Regge behavior of scattering of unequal-mass particles, Freedman and Wang[14] introduced the notion of daughter trajectories. The first daughter trajectory of the Pomeranchuk has $Y=T=0$, $G=+1$, and odd signature. So the lowest physical particle on this trajectory (which has $J^P=1^-$) has the same quantum numbers as the h meson (see, e.g., Low[15]).

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[1]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967); T. D. Lee and B. Zumino, "Field Current Identities and The Algebra of Fields," Phys. Rev. (to be published).

[2]T. D. Lee, in Proceedings of the Second Hawaii Topical Conference on Particle Physics, 1967 (University of Hawaii Press, Honolulu, Hawaii, 1967).

[3]See, for instance, M. Gell-Mann and Y. Ne'eman, The Eightfold Way (W. A. Benjamin, Inc., New York, 1964), p. 98.

[4]S. F. Tuan and T. T. Wu, Phys. Rev. Letters 18, 349 (1967).

[5]M. Gell-Mann, Phys. Rev. Letters 14, 334 (1965); R. H. Dalitz, in Proceedings of the Oxford International Conference on Elementary Particles, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), p. 157.

[6]See, for instance, Eqs. (33)-(35) of T. D. Lee, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Stanford, California, 1967 (to be published).

[7]M. Crozon (private communication) informs us that the production cross section of δ^+ from $pp\to d\delta^+$ [M. Banner et al., Phys. Letters 25B, 569 (1967)] is consistent with an upper limit of order 1 μb $(\mathrm{GeV}/c)^{-2}$. This lends some support to the hypothesis that the δ^+ (if it exists) is abnormal, since nucleon exchange is forbidden here.

[8]See, for instance, R. H. Dalitz, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967), p. 230.

[9]S. Pakvasa, S. F. Tuan, and T. T. Wu, "Some Considerations on the Intermediate Boson Hypothesis and CP Violation" (to be published).

[10]S. Adler, Phys. Rev. Letters 18, 519, 1036(E) (1967).

[11]V. L. Fitch, in Proceedings of the Second Hawaii Topical Conference on Particle Physics, 1967 (University of Hawaii Press, Honolulu, Hawaii, 1967).

[12]A model of this type has been suggested by R. G. Sachs [Phys. Rev. Letters 13, 286 (1964)] which in addition predicts that $|\eta_{00}|=|\eta_{+-}|$.

[13]G. Goldhaber, in Proceedings of the Second Hawaii Topical Conference on Particle Physics, 1967 (University of Hawaii Press, Honolulu, Hawaii, 1967).

[14]D. Z. Freedman and J.-M. Wang, Phys. Rev. Letters 17, 569 (1966).

[15]F. E. Low, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967).

K_1^0-K_2^0 MASS DIFFERENCE AND THE INTERMEDIATE VECTOR BOSON*

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The K_1^0-K_2^0 mass difference is calculated to lowest order in the intermediate-boson model, using Weinberg's first sum rule. A finite result is obtained without requiring equal coupling of strange and nonstrange mesons to the corresponding currents.

The K_1^0-K_2^0 mass difference has recently been calculated by Biswas and Smith[1] from the current-current form of the weak Hamiltonian, convergence being obtained by the use of the Weinberg sum rules[2] for the spectral functions of the propagators of vector and axial-vector currents of chiral SU(3)⊗SU(3). Doubts have been expressed[3] concerning the validity of Weinberg's second sum rule applied to SU(3). In this note we point out that a convergent result may be obtained without this second sum rule, if we assume the existence of an intermediate vector boson of finite mass.[4] An estimate can be made of this mass from the observed K_1^0-K_2^0 mass difference, but depends on some poorly determined parameters.

We follow the method[5] used by Biswas and Smith.[1] The mass difference is[6]

$$\Delta M=\Delta E(K_1^0)-\Delta E(K_2^0)=\mathrm{Re}(2\pi)^3 i\int d^4z\langle K^0|T\{H_w(z)H_w(0)\}|\bar{K}^0\rangle. \quad (1)$$

The second-order weak coupling will be taken as

$$T\{H_w(z)H_w(0)\}=2G^2M_B^{\,4}\int d^4x d^4y\,\Delta_{\alpha\beta}^{\ \ B}(y-z)\Delta_{\gamma\delta}^{\ \ B}(x)T\{J^\alpha(y)J^{+\beta}(z)J^\gamma(x)J^{+\delta}(0)\}, \quad (2)$$

where[7]

$$\Delta_{\alpha\beta}^{B}(y) = -(2\pi)^{-4}\int d^4p\, e^{ipy}(g_{\alpha\beta}-p_\alpha p_\beta/M_B^2)(p^2-M_B^2)^{-1} \quad (3)$$

and

$$J_\mu = (V_\mu^{(\pi+)} + A_\mu^{(\pi+)})\cos\theta + (V_\mu^{(K+)} + A_\mu^{(K+)})\sin\theta, \quad (4)$$

$$V_\mu^{(\pi+)} = -2^{-\frac{1}{2}}(\mathfrak{F}_{1\mu} + i\mathfrak{F}_{2\mu}), \text{ etc.}$$

As in Ref. 1, the kaons are now reduced out using the hypothesis of partial conservation of axial-vector current, viz. $\partial_\mu A^\mu(K) = \frac{1}{2}F_K\varphi(K)m_K^2$, and taking the soft-kaon limit. This gives

$$\Delta(M^2) = -\mathrm{Re}[8iF_K^{-2}(GM_B^2\sin\theta\cos\theta)^2\int d^4x d^4y d^4z\, \Delta_{\alpha\beta}^{B}(y-z)\Delta_{\gamma\delta}^{B}(x)M^{\alpha\beta\gamma\delta}(x,y,z)], \quad (5)$$

where $M_{\alpha\beta\gamma\delta}(x,y,z)$ is a sum of terms like

$$\langle 0|T\{V_\alpha^{(\pi+)}(y)V_\beta^{(\pi-)}(z)V_\gamma^{(\pi+)}(x)V_\delta^{(\pi-)}(0)\}|0\rangle. \quad (6)$$

These are approximated to lowest order by the contribution of a vacuum intermediate state, in this case

$$\langle 0|T\{V_\alpha^{(\pi+)}(y)V_\delta^{(\pi-)}(0)\}|0\rangle\langle 0|T\{V_\beta^{(\pi-)}(z)V_\gamma^{(\pi+)}(x)\}|0\rangle \equiv \Delta_{\alpha\delta}^{V}(y)\Delta_{\beta\gamma}^{V}(z-x). \quad (7)$$

We can throw away terms like $\Delta_{\alpha\beta}^{V}(y-z)\Delta_{\gamma\delta}^{V}(x)$ [which gives a divergent contribution], because it is not a continuation of any term than can occur when the kaons have nonzero mass.[8]

The current propagators $\Delta_{\alpha\beta}^{V}$, $\Delta_{\alpha\beta}^{A}$, $\Delta_{\alpha\beta}^{KV}$, and $\Delta_{\alpha\beta}^{KA}$ have spectral representations[9] such as

$$\Delta_{\alpha\beta}^{A}(x) = -i(2\pi)^{-4}\int d^4p\, e^{ipx}\int d\mu^2\{\rho_A^{(1)}(\mu^2)[(g_{\alpha\beta}-p_\alpha p_\beta/\mu^2)(p^2-\mu^2)^{-1}+\delta_{\alpha4}\delta_{\beta4}\mu^{-2}]$$
$$+\rho_A^{(0)}(\mu^2)[p_\alpha p_\beta(p^2-\mu^2)^{-1}-\delta_{\alpha4}\delta_{\beta4}]\}. \quad (8)$$

On substituting (7) and (8) in (5), divergent terms can be removed by use of Weinberg sum rules of the form

$$\int d\mu^2\{[\rho_A^{(1)}(\mu^2)-\rho_{KA}^{(1)}(\mu^2)]\mu^{-2}+[\rho_A^{(0)}(\mu^2)-\rho_{KA}^{(0)}(\mu^2)]\}=0 \quad (9)$$

and similarly for the vector terms; $\rho^{(0)}(\mu^2)$ are assumed to be saturated by massless particles, e.g., $\rho_A^{(0)}(\mu^2) \simeq \frac{1}{4}F_\pi^2\delta(\mu^2)$.

Hence

$$\Delta(M^2) = \mathrm{Re}\Big[2iF_K^{-2}(GM_B^2\sin\theta\cos\theta)^2 12(2\pi)^{-4}\int\frac{d^4p\,d\mu_1^2 d\mu_2^2\,\sigma(\mu_1^2)\sigma(\mu_2^2)}{(p^2-M_B^2)^2(p^2-\mu_1^2)(p^2-\mu_2^2)}\Big], \quad (10)$$

where

$$\sigma(\mu^2) = \rho_V^{(1)}(\mu^2)+\rho_A^{(1)}(\mu^2)-\rho_{KV}^{(1)}(\mu^2)-\rho_{KA}^{(1)}(\mu^2). \quad (11)$$

These spectral functions are approximated by assuming single-particle dominance in the usual way, e.g., $\rho_V^{(1)}(\mu^2) \simeq g_\rho^2\delta(\mu^2-m_\rho^2)$. Particles used are $\rho(760)$, $A_1(1080)$, $K^*(890)$, and $K_A^*(1320)$.

The integral in (10) converges without the requirements that $g_\rho^2 = g_{KV}^2$ and $g_A^2 = g_{KA}^2$. However, since these quantities are not well determined by experimental data, we shall use the results of vector-meson dominance applied to the second Weinberg sum rule, for $SU(2)\otimes SU(2)$,[10] i.e.,

$$g_A^2 = g_\rho^2;\quad g_{KA}^2 = g_{KV}^2 \quad (\equiv g_K^2). \quad (12)$$

We also take[11] $g_\rho^2 \simeq \frac{1}{2}F_\pi^2 m_\rho^2$; this leaves $(g_K/g_\rho)^2$ to be estimated. From sum rules of type (9),

$$g_K^2/g_\rho^2 = (m_{KA}^2/m_\rho^2)\{1-(F_K^2/F_\pi^2)(m_{A_1}^2 - m_\rho^2)/m_{A_1}^2\} \tag{13}$$

and

$$(g_K^2/m_{KV}^2) - (g_\rho^2/m_\rho^2) = -F_{ks}^2, \tag{14}$$

where we have used $\rho_{KV}^{(0)}(\mu^2) \simeq \frac{1}{4}F_{ks}^2\delta(\mu^2)$ (corresponding to the so-called kappa meson).

We consider the following possibilities:

(a) We put the experimental result $F_K/F_\pi \simeq 1.28$ in (13). This gives $g_K^2 = 0.575 g_\rho^2$.

(b) F_{ks} in (14) is assumed negligible; so $g_K^2 = (m_{KV}/m_\rho)^2 g_\rho^2$. This agrees, in the vector-meson–dominance model,[3] with the observed ratio $\Gamma(K^* \to K\pi)/\Gamma(\rho \to \pi\pi)$.

(c) Assume $g_K^2 = g_\rho^2$, as in Ref. 1.

The result in each case is compared with the experimental result $\Delta M\tau(K_1^0) = -0.48$ to attempt to estimate M_B. We take $F_\pi = 187$ MeV, $\sin\theta = 0.21$, $\tau(K_1^0) = 0.87 \times 10^{-10}$ sec. Then (a) gives $M_B \sim 7$ BeV, (b) gives $M_B \sim 30$ BeV, and (c) does not come within an order of magnitude of the experimental result for any M_B.

The above values of M_B are obviously very sensitive to the value of $(g_K/g_\rho)^2$ taken. We would also expect correction terms, ignored in approximating expressions like (6), to have an appreciable effect (possibly by an order of magnitude) on the estimate of M_B.

In case (c), we obtain $\Delta M\tau(K_1^0) \simeq -0.02$ in the limit $M_B \to \infty$. However, there is some further ambiguity in this case, since the full set of Weinberg sum rules for asymptotic SU(3)⊗SU(3), giving $g_\rho^2 = g_{A_1}^2 = g_{KA}^2 = g_{KV}^2$, is inconsistent with the experimental particle masses; consequently different subsets of these sum rules will give us different estimates of $g_\rho^2 = g_K^2$. One such alternative estimate is[12]

$$g_K^2 = F_K^2 m_{KV}^2\{1-(m_{KV}^2/m_{KA}^2)\}^{-1} \simeq \tfrac{1}{2}F_K^2 m_{KV}^2, \tag{15}$$

which was used in Ref. 1. Taking the value of $F_K^2 \sin^2\theta$ from the decay rate $\Gamma(K \to \mu\nu)$, we obtain $\Delta M\tau(K_1^0) \simeq -0.08$ in the limit $M_B \to \infty$. (The somewhat different value obtained in Ref. 1 is due to the use of larger numerical values for F_K and θ.)

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[1]S. N. Biswas and J. Smith, Phys. Rev. Letters 19, 727 (1967), who give further references.

[2]S. Weinberg, Phys. Rev. Letters 18, 507 (1967); S. L. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967).

[3]J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967).

[4]Intermediate bosons are used in similar circumstances by S. L. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 205 (1967).

[5]This type of procedure was previously used to calculate the pion mass difference by T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).

[6]V. Barger and E. Kazes, Nuovo Cimento 28, 394 (1963).

[7]The results are not affected if we use the alternative "mixed-spin" intermediate boson with only $g_{\alpha\beta}$ in the numerator of (3), as used, for example, by Z. Bialynicka-Birula, Nuovo Cimento 21, 571 (1961).

[8]This and the preceding statement are evident on considering the relevant vacuum-polarization diagrams.

[9]Given, for example, by Das et al., Ref. 5.

[10]This is on a firmer basis than the corresponding sum rule for SU(3) generally; see Ref. 3.

[11]This value agrees with the experimental value of the ρ width in the vector-meson–dominance model; see, for example, Riazuddin and Fayyazuddin, Phys. Rev. 147, 1072 (1966). It is also a necessary condition, from Weinberg's first sum rule, if we are to saturate $\rho_A^{(1)}$ with the A_1(1080).

[12]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967). Use of this result in the present case was pointed out by the authors of Ref. 1 (private communication).