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[14]For discussions on this subject, see the references cited in footnote 1, and J. Schwinger, 1967 Brandeis Summer Institute Lectures (unpublished); S. Weinberg (to be published); W. A. Bardeen (to be published).

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S-WAVE π-N SCATTERING LENGTHS, CURRENT ALGEBRA, AND $\rho+\rho'$ DOMINANCE

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Various predictions of the $\rho\pi\pi$ coupling constant $f_\rho^2/4\pi$ are discussed. A $\rho+\rho'$ meson dominance model is considered with $M_{\rho'}\simeq 1600$ MeV and $\Gamma_{\rho'}\simeq 200$ MeV. This model predicts a value of the S-wave π-N scattering length formula a_1-a_3 in better agreement with experiment than the ρ-dominance model when $\Gamma_\rho=128$ MeV, and the modified Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation is well satisfied.

A number of people[1] have used the SU(2)⊗SU(2) commutation relations (CCR)[2] and the partially conserved axial-vector current (PCAC) hypothesis[3] to derive an interesting expression for the scattering length a_T of pion scattering on any isospin-carrying target:

$$a_T=(\mu_{\pi t}/4\pi F_\pi^2)(\vec{T}_\pi\cdot\vec{T}_t), \tag{1}$$

where $\vec{T}_\pi$ and $\vec{T}_t$ are the isospins of the pion and the target particle, and F_π is defined by $\langle 0|A_\mu^1-iA_\mu^2|\pi^+\rangle=-i\sqrt{2}q_\mu F_\pi$. The reduced mass is denoted by $\mu_{\pi t}=m_\pi m_t/(m_\pi+m_t)$. If we adopt the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF)[4] relation $F_\pi^2=F_\rho^2/2m_\rho^2$, and the ρ-dominance and universality relation $F_\rho^2=m_\rho^4/f_\rho^2$, then in place of Eq. (1) we obtain

$$a_T=-2(f_\rho^2/4\pi)(\mu_{\pi t}/m_\rho^2)(\vec{T}_\pi\cdot\vec{T}_t). \tag{2}$$

This relation was derived directly from ρ dominance and universality by Sakurai.[5] It can be derived simply by considering the "effective" Yukawa potential for single ρ exchange between the pion and the t particle. In the Born approximation for the scattering amplitude, the result (2) follows in the limit that the momentum tends to zero. In this derivation the rescattering corrections are neglected, but these corrections are expected to be small.

By comparing (1) and (2) and using the Goldberger-Treiman (GT) relation, it follows that[5]

$$f_\rho^2/4\pi=(g_{\pi N}^2/4\pi)(m_\rho^2/2g_A^2m_N^2), \quad (3)$$

which is another form of the KSRF relation.[4] The latest value for F_π based on the π^+ decay mode is $F_\pi=0.10M_p=0.67m_\pi$, while the GT relation predicts $F_\pi=(1.18M_p)/g_{\pi N}=0.087M_p$. Whenever F_π^2 is used in a calculation there will occur a 24% difference between these two values of F_π^2.

There are several ways of determining the constant $f_\rho^2/4\pi$.[6] In terms of the decay width of the ρ meson into two pions, we get

$$\Gamma(\rho\to\pi+\pi)=\tfrac{2}{3}(f_\rho^2/4\pi)|\vec{p}_{\pi\pi}|^3/m_\rho^2. \quad (4)$$

The currently quoted average values for the mass and width of the ρ meson are $M_\rho=774$ MeV and $\Gamma_\rho=128$ MeV.[7] These values yield $f_\rho^2/4\pi=2.4$. In the colliding-beam experiment for the process $e^++e^-\to\pi^++\pi^-$, performed at Novosibirsk,[8] the quoted ρ width is as low as $\Gamma_\rho=93\pm15$ MeV. From Eq. (4) this gives the value $f_\rho^2/4\pi=1.8\pm0.26$. If we use $m_\rho^2=31m_\pi^2$, $g_{\pi N}^2/4\pi=14.6$, and $g_A=1.18$ in (3), then $f_\rho^2/4\pi=3.6$; and from (4) it follows that $\Gamma_\rho=190$ MeV. This latter value of Γ_ρ is several standard deviations from $\Gamma_\rho=128$ MeV. The latest photoproduction experiment on carbon nuclei at Deutsches Elektronen-Synchrotron[9] gives $f_\rho^2/4\pi=1.96\pm0.48$, and by virtue of (4) this corresponds to $\Gamma_\rho=103\pm25$ MeV. This is consistent with both the "average" value $\Gamma_\rho=128$ MeV and the colliding-beam experiment value $\Gamma_\rho=93\pm15$ MeV. Thus $f_\rho^2/4\pi$ ranges all the way from 1.8 to 3.6, and such a large fluctuation makes it difficult to establish any particular theoretical prediction at this time, e.g., the ρ-dominance hypothesis[5] or the calculations of the decay width $\Gamma(A_1\to\rho+\pi)$.[10]

Let us consider the scattering-length formula in S-wave π-N scattering,

$$a_1-a_3=3(f_\rho^2/4\pi)(\mu_{\pi N}/m_\rho^2), \quad (5)$$

where $\mu_{\pi N}=m_\pi m_N/(m_\pi+m_N)$. The experimental value is[11]

$$(a_1-a_3)_{\text{expt}}=(0.271\pm0.006)m_\pi^{-1}. \quad (6)$$

In a careful analysis of π-N scattering based on dispersion relations,[12] it has been shown that the long-range Born effects and crossed N^*-exchange effects are quite small in the S-wave scattering. In particular, it was found that the short-range effects including the Born terms almost cancel in the difference a_1-a_3, whereas these effects are important in a_1+2a_3 and the individual scattering lengths a_1 and a_3. Moreover, the $T=0$ π-π effects cancel in a_1-a_3. This means that, apart from small rescattering corrections, the ρ channel should dominate the evaluation of a_1-a_3.[13,14]

If we include the reduced-mass correction $\mu_{\pi N}$ in formula (5) (which seems natural, since it occurs in the statement of a_T) and use the value $f_\rho^2/4\pi=2.4$ corresponding to $\Gamma_\rho=128$ MeV, we obtain $a_1-a_3=0.20m_\pi^{-1}$. This differs from the experimental value by 26%.[15] In order to get agreement with $(a_1-a_3)_{\text{expt}}$, we would require $\Gamma_\rho\simeq170$ MeV corresponding to $f_\rho^2/4\pi=3.2$. If we use the formula (1) obtained from CCR and PCAC, then $a_1-a_3=0.23m_\pi^{-1}$; and assuming the KSRF relation and ρ dominance, we also find $f_\rho^2/4\pi=2.7$ corresponding to $\Gamma_\rho=143$ MeV, i.e., a ρ width somewhat larger than the average value $\Gamma_\rho=128$ MeV, although the agreement with $(a_1-a_3)_{\text{expt}}$ is better. Replacing F_π in (1) by the GT formula predicts $a_1-a_3=0.3m_\pi^{-1}$, but this corresponds to $\Gamma_\rho=190$ MeV if we assume the KSRF relation. Returning to the ρ-dominance formula (5), we find that, in terms of the colliding-beam experiment value $\Gamma_\rho=93\pm15$ MeV, the predicted scattering length difference is $a_1-a_3=(0.15\pm0.02)m_\pi^{-1}$, which disagrees with the experimental value by ~50%.

If we adopt the attitude that the "world average" value $\Gamma_\rho=128$ MeV is the correct ρ width, then we are faced with the question as to why the ρ-dominance prediction of a_1-a_3, based on Eq. (5), disagrees by ~26% from $(a_1-a_3)_{\text{expt}}$. The rescattering corrections are expected to produce ~6-10% of this discrepancy. The origin of this discrepancy may be related to the possible existence of a ρ' meson.

In a recent Regge-pole analysis of high-energy π-N scattering,[16] it was found that the inclusion of a ρ'-meson trajectory was necessary to fit the data. The exchange of a single ρ meson predicts zero polarization in disagreement with the current polarization data. A solution was found with $\alpha_{\rho'}(0)<0$ and a ρ'-trajectory slope consistent with the existence of a ρ' meson at ~1.6-1.7 BeV with $J^P=1^-$, and also a ρ-trajectory recurrence at about the same energy with $J^P=3^-$.[17]

Let us consider a model in which both the ρ and ρ' dominate the S-wave π-N scattering for the difference of the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ amplitudes. We shall assume in analogy with the ρ-dominance model that the ρ' meson is coupled universally to the isospin, i.e., $f_{\rho'NN} \simeq f_{\rho'\pi\pi}=f_{\rho'}$. In place of (4) and (5) we have

$$\Gamma(\rho'\to\pi+\pi)=\tfrac{2}{3}(f_{\rho'}^{\,2}/4\pi)|\vec{p}_{\pi\pi}|^3/m_{\rho'}^{\,2}, \tag{7}$$

and

$$a_1-a_3=3\mu_{\pi N}\left(\frac{f_\rho^{\,2}}{m_\rho^{\,2}}+\frac{f_{\rho'}^{\,2}}{m_{\rho'}^{\,2}}\right). \tag{8}$$

We shall adopt a ρ' width $\Gamma_{\rho'}\simeq 200$ MeV and also a mass $m_{\rho'}\simeq 1600$ MeV[17] consistent with the Regge-pole analysis.[16] Then from (7) it follows that $f_{\rho'}^{\,2}/4\pi=1.7$, and from (8) using $m_\rho=774$ MeV and $\Gamma_\rho=128$ MeV it is predicted that $a_1-a_3=0.24m_\pi^{-1}$ in better agreement with $(a_1-a_3)_{\rm expt}$.

The KSRF relation in the case of ρ dominance and universality (together with PCAC) can be expressed in terms of the parameter $\xi=1-2F_\pi^{\,2}f_\rho^{\,2}/m_\rho^{\,2}$, which is zero when the relation is exactly satisfied.[10] No satisfactory derivation of this relation exists as yet.[18] However, it can be obtained by an Ansatz of the type used by Sakurai[19] based on CCR and ρ exchange in π-N scattering. It is well satisfied for $f_\rho^{\,2}/4\pi=2.7$, $F_\pi=0.67m_\pi$, and $\Gamma_\rho=143$ MeV. The derivation of the KSRF relation can be extended to the $\rho+\rho'$ model. Assuming ρ and ρ' exchange in π-N scattering and CCR, we have

$$\frac{f_{\rho NN}f_{\rho\pi\pi}}{m_\rho^{\,2}}+\frac{f_{\rho'NN}f_{\rho'\pi\pi}}{m_{\rho'}^{\,2}}=\frac{V_1(0)}{2F_\pi^{\,2}}, \tag{9}$$

where $V_1(t)$ is the nucleon form factor and $V_1(0)=1$. By assuming universality $f_\rho^{\,2}\approx f_{\rho NN}f_{\rho\pi\pi}$ and $f_{\rho'}^{\,2}\approx f_{\rho'NN}f_{\rho'\pi\pi}$, we obtain a modified KSFR relation

$$\xi'=1-2F_\pi^{\,2}\left(\frac{f_\rho^{\,2}}{m_\rho^{\,2}}+\frac{f_{\rho'}^{\,2}}{m_{\rho'}^{\,2}}\right). \tag{10}$$

We find $\xi'=-0.02$ when $f_\rho^{\,2}/4\pi=2.4$, $m_\rho=774$ MeV, $f_{\rho'}^{\,2}/4\pi=1.7$, and $m_{\rho'}=1600$ MeV.

Our results are summarized in Table I. The various ways of calculating $f_\rho^{\,2}/4\pi$ described here clearly depend upon knowing Γ_ρ more accurately. If the colliding-beam experiment value for Γ_ρ is correct, then it is difficult to see how the ρ-dominance hypothesis can be saved unless there is a ρ' meson with $J^P=1^-$ at a much lower energy ($\lesssim$1.2 BeV) than considered here. On the other hand, if the currently quoted average value $\Gamma_\rho=128$ MeV is valid, then the $(\rho+\rho')$-dominance model provides a good value for a_1-a_3 and $f_\rho^{\,2}/4\pi$. In addition the modified KSRF formula (10) is well satisfied.

Table I. Comparison of various predictions for $f_\rho^{\,2}/4\pi$, Γ_ρ, $(a_1-a_3)m_\pi$, and ξ.

$f_\rho^{\,2}/4\pi$	Γ_ρ (MeV)	$(a_1-a_3)m_\pi$	ξ	Theoretical predictions tested
2.4	128	0.20	0.12	Fitting $f_\rho^{\,2}/4\pi$ to Rosenfeld et al. Γ_ρ, ρ-dominance, and universality.
1.96 ± 0.48	103 ± 25	0.17 ± 0.04	0.18	$f_\rho^{\,2}/4\pi$ from DESY γ-production experiment on C nuclei, ρ dominance.
1.8 ± 0.29	93 ± 15	0.15 ± 0.02	0.34	Fitting $f_\rho^{\,2}/4\pi$ to colliding-beam experiment Γ_ρ, ρ dominance.
3.6	190	0.30	0	Validity of GT relation using CCR, PCAC, KSRF, and universality.
2.7	143	0.23	0	Eq. (1) for a_1-a_3 from CCR and PCAC. Validity of KSRF.
2.4	128	0.24	···	$\rho+\rho'$ dominance and universality using $M_{\rho'}\simeq 1.6$ BeV and $\Gamma_{\rho'}\simeq 200$ MeV. Modified KSRF gives $\xi'=\xi-2F_\pi^{\,2}f_{\rho'}^{\,2}/m_{\rho'}^{\,2}=-0.02$.

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