most recent phase-shift analyses.[3]

This scheme has, however, two striking failures: (i) the nonexistence of a stable S_{11} particle as partner of the nucleon and (ii) the nonexistence of a low-energy D_{33} resonance as partner of the P_{33} (1236). Barger and Cline explain these failures by postulating that the corresponding Regge residues vanish at just the relevant energies. In this note I want to put forward a more natural explanation, and also to draw attention to the importance of the whole question.

First, it will be shown that $\alpha(w)$ cannot be an exactly even function of w (i.e., a function of s alone) if the usual threshold behavior is to be satisfied.[2] Indeed, just above $w = (M+\mu) \equiv w_0$ (the πN threshold), one has for the N trajectories

$$\mathrm{Im}\,\alpha(w) \sim q^{|2l(w_0)+1|} \sim (w-w_0)^{|\alpha(w_0)+1|}, \tag{3}$$

whereas just below $w = -w_0$ one has

$$\mathrm{Im}\,\alpha(w) \sim q^{|2l(-w_0)+1|} \sim (w+w_0)^{|\alpha(-w_0)|}. \tag{4}$$

(Here l denotes the orbital angular momentum corresponding to spin α, so $l = \alpha \pm \frac{1}{2}$ for $w \gtrless 0$; for the Δ trajectory, $l = \alpha \mp \frac{1}{2}$.) Hence, if we start off by assuming that $\alpha(w) = \alpha(-w)$, (3) and (4) lead to a contradiction.

Having realized this, one feels free to consider the possibility that quite large deviations from the parabolic form (2) might occur. Then the nonexistence of low-energy S_{11} and D_{33} objects leads one to postulate that the trajectories are considerably depressed for $w < 0$, so that the S_{11} and D_{33} objects have fairly high energies. It is in fact quite easy to find a suitable pair of resonances, and we have plotted modified trajectories for the $N_{\alpha,\beta}$ and $\Delta_{\gamma,\delta}$ cases in Figs. 1 and 2.

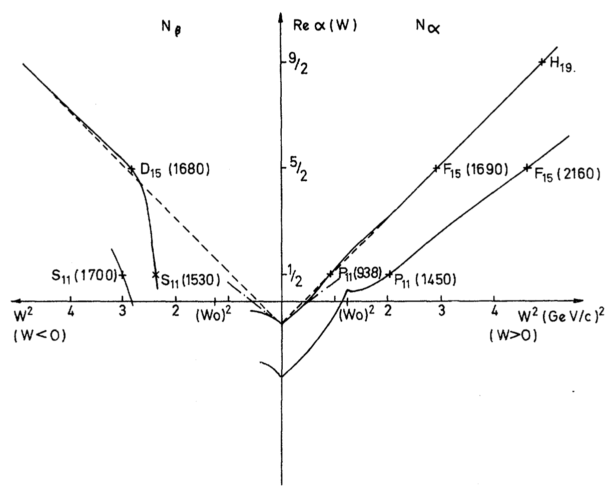

FIG. 1. Conjectured $N_{\alpha,\beta}$ trajectory. The dashed line is conjectured trajectory of Barger and Cline (Ref. 4). The dash-dotted line indicates the experimental slope and intercept at $w = 0$ (Ref. 5). The full lines are our conjectured leading and daughter trajectories.

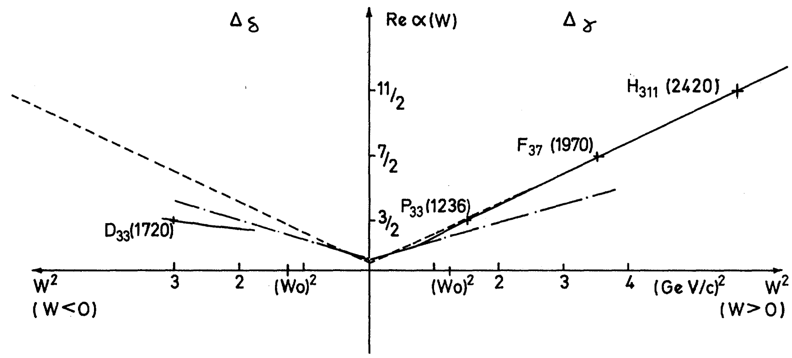

FIG. 2. Conjectured $\Delta_{\gamma,\,\delta}$ trajectory. The lines have the same significance as in Fig. 1.

These trajectories are partially supported by independent experimental evidence concerning high-energy backward πN scattering. By fitting the diffraction peak with Regge-pole terms, positions and slopes of the trajectories at $w=0$ have recently been estimated,[5] and we show them in Figs. 1 and 2. The slopes are smaller than those estimated from the Chew-Frautschi plot, which is just what we need. As no reliable determination of the coefficient of w [Eq. (1)] has yet been published, Figs. 1 and 2 have been drawn as if it were zero (i.e., no cusp at $w=0$). This coefficient could be quite large without affecting the usual interpretation[4] of the backward dip because of the small value of w at this dip.

In their work, Barger and Cline[4] have also assigned further resonances to "daughter" trajectories and this is still possible in the present scheme as indicated in Fig. 1. Of course, the present scheme differs in assigning the lowest S_{11} and D_{33} states to the leading trajectory rather than the first daughter.

In Figs. 1 and 2 the trajectories have not been plotted in the vicinity of the negative parity thresholds because their detailed form here is not clear. One has to satisfy the following requirements: (i) For small Imα, Im$\alpha(d\,\mathrm{Re}\alpha/dw)^{-1}>0$ on top side of the cuts[6]; (ii) if Reα remains finite at threshold, Imα satisfies the threshold behavior (4); (iii) the function α is analytic except for the cuts; and (iv) the trajectories Reα are depressed (relative to their parabolic large-w form) around the threshold, not raised. These requirements are quite difficult to satisfy simultaneously, and the question deserves further consideration.

Finally, let us examine the compatibility of the postulated trajectories with low-energy πN dynamics. Although single-channel N/D calculations are not quantitatively reliable, they do predict the P_{11} (nucleon) and P_{33} (1236) particles in the sense that a first sheet or low-energy second-sheet pole always appears.[7] (The reason for this is that the left-hand cuts coming from N and ρ exchange are so large for these partial waves that unitarity is violated unless there is a particle pole to cancel them; this conclusion is not affected by possible CDD pole terms.[7]) In contrast, the left-hand cuts for the remaining S, P, and D waves are small[8] and no low-energy resonances are expected in these waves unless CDD pole terms are important. For nonphysical J and l values no explicit calculations seem to have been performed, but the left-hand cuts coming from single-particle exchange are smooth functions of J. Hence we may summarize the position by saying that low-energy πN dynamics predict strongly asymmetric N and Δ trajectories in the region $\frac{1}{2}\lesssim J\lesssim\frac{3}{2}$, $0\lesssim w\lesssim 1$ GeV.

It is amusing to note that if one blindly extended the calculation to large J (e.g., by analytically continuing the N/D solution), one would presumably obtain symmetric trajectories because the asymmetry arises purely from the different orbital angular momenta ($l=J\pm\frac{1}{2}$) associated with the two cuts. However, one would never obtain indefinitely rising trajectories from such a procedure.

I would like to thank Dr. H. F. Jones (Imperial College, London) for pointing out a major error in an earlier version of this work and for several stimulating communications.

[1]S. MacDowell, Phys. Rev. 116, 774 (1960).
[2]For general Regge theory we refer the reader to the following books: E. J. Squires, Complex Angular Momenta and Particle Physics (W. A. Benjamin, Inc.,

New York, 1963); S. C. Frautschi, Regge Poles and S-Matrix Theory (W. A. Benjamin, Inc., New York, 1963).

[3]C. Lovelace, in Proceedings of the Heidelberg International Conference on High Energy Physics, 1967 (to be published).

[4]V. Barger and D. Cline, Phys. Rev. Letters 20, 298 (1968); C. B. Chiu and J. D. Stack, Phys. Rev. 153, 1575 (1967).

[5]V. Barger and D. Cline, Phys. Rev. Letters 19, 1504 (1967).

[6]Compare the contributions with a partial-wave amplitude coming first from a J-plane Regge pole and then from an ordinary s-plane pole.

[7]D. H. Lyth, "A simple graphical method for predicting resonances" (to be published); J. Allcock, H. Burkhardt, D. H. Lyth, and G. McCauley, "Semi-phenomenological calculations with partial-wave dispersion relations" (to be published). There are numerous other calculations giving similar results, and those that fail to do so seem either to (a) miss the ρ-exchange contribution (for P_{11}) or (b) use a kinematic factor which introduces spurious poles at $w=0$ (especially for the P_{33}).

[8]E.g., A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. 135, B515 (1964).

PHOTOPRODUCTION OF CHARGED PIONS IN THE FORWARD DIRECTION AND REGGE POLES*

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We propose here a test, using linearly polarized photons, of the hypothesis that the forward peak in charged-pion photoproduction is due to a conspiracy between the pion and an opposite-parity conspirator. The energy dependence of the differential cross section for photons linearly polarized in the plane of scattering can distinguish between a nucleon pole leading to the forward peak or a pion conspirator.

Recent experiments at Stanford Linear Accelerator Center and Deutsches Elektronen-Synchrotron indicate that the charged-pion differential cross section has a sharp peak in the forward direction.[1,2] This differential cross section has the following properties: If we plot $s^2(d\sigma/dt)$ vs t we find that it is roughly independent of energy and it can be fitted by the electric Born terms up to about $t=-0.6$ BeV.[3] For $t<-m_\pi^2$ the Born terms are much too large and the data can be fitted by a form-factor model,[1]

$$d\sigma/dt=e^{3t}(d\sigma/dt)_{\rm Born}. \tag{1}$$

Models using absorption in the manner of Gottfried and Jackson are unable to fit the data.[1] The zero-degree cross section, however, is insensitive to how absorption is done[3] and so perhaps one could cook up some sort of absorption model to get the proper t dependence. In this paper we would like to discuss how one can explain the above data by using a pion-conspiracy version of the Regge-pole model, and how the data should change in terms of the energy dependence of $s^2(d\sigma/dt)$ if the conspiracy model is operating and we use linearly polarized photons. This change in energy dependence, as we shall see, will not occur if the forward peak is due to a nucleon pole or to cuts in the angular-momentum plane.

In the Regge-pole model we can understand this peak and its energy dependence only if we introduce a trajectory which is assumed to be degenerate with the pion trajectory at $t=0$. This trajectory is suggested by the symmetry group O(4)[4] and is necessary in order for the cross section to have a peak in the forward direction in the Regge model. In the usual Regge model we find we can exchange the ρ, A_2, π, B, and A_1 mesons. The trajectories of these mesons can be parametrized as follows[5]:

$$\alpha_\rho(t)\approx 0.57+t,\quad \alpha_{A_2}\approx 0.35+t,$$
$$\alpha_B(t)\approx -0.32+t,\quad \alpha_\pi\approx t-m_\pi^2,$$
$$\alpha_{A_1}(0)\approx 0. \tag{2}$$

The energy dependence of the π^+ photoproduction data suggests that at least near the forward direction the pion is the dominating trajectory. The unimportance of ρ and B in this region can be inferred from neutral-pion photoproduction data. By SU(3) arguments the ρ contribution to π^+ photoproduction should be $\frac{1}{9}$ that of the ω to π^0 photoproduction; the B contribution is $\sqrt{2}$ times that found in π^0 photoproduction. By examining the data[3] we see that the contribution of these trajectories compared with the experimental cross section for $t>-0.6$ is small. We neglect the A_1 and assume in this t region